A nonlinear analytical analysis of terrain-induced canopy flows

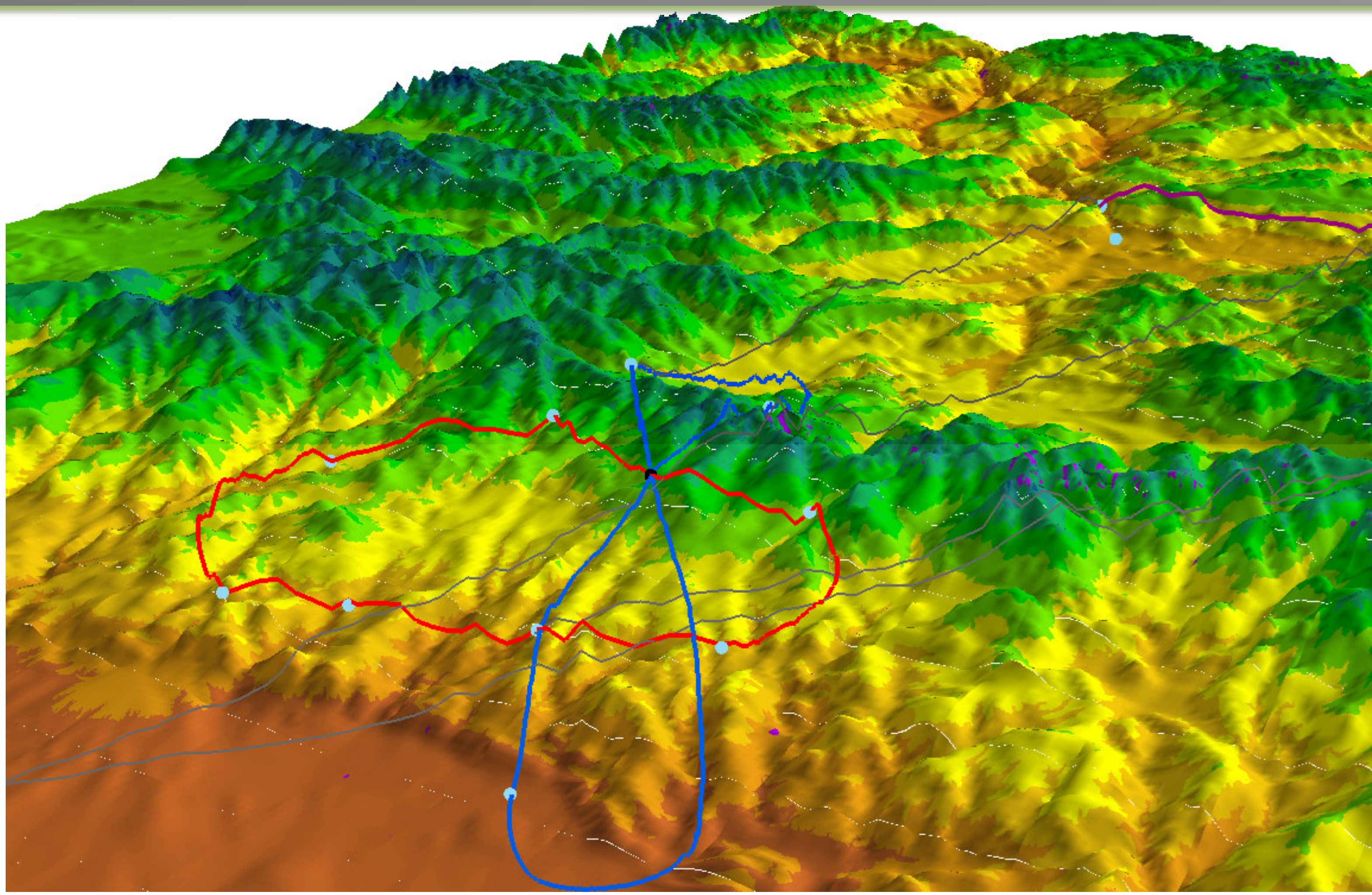
Chuixiang (Tree) Yi and Weiguo Wang

School of Earth and Environmental Sciences
Queens College, City University of New York

