Testing the limits of quasi-geostrophic theory

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1. Motivation

- Quasi-geostrophic theory (Charney, Fjørtoft & von Neumann 1950) is an approximation to the rotating shallow-water equations; it formally applies only to shallow flows with small Rossby numbers and it does not capture interactions with ageostrophic motions

- To what extent is quasi-geostrophic theory able to capture the full fluid dynamics, especially outside these limits?

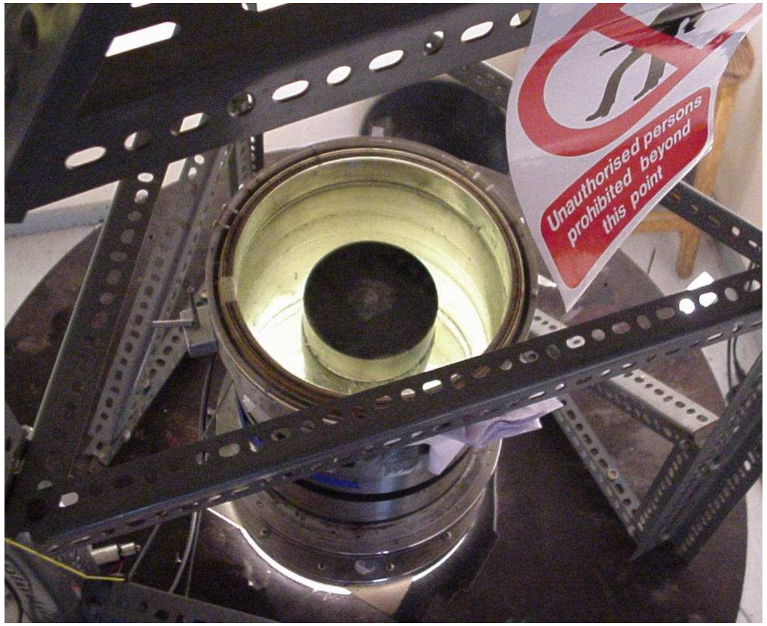
- Quasi-geostrophic theory is found to perform quite well far beyond its expected range of validity compared with a shallow-water equations control run (Mundt, Vallis & Wang 1997)

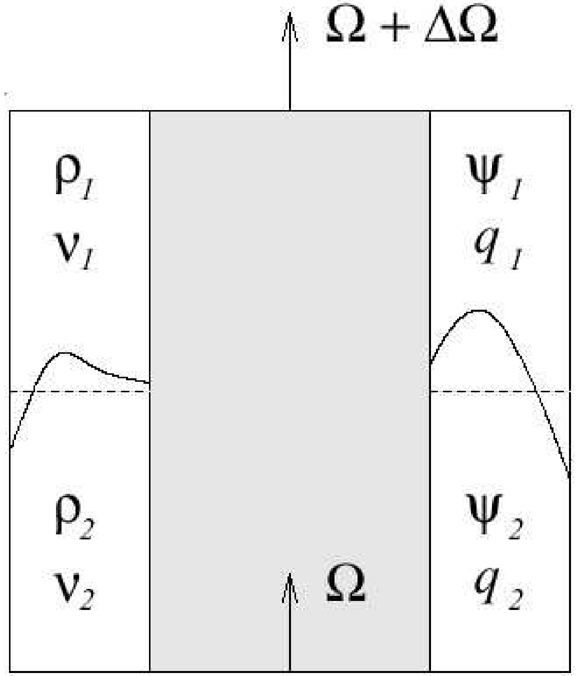
- Also, hydrostatic-primitive-equation and quasi-geostrophic simulations of the equilibration of baroclinic turbulence agree reasonably well over broad parameter ranges (e.g. f_0, β, N; Zurita-Gotor & Vallis 2009)

- But what about deep flows rather than shallow, hydrostatic flows? And what about the comparison with real flows rather than simulated flows?

- Plan for this poster: use the laboratory rotating annulus to find out…

2. The rotating two-layer annulus





Laboratory apparatus

- Rossby number $\Delta\Omega/2\Omega \sim 0.1 - 1$
- aspect ratios $D/L = 4$ and $f/N \sim 10$
- presence of inertia-gravity waves

Quasi-geostrophic model

- assumes Rossby number << 1
- assumes aspect ratios << 1
- absence of inertia-gravity waves

⇨ how well does the model capture equilibrated regular baroclinic waves observed in the laboratory?

