4A.5 EVALUATION OF ELECTROMAGNETIC SCATTERING MODELING TECHNIQUES FOR IRREGULAR ICE HYDROMETEORS

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1. INTRODUCTION

Modeling of electromagnetic scattering properties of ice hydrometeors such as pristine ice crystals and their aggregates provides necessary information to correctly interpret and evaluate radar measurements of ice clouds and precipitation. In general, hydrometeor scattering properties depend on their mass, size, shape, composition and on the radar wavelength. Due to the complexity of their shapes, developing accurate electromagnetic models of ice hydrometeors can be challenging.

Several models for pristine ice crystals and aggregates are available in the literature. Electromagnetic techniques such as the Finite Difference Time Domain (FDTD) or the Discrete Dipole Approximation (DDA) have been applied to develop highly detailed models of pristine ice crystals (e.g., Aydin and Walsh, 1999; Liu, 2008) and aggregates (e.g., Ishimoto, 2008; Petty and Huang, 2010). Computational considerations typically limit the applicability of these two methods to particles that are larger than the wavelength. Draine and Flatau (1994) showed degradation in the backscattering cross-sections obtained using DDA for pseudo-spheres above a certain size. Moreover, DDA is usually not applicable to water particles.

Simplified models based on a "bulk" approximation of the ice particles are also widely used. Bulk models consist in approximating the shape of an ice crystal or aggregate with a sphere or spheroid of similar size by defining an effective dielectric constant. The effective dielectric constant is usually computed as a mixture of ice and air (e.g., Haynes, 2007) or ice, water and air in case of melting particles (e.g., Fabry and Szyrmer, 1999). Although this approach is broadly used it runs into problems for particles with size bigger

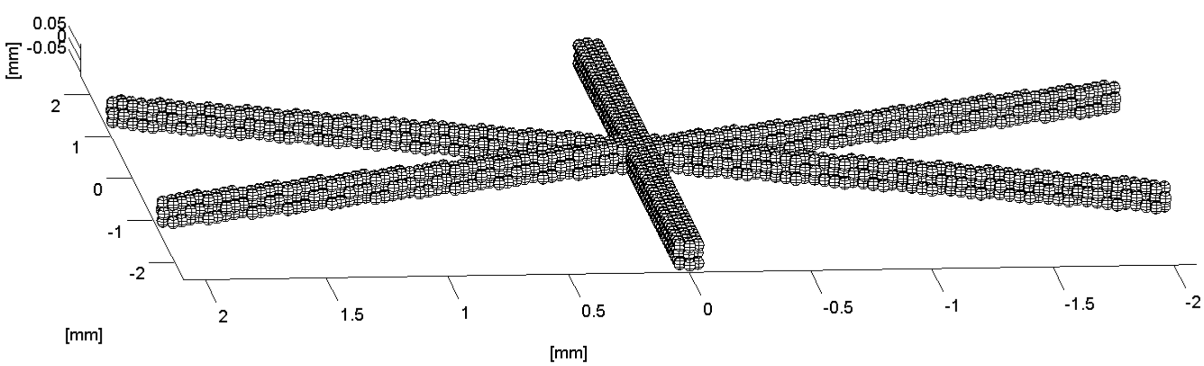

Figure 1: Example of a stellar crystal model composed of about 1800 ice spheres with diameter equal to 0.027 mm. The maximum dimension of the crystal is about 4 mm.

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than half the radar wavelength. Petty and Huang (2010) and Botta et al. (2010; 2011) pointed out some of these issues and concluded that bulk models aren't suitable for reproducing the scattering properties of aggregates, especially when multiple frequencies are considered.

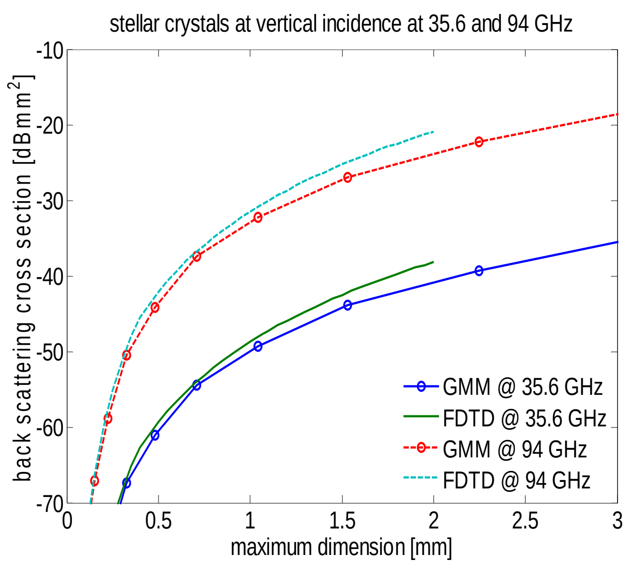

Figure 2: Backscattering cross-sections for stellar crystals at 35.6 and 94 GHz computed using the GMM and the FDTD methods (results available in Aydin and Walsh, 1999).

A new approach for evaluating the electromagnetic scattering characteristics of pristine ice crystals and aggregate was recently introduced by Botta et al. (2010; 2011). This approach employs an analytical solution for electromagnetic scattering by a cluster of non-overlapping spheres with arbitrary position, size and composition. This method, known as Generalized Multiparticle Mie (GMM) method (Xu, 1995), is in very good agreement with measurements (Xu and Gustafson, 2001). Since GMM is an analytical solution it doesn't have restrictions on the size of the target considered or on its composition, allowing for the simulations of large ice aggregates or melting particles (Botta et al., 2010).

In the following sections complex models for pristine ice crystals and aggregates composed of clusters of tiny ice spheres will be presented and the corresponding electromagnetic scattering results at Ka and W band will be shown.