Outline

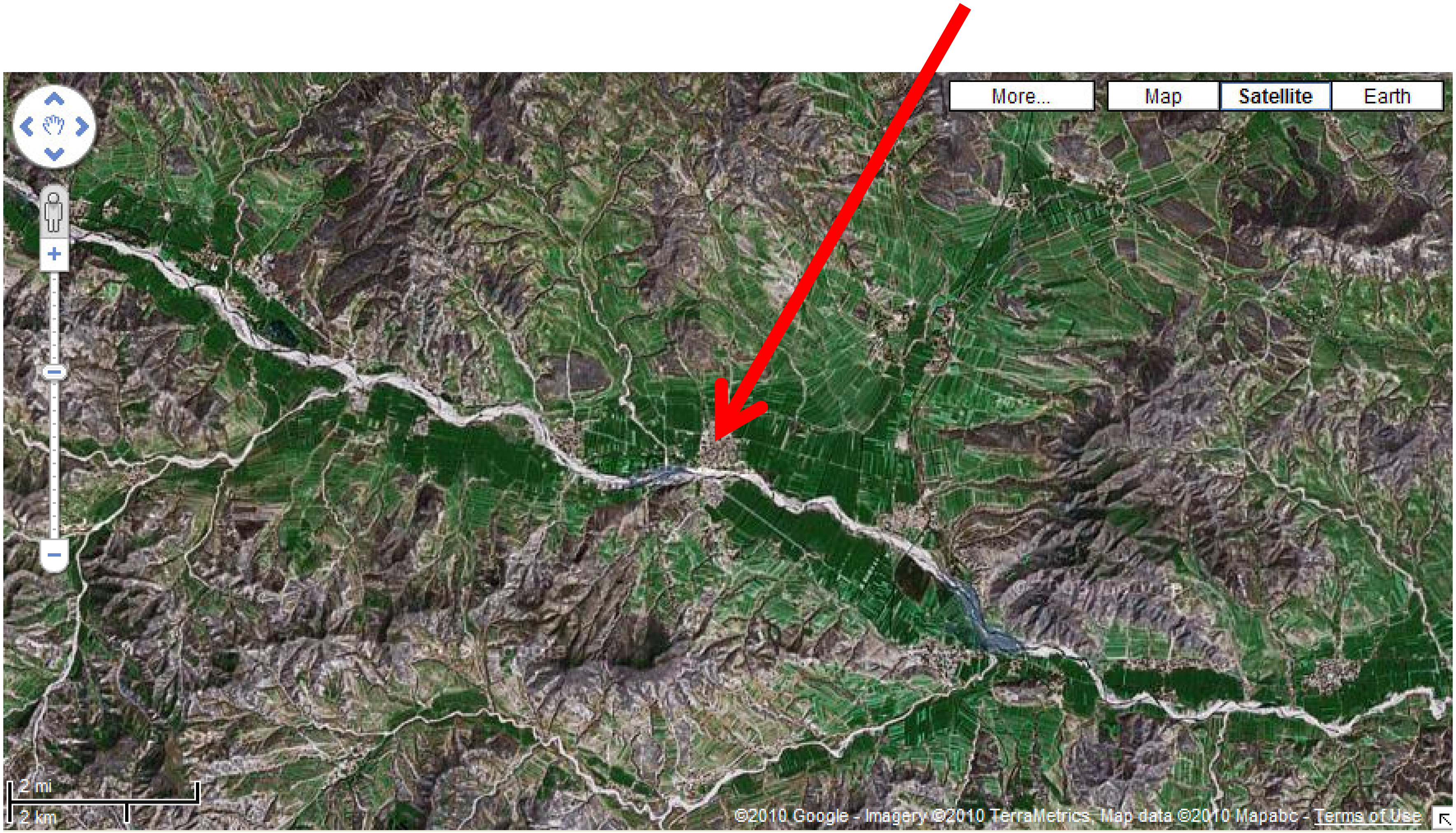
- Complex terrain
- Analytical approach

Real world surface is not flat



Courtesy of David Schimel

Complex terrain is pretty much everywhere
I was born from a complex terrain



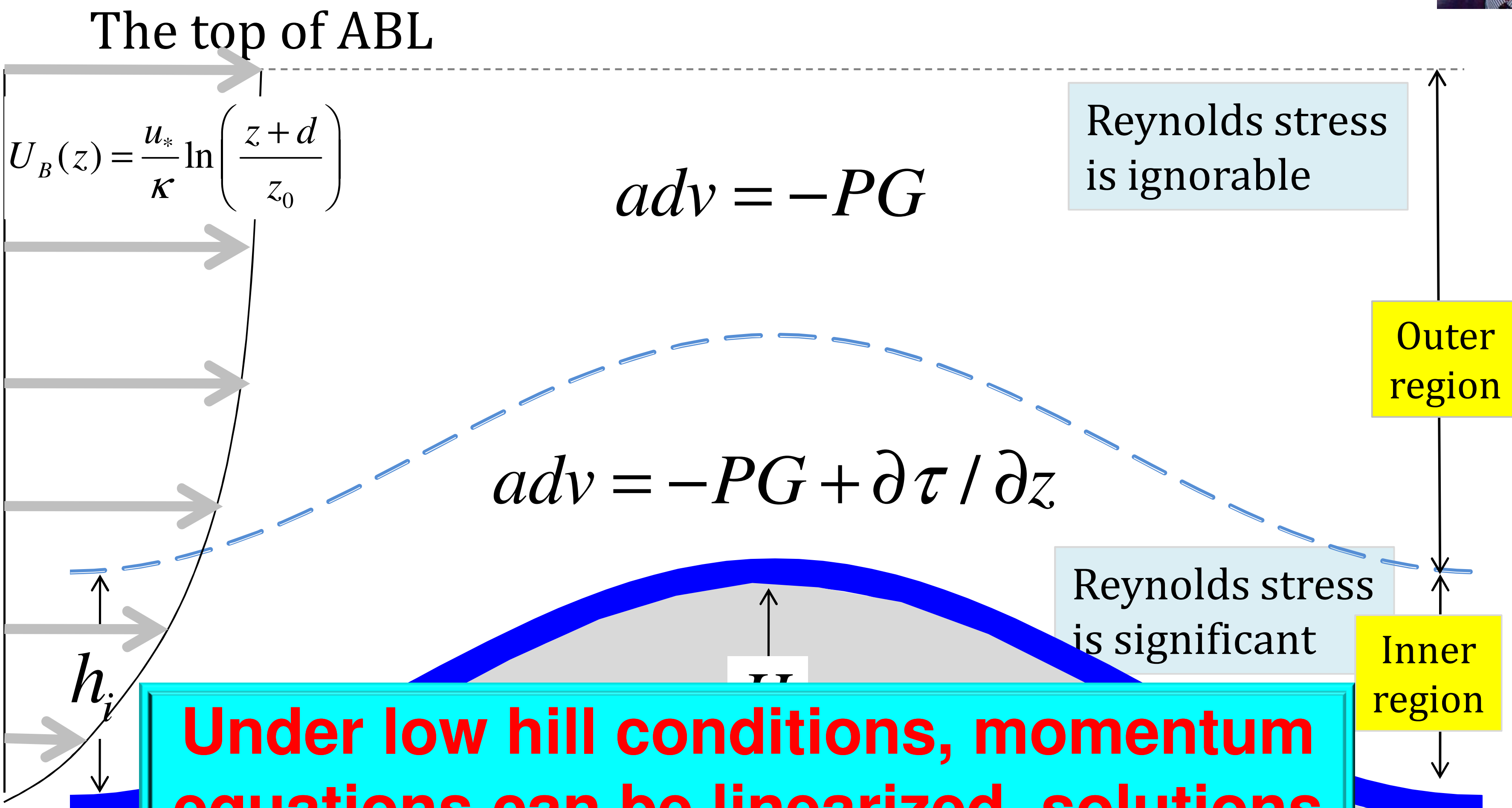
Canopy flows over a forested hill
Analytical theory milestone

A theoretical foundation (no trees) was established by Jackson and Hunt (1975)

Analytical solutions of canopy flows over a forested hill was first obtained by Finnigan and Belcher (2004)

Intensive laboratory studies of canopy flows over a forested hill has been Conducted by Katul and Poggi (2008a,b,c,d,e)

JH75 → A linear theory



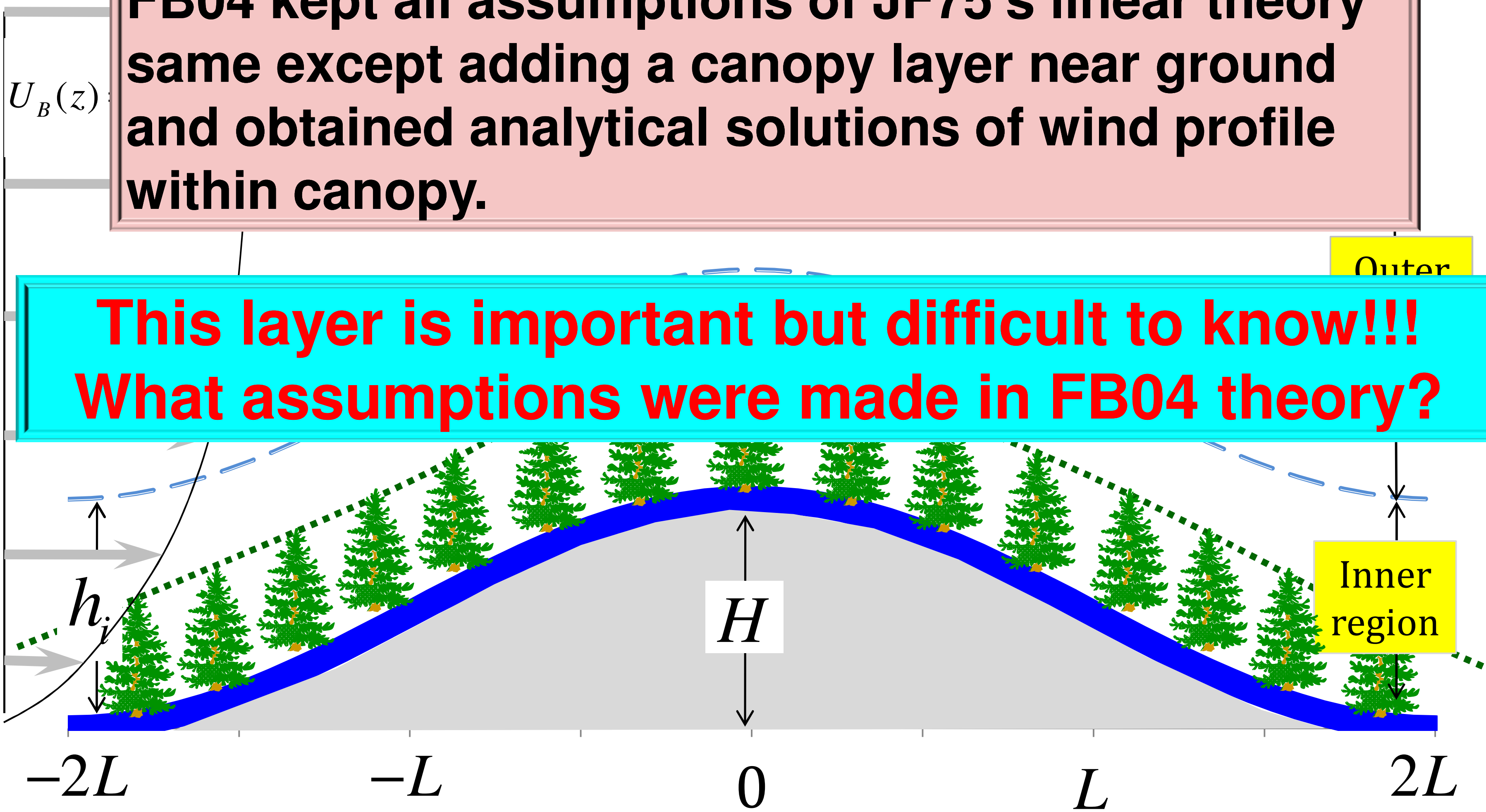
Under low hill conditions, momentum equations can be linearized, solutions can be analytically solved.

$adv = u\partial u / \partial x + w\partial u / \partial z$ $PG = \partial P / \partial x$ $\tau = -\overline{u'w'}$

FB04 → analytical solution within canopy

FB04 kept all assumptions of JF75's linear theory same except adding a canopy layer near ground and obtained analytical solutions of wind profile within canopy.

This layer is important but difficult to know!!!
What assumptions were made in FB04 theory?