3. Quasi-geostrophic model equations

advection | parameterized Ekman layer damping | generation of PV by rotating lid

$$\frac{\partial q_1}{\partial t} = \frac{1}{r}\frac{\partial \psi_1}{\partial \theta}\frac{\partial q_1}{\partial r} - \frac{1}{r}\frac{\partial \psi_1}{\partial r}\frac{\partial q_1}{\partial \theta} - \frac{\sqrt{\Omega \nu_1}}{H}\left[\nabla^2 \psi_1 + \chi_2 \nabla^2(\psi_1 - \psi_2)\right] + \frac{2\Delta\Omega\sqrt{\Omega\nu_1}}{H}$$

$$\frac{\partial q_2}{\partial t} = \frac{1}{r}\frac{\partial \psi_2}{\partial \theta}\frac{\partial q_2}{\partial r} - \frac{1}{r}\frac{\partial \psi_2}{\partial r}\frac{\partial q_2}{\partial \theta} - \frac{\sqrt{\Omega \nu_2}}{H}\left[\nabla^2 \psi_2 + \chi_1 \nabla^2(\psi_2 - \psi_1)\right]$$

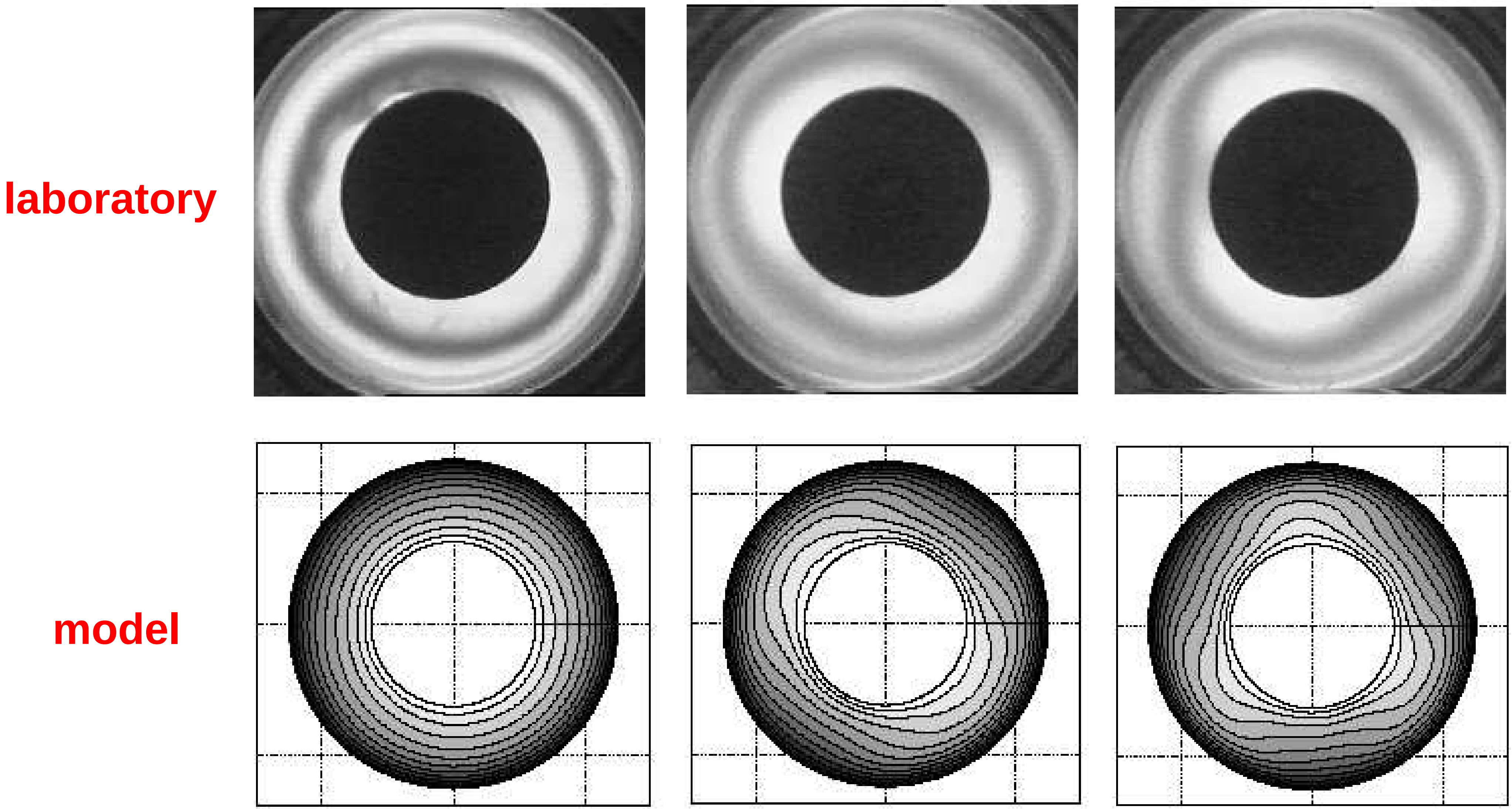
relative vorticity | vortex stretching/ compression | centripetal β-effect