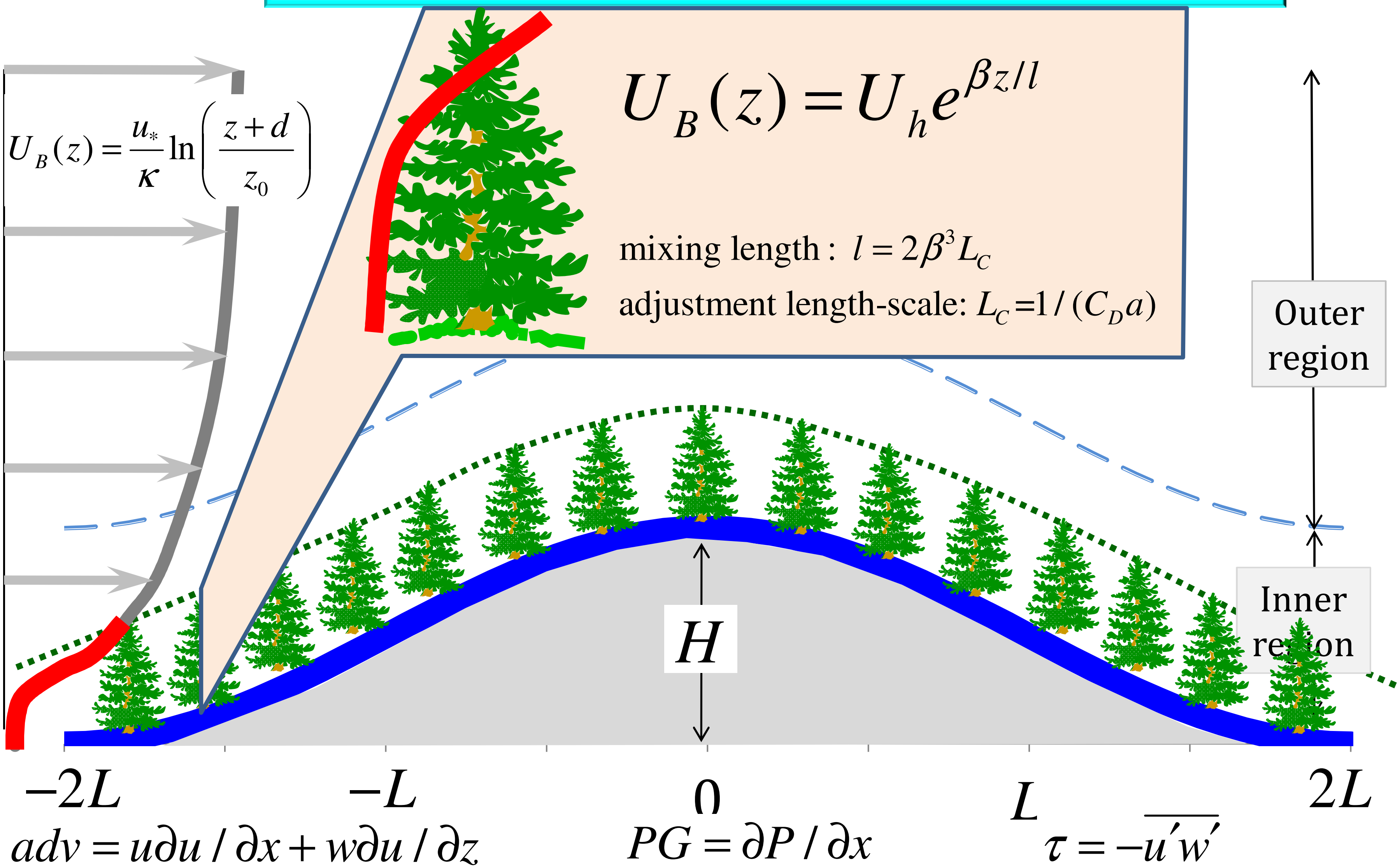
$adv = u\partial u / \partial x + w\partial u / \partial z$

$PG = \partial P / \partial x$

$\tau = -\overline{u'w'}$

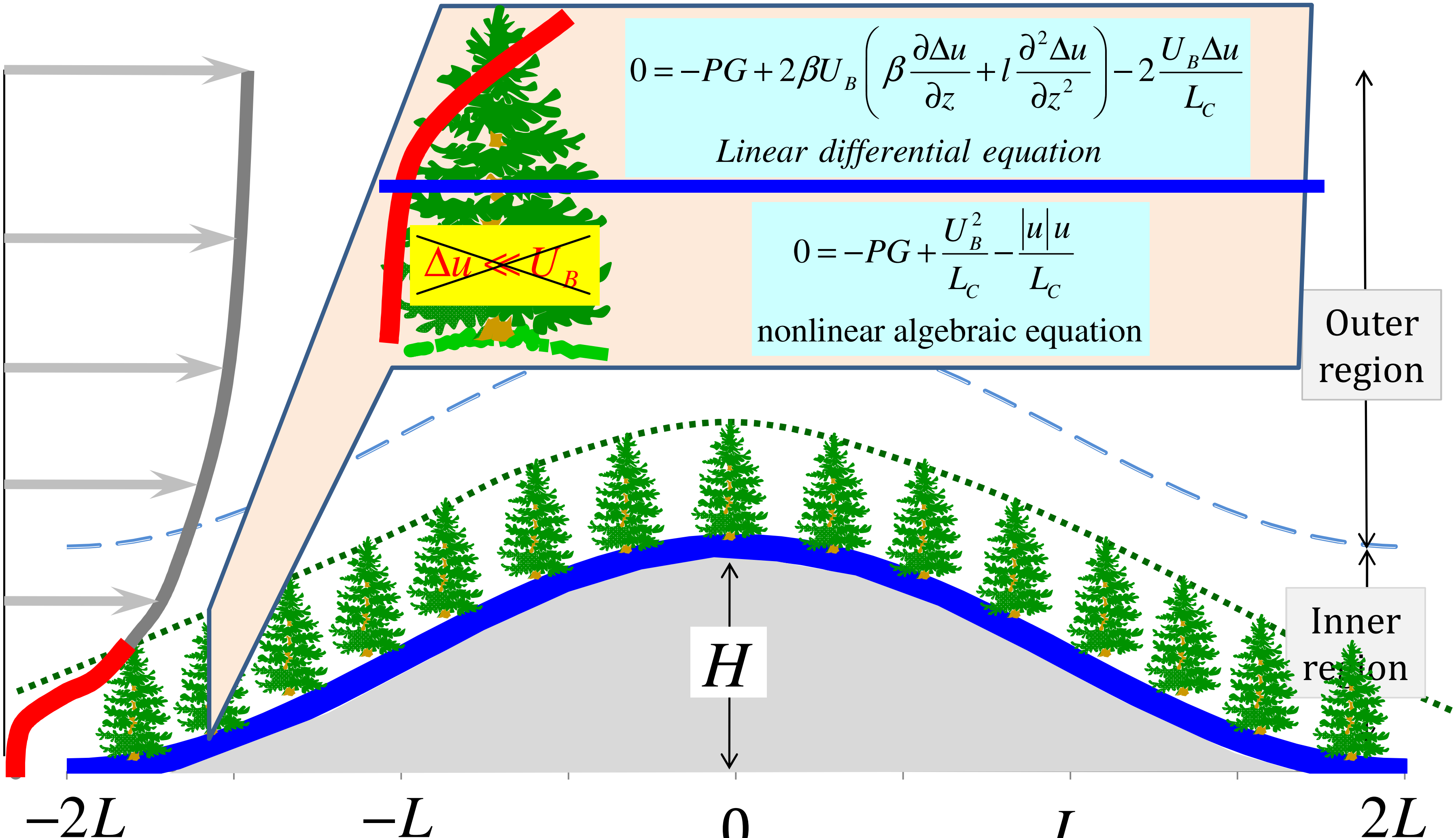
FB04 → analytical solution within canopy

Basic state within canopy: Inoue's model



To obtain analytical solution $u(z) = U_B(z) + \Delta u(z)$

Assumed: ~~adv~~ $\Rightarrow$ 3-force balance; $\Delta u \ll U_B \Rightarrow$ *linearization*



$adv = u\partial u / \partial x + w\partial u / \partial z$ $PG = \partial P / \partial x$ $\tau = -\overline{u'w'}$

FB04 → analytical solution within canopy

$$u = U_B + \Delta u = \left| U_B^2 - L_C PG \right|^{1/2} \operatorname{sgn}\left(U_B^2 - L_C PG \right) + A_C U_C e^{\beta z/l}$$

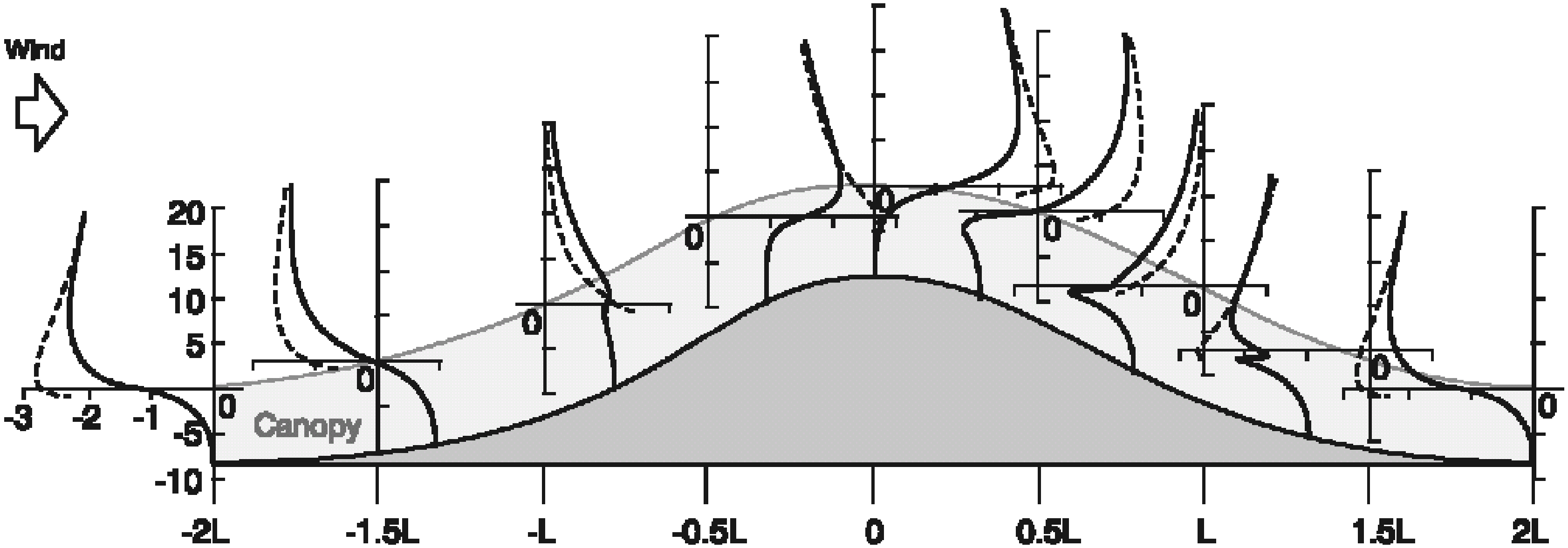


Figure 2. Comparison of canopy velocity perturbation, $\Delta u/U_{SC}$ with the no-canopy solution of Hunt *et al.* (1988; dotted line). Note the Hunt *et al.* solution is only valid to $z = -d + z_0$. Profiles are plotted at a series of X/L values between $X/L = -2$ (upwind trough) and $X/L = 2$ (downwind trough). The units of Z are m, and the vertical range is from $2h_i > Z > L_c$. See text for details.

FB04 made great progresses but some weaknesses need to be improved

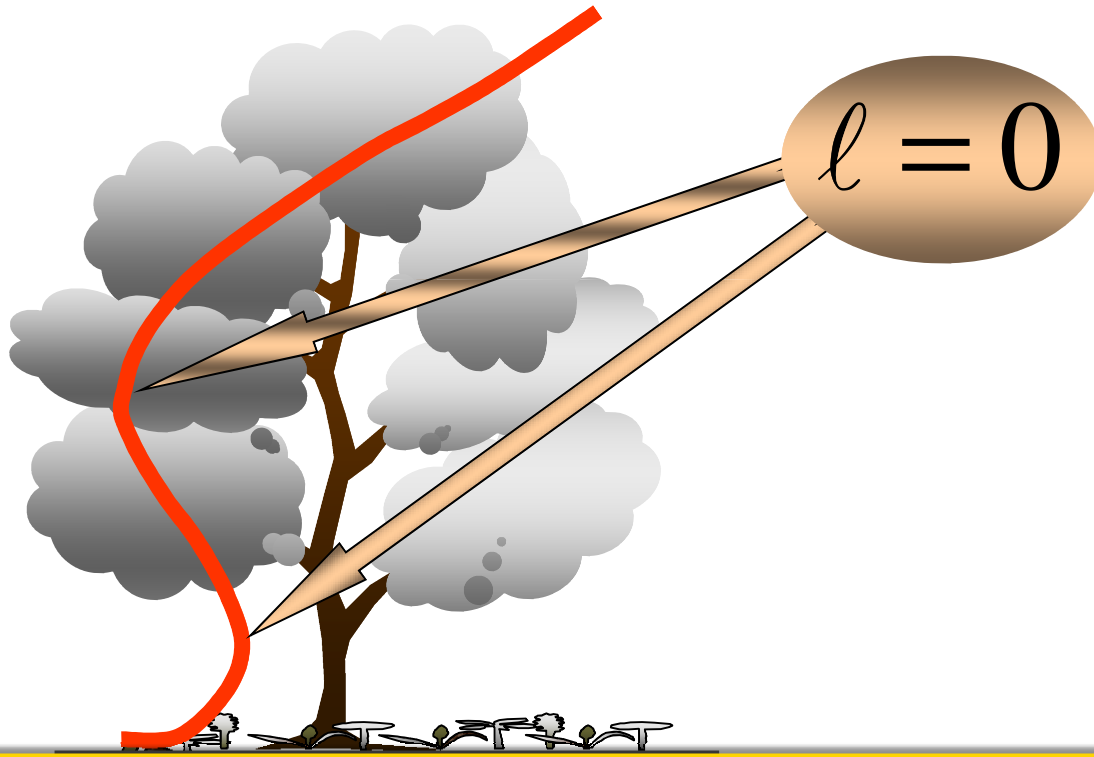
What are weaknesses of FB04?

- **Physical inconsistency** -- Constant mixing-length assumption leading to a varying mixing-length.
- **A non-slip boundary condition** is not able to apply to the nonlinear algebraic equation.
- It is unnecessary to divide a canopy layer into **two layers:** a linear layer and a nonlinear layer.
- No need to assume $\Delta u \ll U_B$

Theoretically, Mixing-length is not constant within canopy!

Von Karman similarity rule

$$\ell = \kappa \left| \frac{d\overline{u}/dz}{d^2\overline{u}/dz^2} \right|$$



- ❑ **Mixing-length is not constant**
- ❑ **Mixing-length theory is not valid within canopy**

Large Eddy Simulation demonstrated "that a constant mixing-length assumption is not strictly valid within the canopy." – Andrew Ross (2008 in BLM)