where $q_1 = \nabla^2 \psi_1 + \frac{f^2}{g' H}(\psi_2 - \psi_1) + \frac{f}{H}\frac{r^2 \Omega^2}{2g}$

$$q_2 = \nabla^2 \psi_2 - \frac{f^2}{g' H}(\psi_2 - \psi_1) - \frac{f}{H}\frac{r^2 \Omega^2}{2g}$$

These equations are integrated using the QUAGMIRE numerical model (Williams et al. 2009).

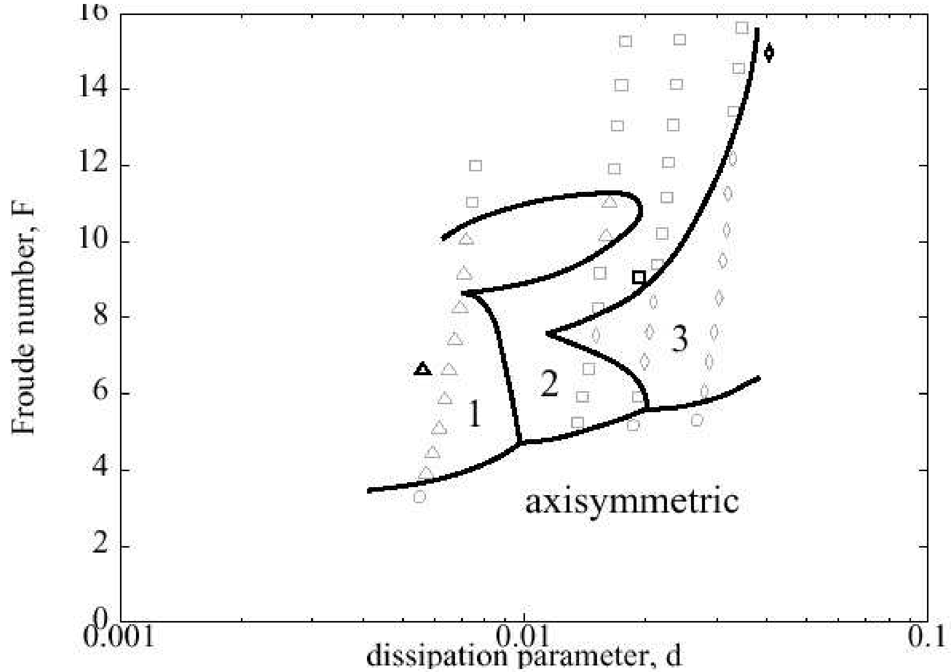
4. Interface height maps



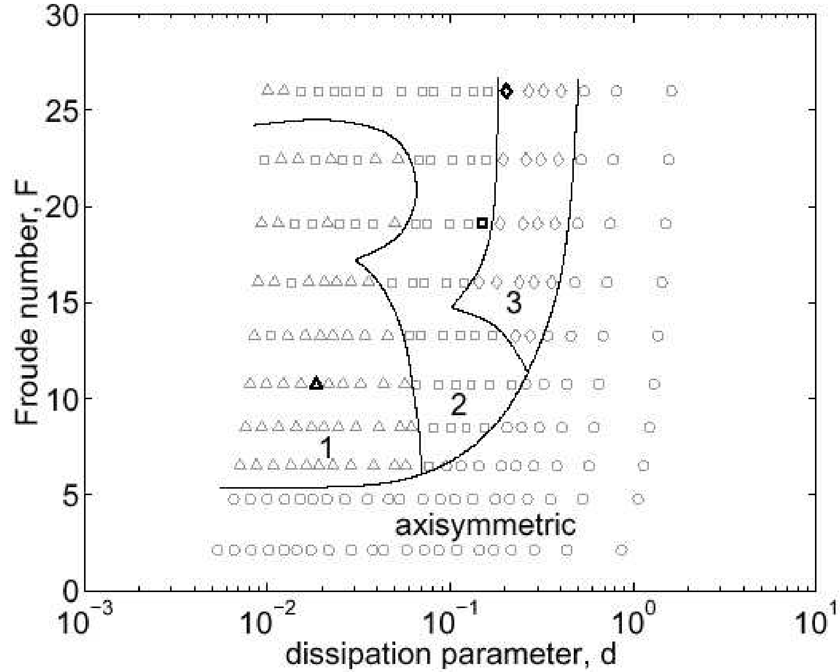
The basic qualitative structures of equilibrated low-wavenumber baroclinic waves are captured well by the model.