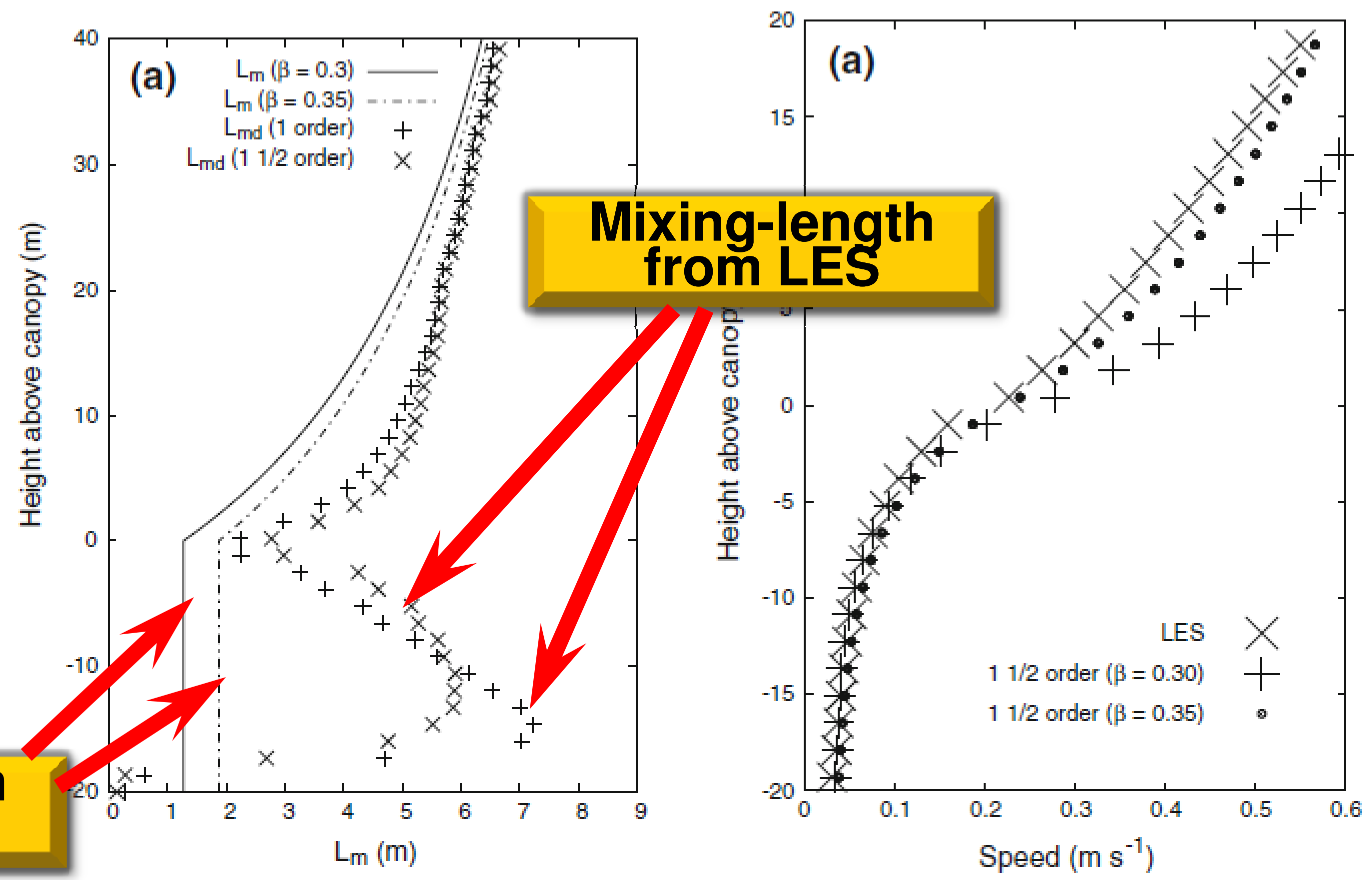
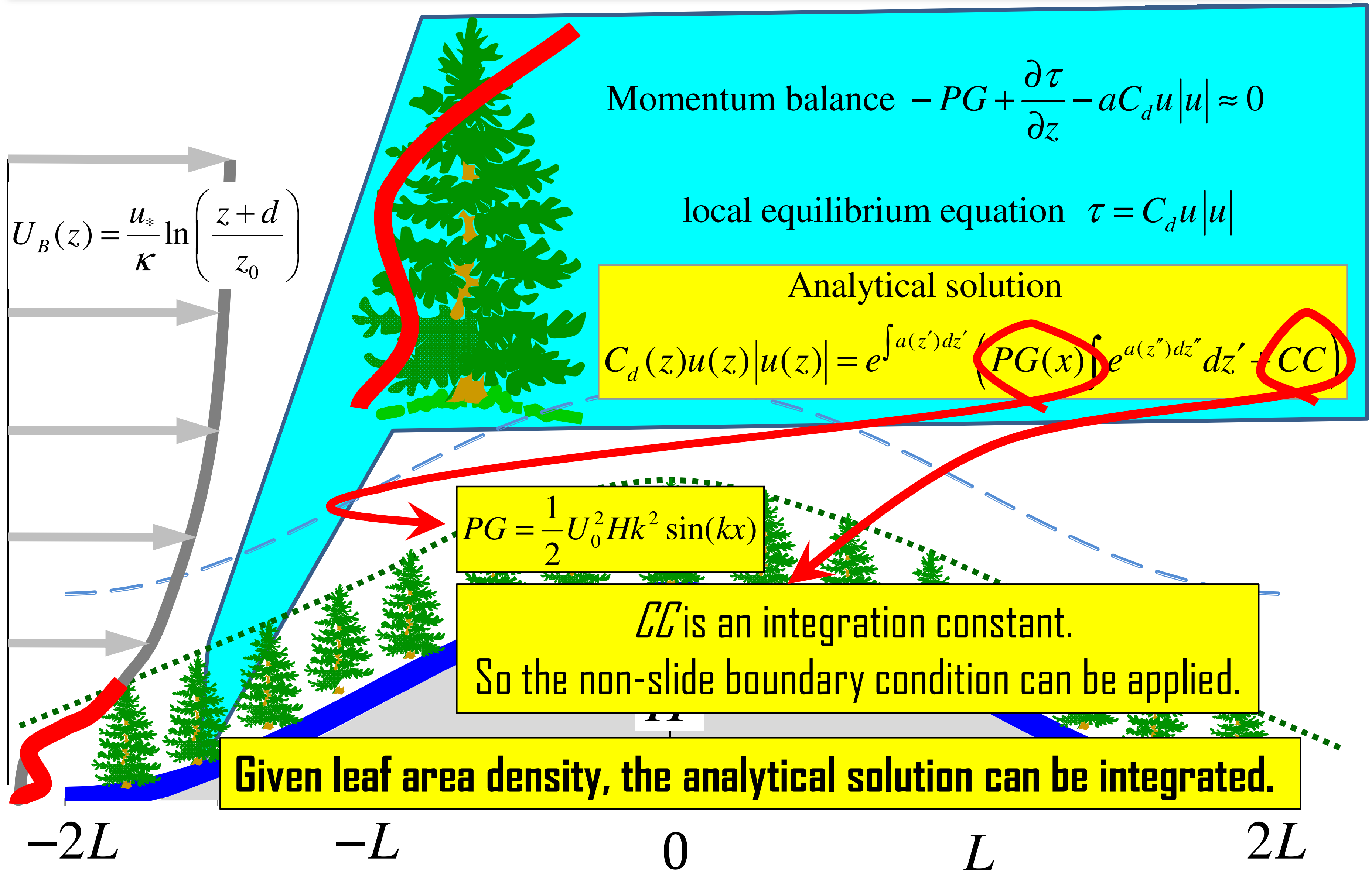


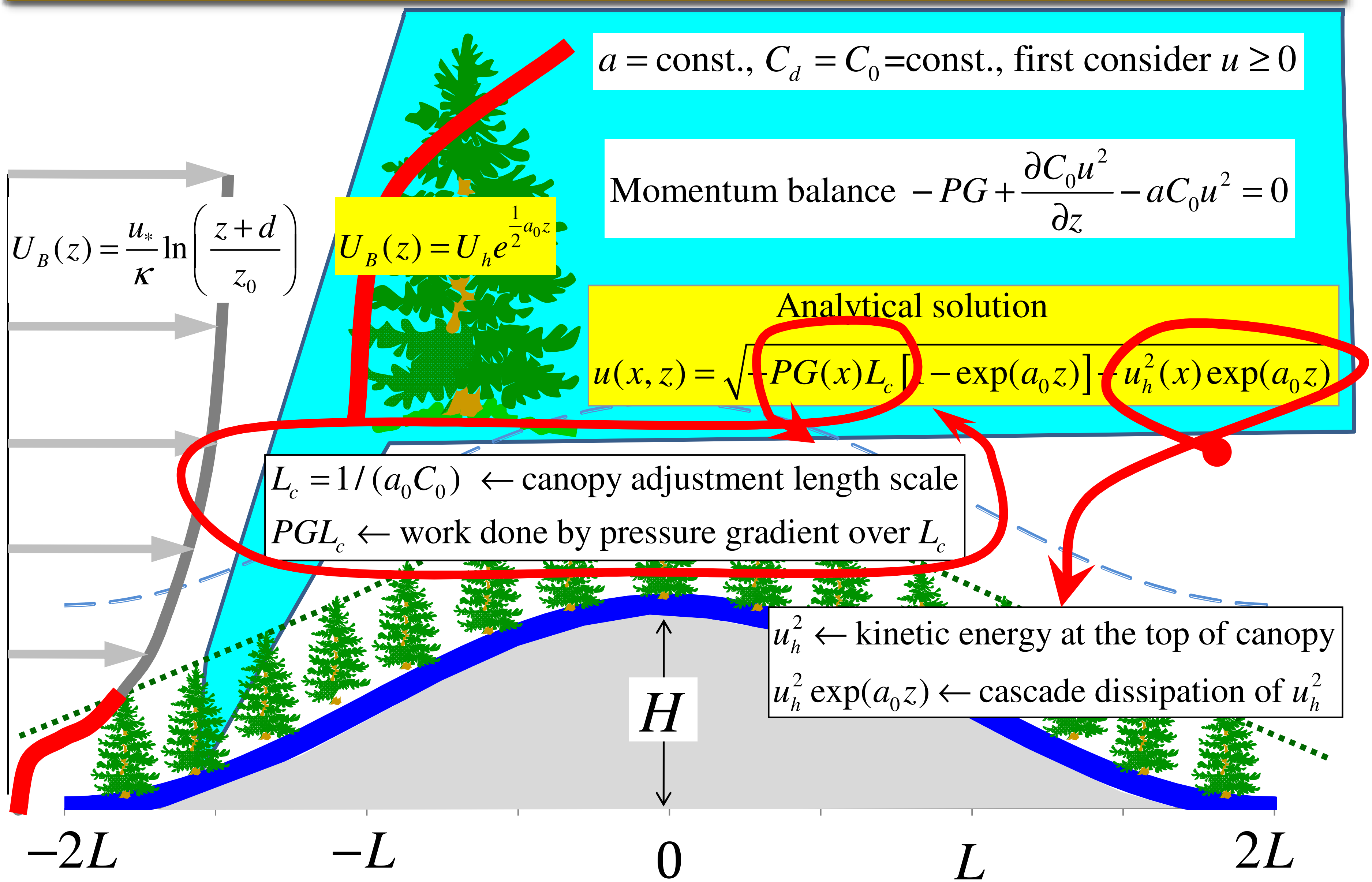
Fig. 7 Profiles of (a) mixing length (L_m) and (b) s