5. Wavenumber regime diagrams

laboratory



model



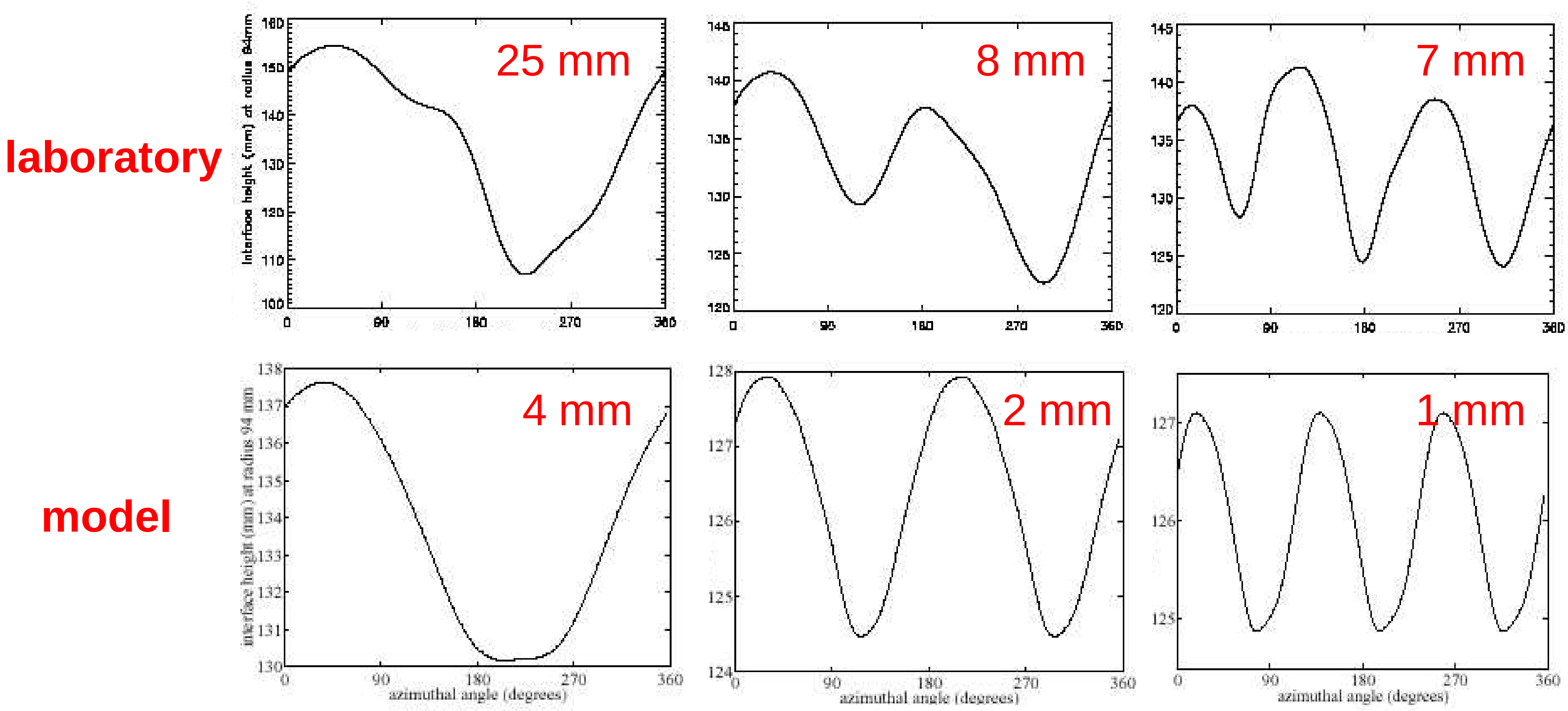
Many aspects of the nontrivial laboratory bifurcation structure are convincingly captured by the model...