A new nonlinear model – forested hill

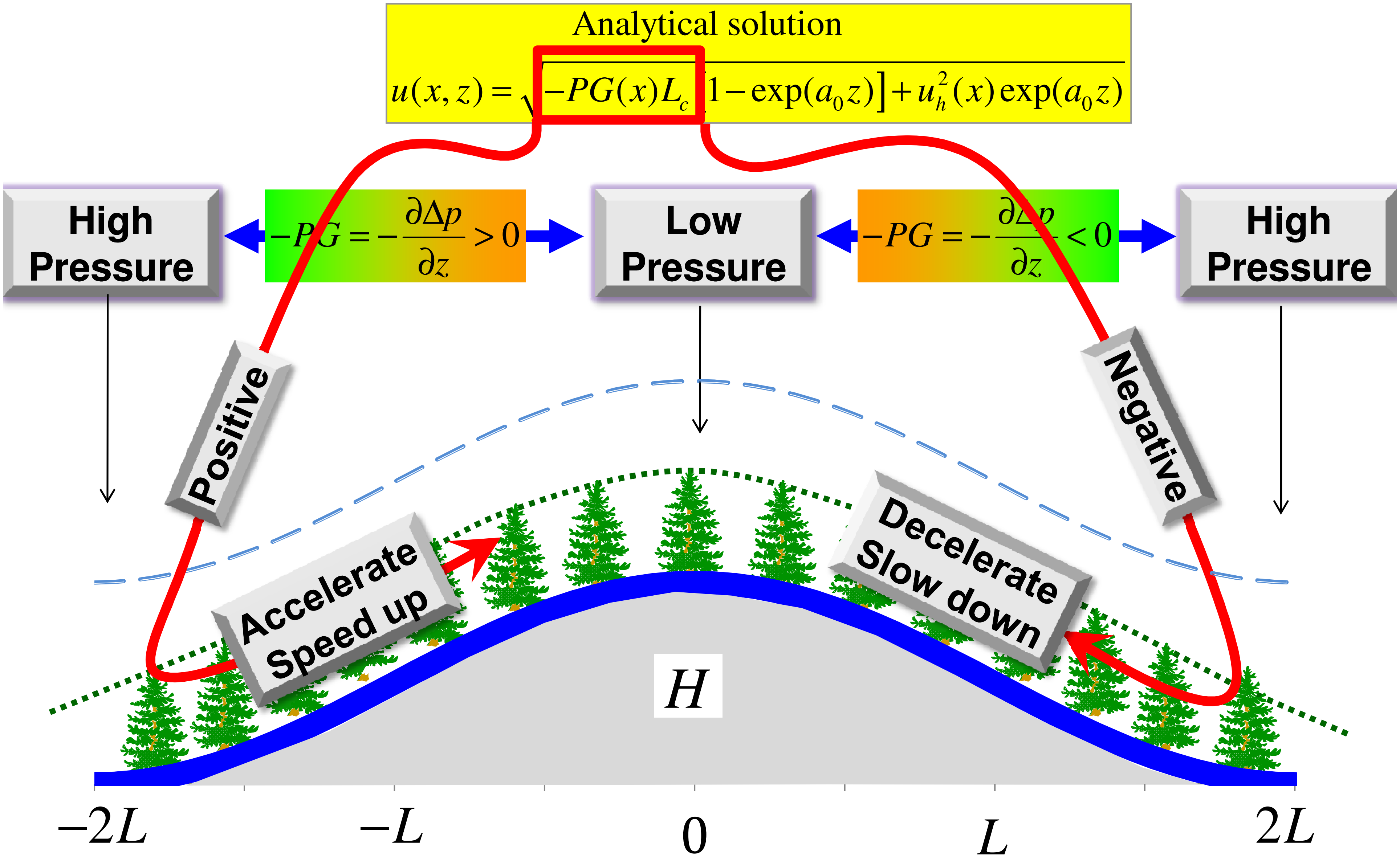


Our model can do what FB04 can do, but even better!

A new nonlinear model – uniform vegetation



A new nonlinear model – uniform vegetation



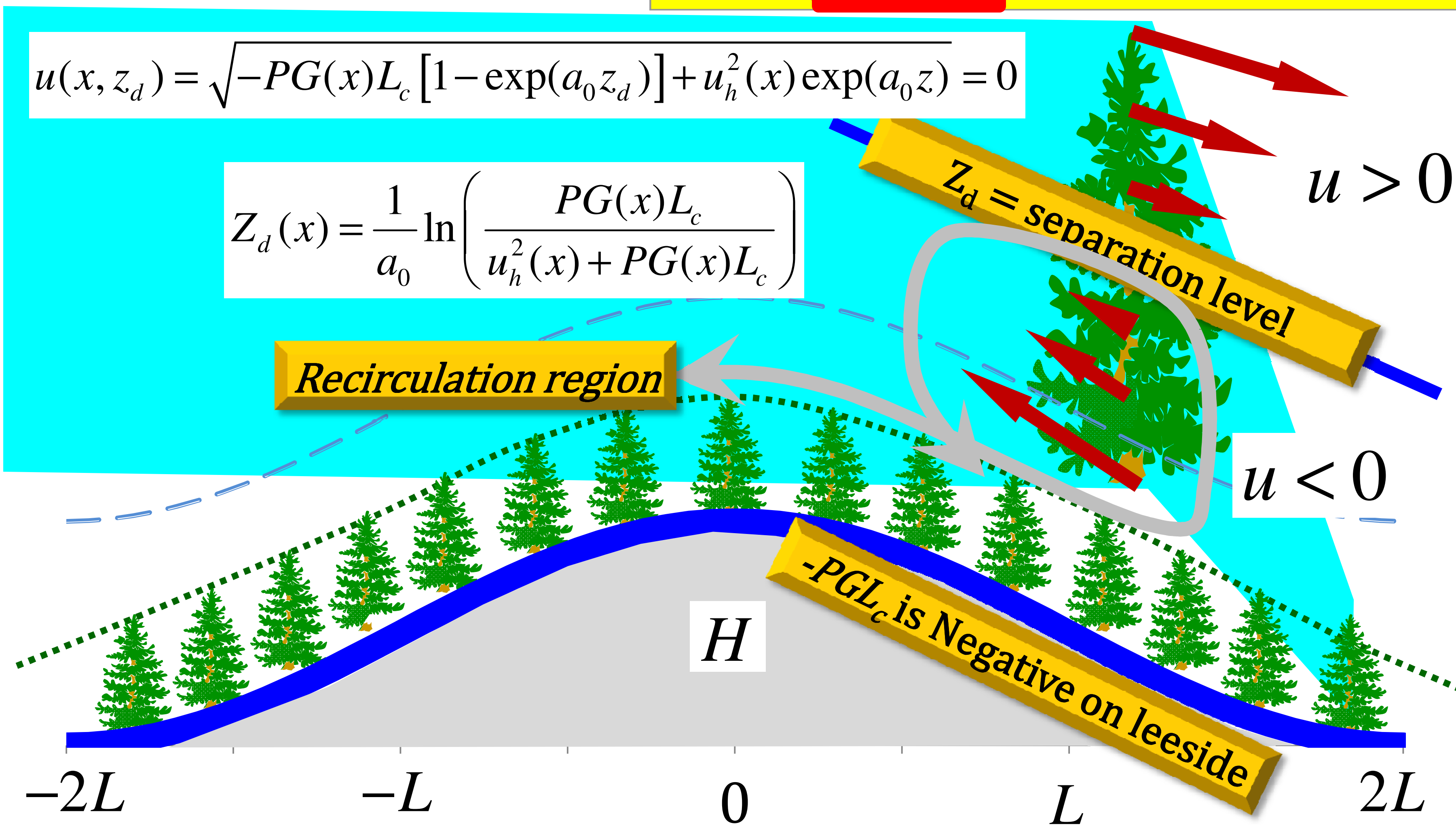
A new nonlinear model – uniform vegetation

Analytical solution

$$u(x,z)=\sqrt{-PG(x)L_c\left[1-\exp(a_0 z)\right]+u_h^2(x)\exp(a_0 z)}$$

$$u(x,z_d)=\sqrt{-PG(x)L_c\left[1-\exp(a_0 z_d)\right]+u_h^2(x)\exp(a_0 z)}=0$$

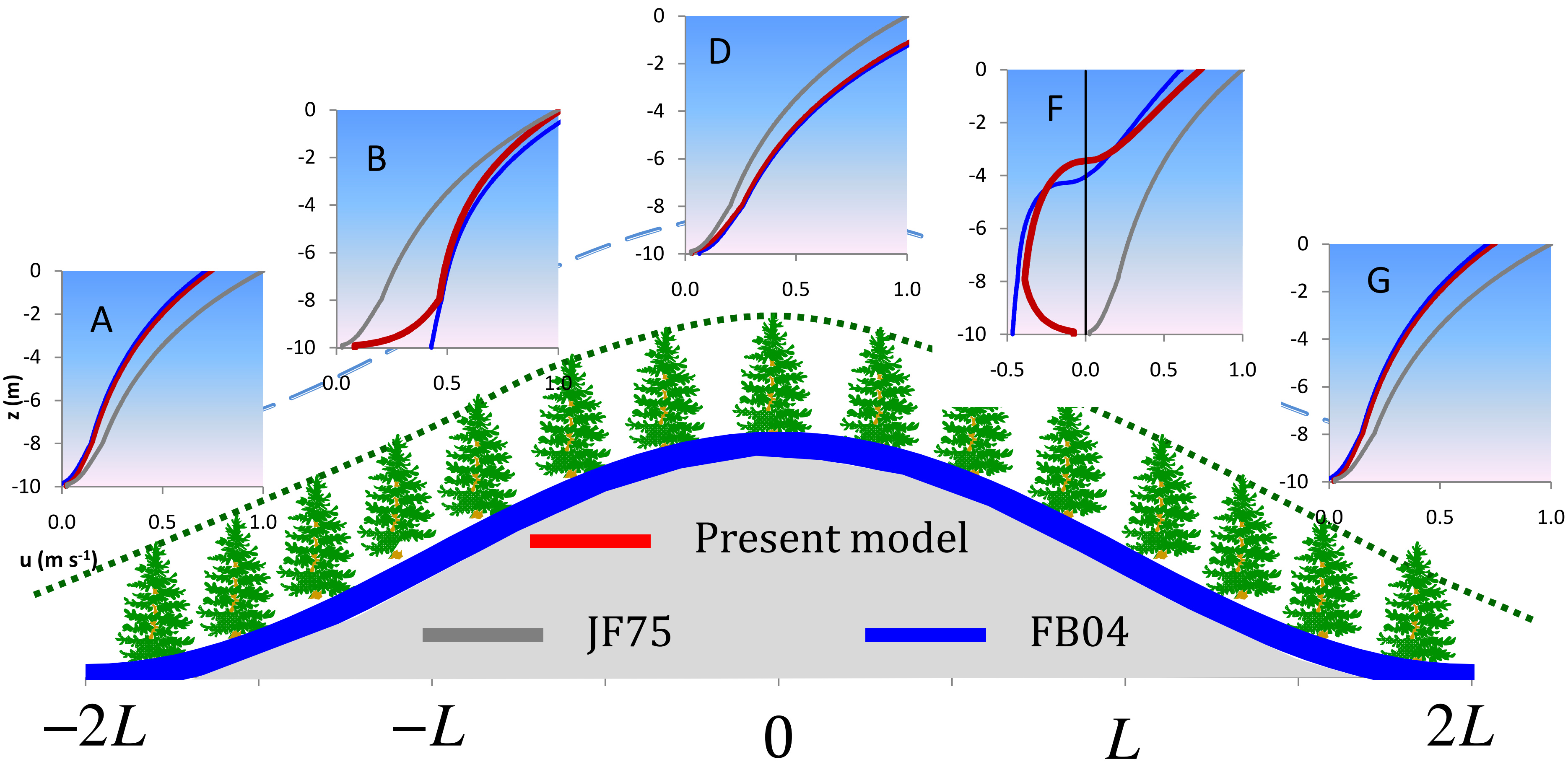
$$Z_d(x)=\frac{1}{a_0}\ln\left(\frac{PG(x)L_c}{u_h^2(x)+PG(x)L_c}\right)$$



A new nonlinear model – uniform vegetation

Excellent agreement between FB04 and present model at the crest and troughs.

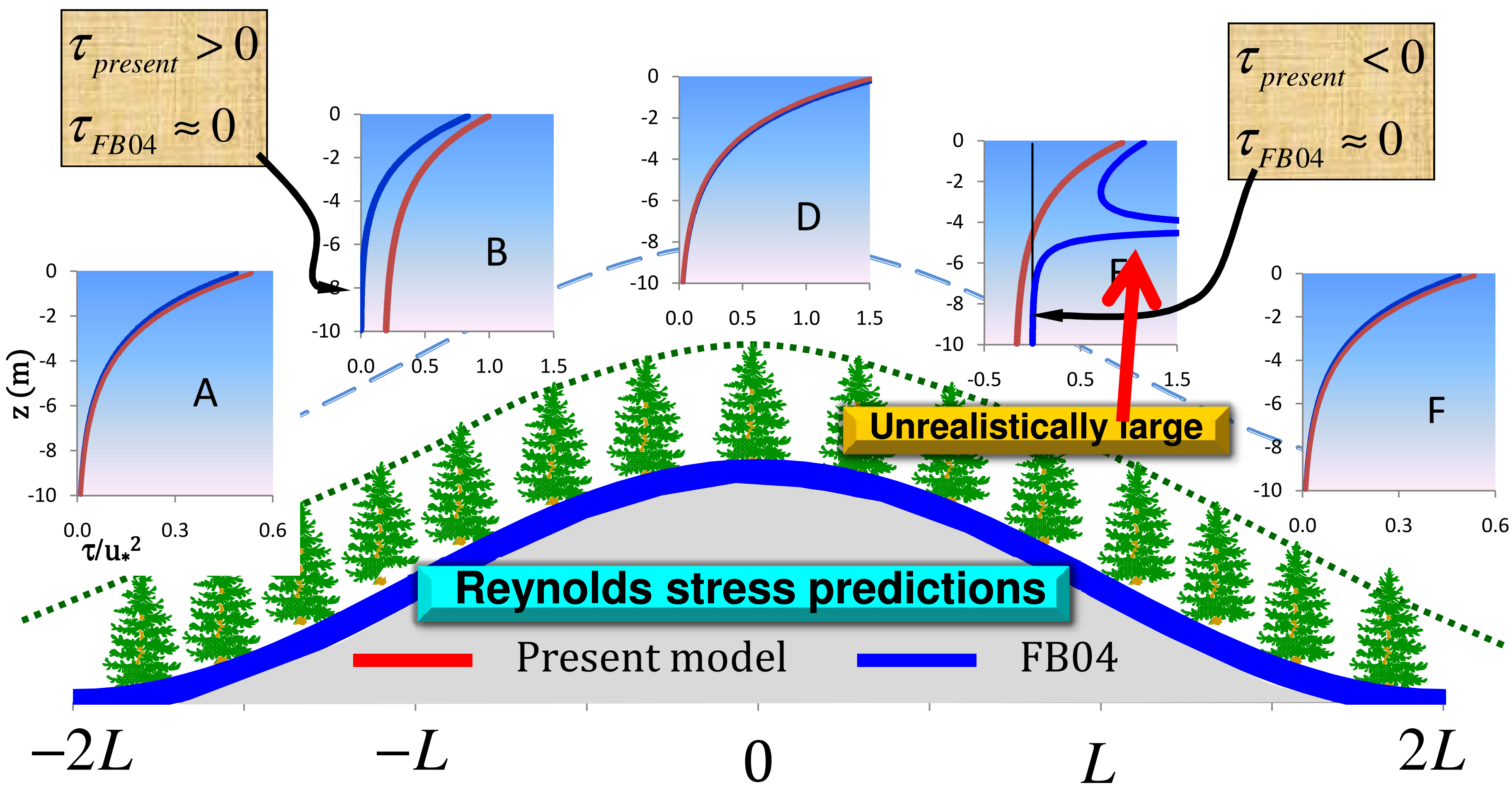
The S-shaped wind profile on windward side and C-shaped wind profile on leeward side are predicted by the new model rather than by FB04.



A new nonlinear model – uniform vegetation

Excellent agreement at the crest and trough

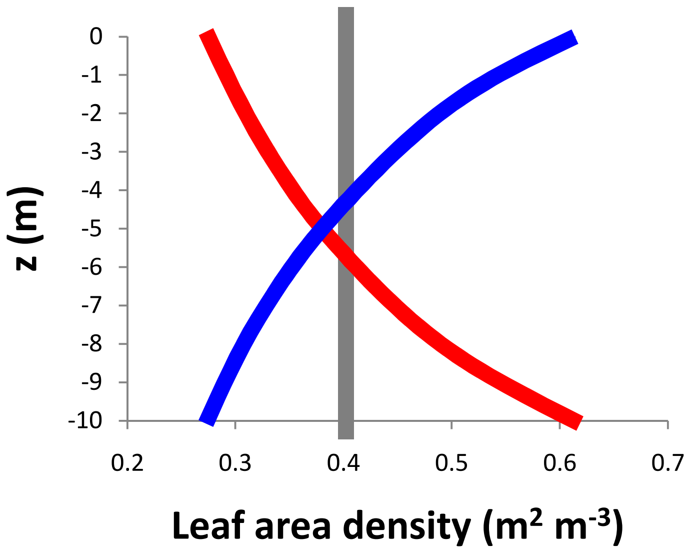
Different on windward and leeward sides



Our model can do what FB04 cannot do!