- e.g., the neutral curve and the relative sizes of the regimes

...but the quantitative agreement is not wholly satisfactory

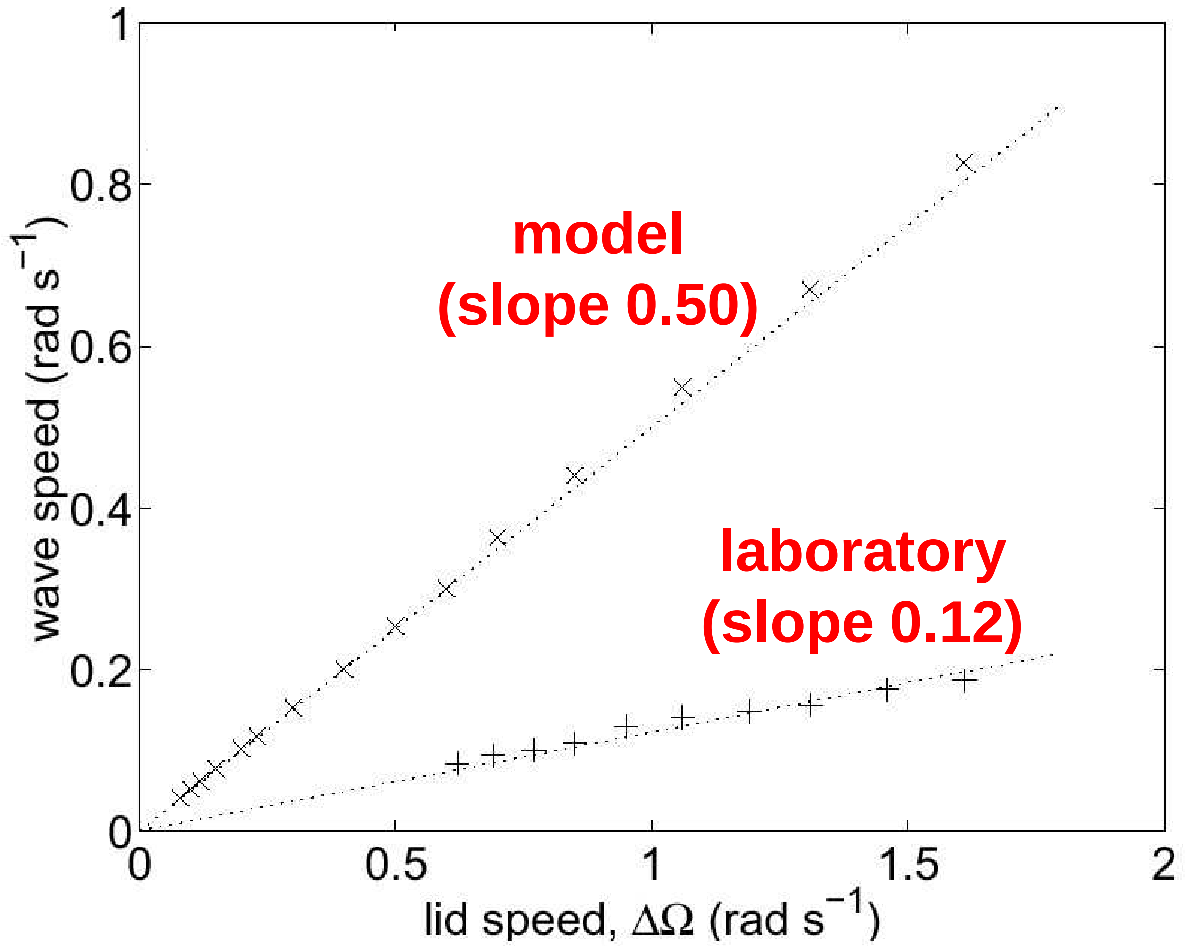
- the model overestimates *F* by a factor of 1-2 and *d* by 5-10

6. Wave amplitudes



Although the model correctly captures the decreasing amplitude with increasing wavenumber, the model waves are a few times weaker than the laboratory waves and are more monochromatic.

7. Azimuthal wave speeds



The wave speed is proportional to the lid speed in both the laboratory and the model, but the model overestimates the wave speeds by a factor of around 4.

8. Discussion

- The model displays three systematic biases compared to the laboratory:
  - overestimation of the dissipation parameter, *d*, suggests that the linear parameterisation of Ekman layers is inadequate
  - underestimation of wave amplitudes is due to the neglect of surface tension; including its leading-order effects in the model fixes the error
  - overestimation of wave speeds is due to the neglect of Stewartson layer drag at the sidewalls
- The three biases may be explained without invoking the dynamical effects of the moderate Rossby number, large aspect ratio or inertia-gravity waves
- We conclude that quasi-geostrophic theory appears to continue to apply well outside its formal bounds