- Varying leaf area density
- Varying drag coefficient

A new nonlinear model – varying LAD

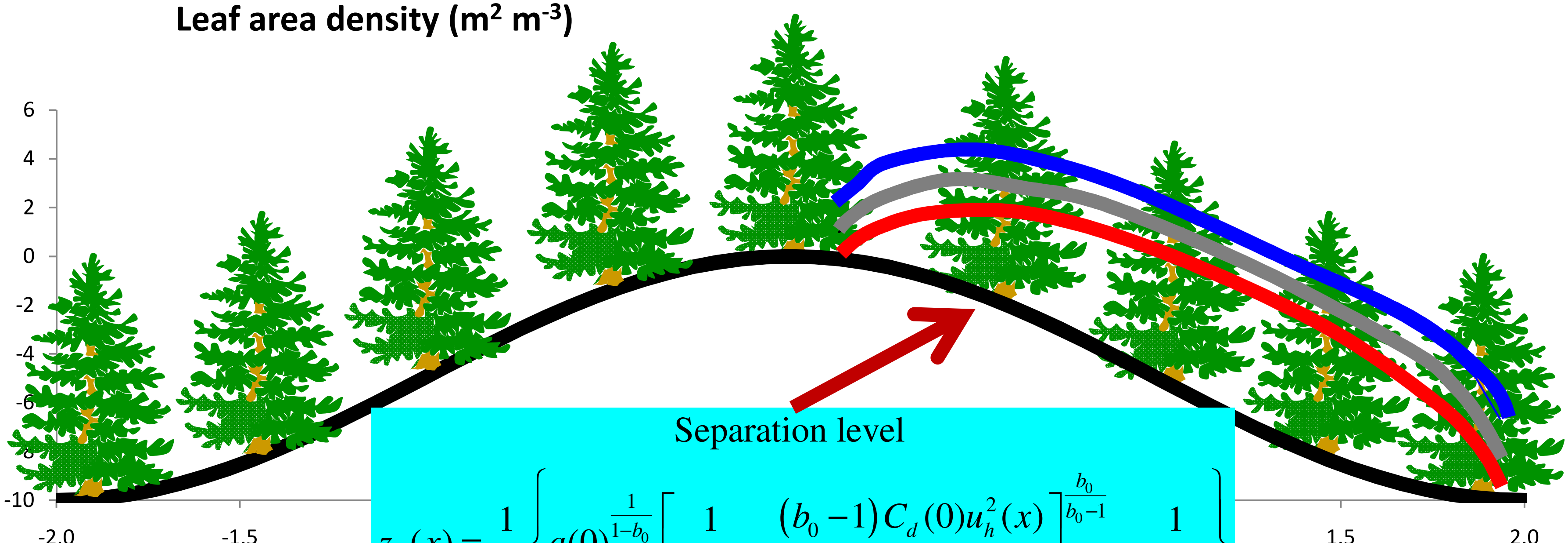


$LAI = 4.0$ for all examples

$a_0 = const. = 0.4 \rightarrow E_1$ black

leaf area density $\rightarrow a = \dfrac{1}{b_0(z+h)+b_1}$

$$\begin{cases} b_0 = -0.2,\ b_1 = 3.63 \rightarrow E_2 \text{ increasing function (blue)} \\ b_0 = 0.2,\ b_1 = 1.63 \rightarrow E_3 \text{ decreasing function (red)} \end{cases}$$

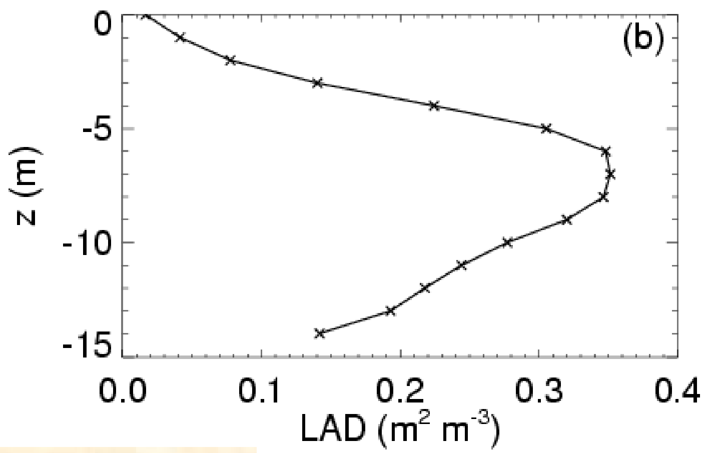


Separation level

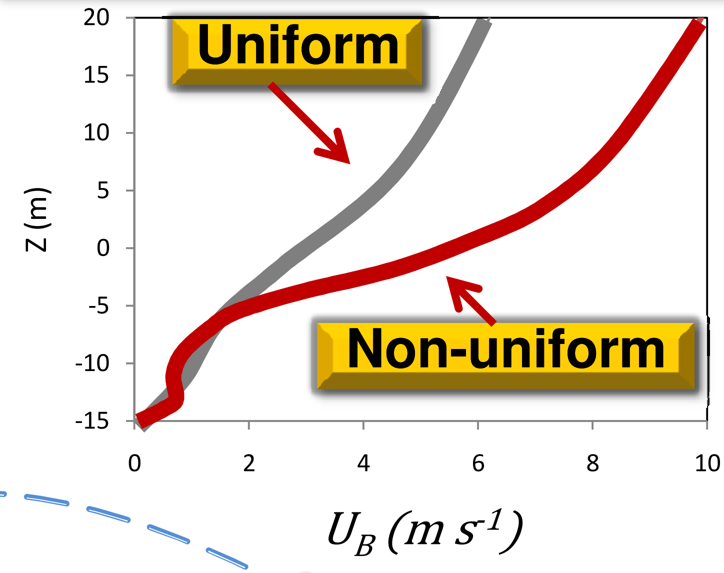
$$z_d(x) = \frac{1}{b_0}\left\{ a(0)^{\frac{1}{1-b_0}}\left[\frac{1}{a(0)} - \frac{(b_0-1)C_d(0)u_h^2(x)}{PG}\right]^{\frac{b_0}{b_0-1}} - \frac{1}{a(0)}\right\}$$

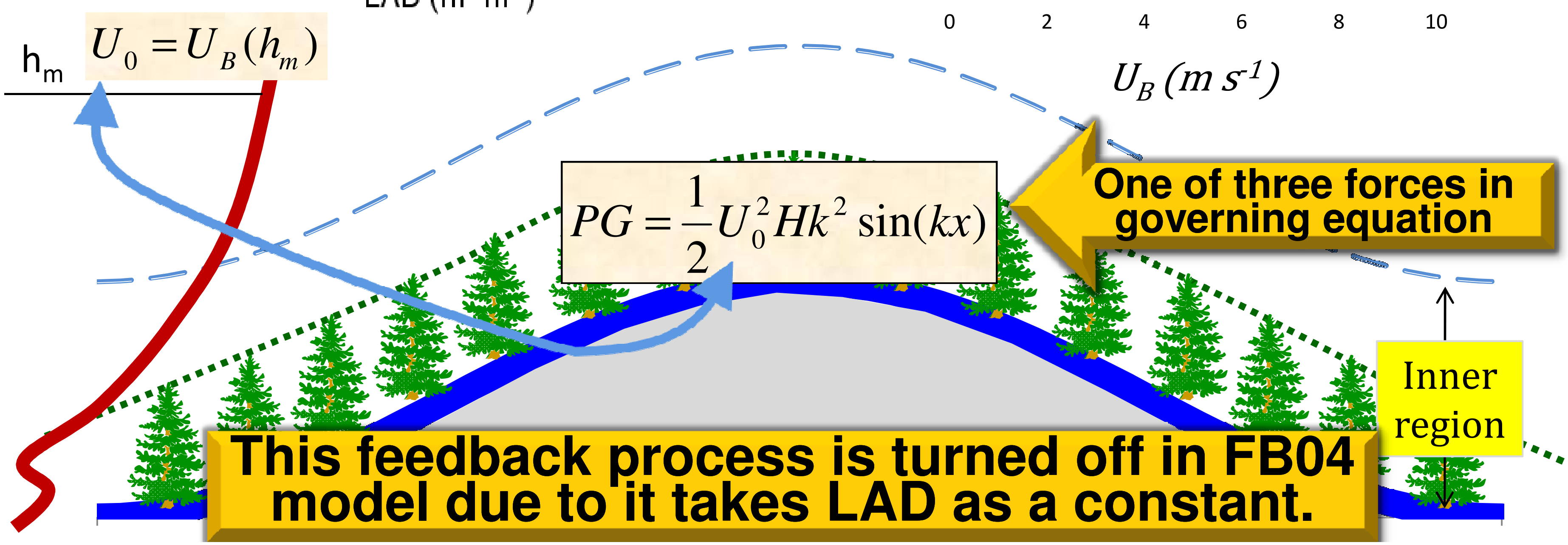
What will be lost if we treat non-uniform LAD as uniform LAD?

Observed LAD at Niwot Ridge AmeriFlux site



Difference in basic state U_B





Summary and Conclusion

Following weaknesses of FB04 have been improved!

- **Physical inconsistency** -- Constant mixing-length assumption leading to a varying mixing-length.
- **A non-slip boundary condition** is not able to apply to the nonlinear algebraic equation.
- It is unnecessary to divide a canopy layer into **two layers:** a linear layer and a nonlinear layer.
- No need to assume $\Delta u \ll U_B$

The new model is simpler but more useful!

- Separation level prediction
- Perform feedback - varying LAD → changing U_B in outer region → affecting PG in the inner region →modifying momentum balance in canopy layer

Thank You!