2. PRISTINE ICE CRYSTAL MODELS

A cluster of tiny ice spheres arranged in the shape of an individual crystal was used to compute the GMM scattering solution. The way the spheres are arranged in the cluster requires careful consideration. In order to approximate a solid ice particle as accurately as possible, the spheres need to be closely packed together so that the ice fills most of the space. Considering an infinite regular arrangement of ice spheres (lattice) in space, the volumetric fraction can be defined as the ratio between the amount of space occupied by the ice spheres and the total space, i.e., as the ratio of the volume occupied by the spheres in a lattice cell divided by the total lattice cell volume. Unfortunately, for any such arrangement the maximum achievable volume fraction is given by the constant value $\pi/\sqrt{18} \approx 0.74$. This result was proved for a regular lattice by Gauss in 1831 and conjectured to be true for irregular lattices as well by Kepler in 1611 and proved by Hales (1998). Thus, a pristine ice crystal model employing a regular lattice of spheres will achieve a volume fraction of about 0.74, which is the best one can obtain with a cluster of spheres approximation.

A stellar crystal model tailored on the stellar crystal model described by Aydin and Walsh (1999) was developed. The model was developed to match the same mass-dimensional relationship used in the cited paper and employs a Face Centered Cubic (FCC) lattice for the arrangement of the spheres in order to achieve the maximum volume fraction of 0.74.

An example of an individual stellar crystal is shown in Figure 1. Figure 2 shows the backscattering cross-section computed using the GMM solution for this stellar crystal model, compared to the FDTD results obtained by Aydin and Walsh (1999) at Ka and W band (35.6 and 94 GHz). The GMM simulations differ by less than 3 dB with respect to the FDTD computations, the difference being larger for larger sizes. This could depend on the differences in the two models that weren't accounted for or in the volumetric fraction of the GMM model being smaller than 1.

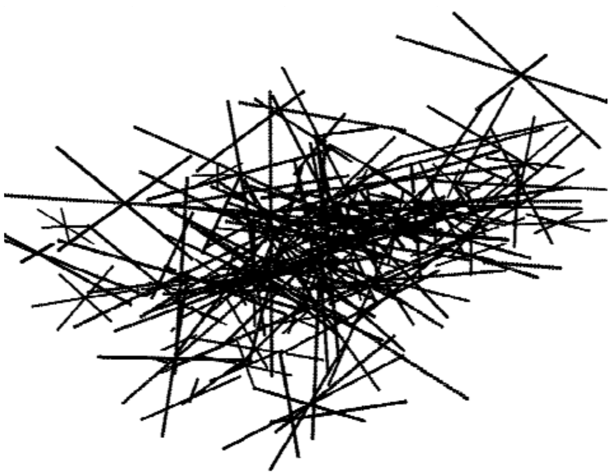

Figure 3: Example of an ice crystal aggregate random realization composed of about 6000 tiny ice spheres.

3. ICE AGGREGATES

An algorithm to generate random realizations of ice crystal aggregates was developed and described by Botta et al. (2010; 2011). This approach was compared to radar measurements by Botta et al. (2011), showing good results at multiple frequencies. The algorithm allows for the generation of ice crystal aggregates from simplified single crystal models (columns, stellar crystals, dendrites, plates, etc.). The algorithm allows assigning a specific mass-dimensional relationship to the aggregate realizations within the prescribed error tolerance. Figure 3 shows an example of such aggregate realizations. For the sake of comparison, a set of bulk models was also defined. Figure 4 shows a random aggregate realization with two corresponding bulk model shapes (in wireframe). The bulk models in the figure are defined by assuming the same maximum dimension as the original aggregate realizations. However, an equivalent diameter based on the root mean square distance of all the spheres from the particle center of mass (Petty and Huang, 2010) will be smaller than the aggregate maximum dimension for low density particles.

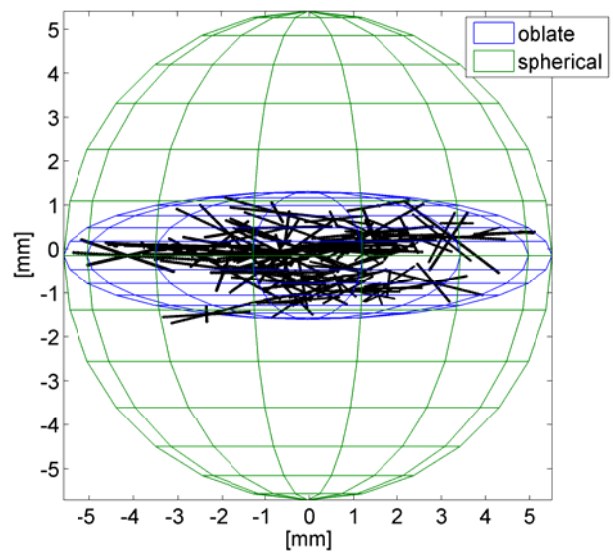

Figure 4: An example of ice crystal aggregate realization and two equivalent bulk model shape representations. Both the spherical and spheroidal (oblate) bulk models have maximum dimension identical to the aggregate maximum dimension.

Figures 5 and 6 show backscattering cross-sections for GMM aggregates. Each size-mass bin contains 10 random aggregate realizations (shown as dots in the figures). The plots also show comparisons with three different bulk models: the oblate spheroid and sphere models were discussed previously (Figure 4), while the model identified as "rms sphere" uses the effective diameter defined by Petty and Huang

(2010). As expected, all the bulk models show resonance effects for particle sizes bigger than half the wavelength (about 4 mm at Ka and 1.6 mm at W band). The resonances are typical of spherical and spheroidal shapes and should be considered an artifact due to the solid smooth shape of these particles. In this particular set of computations, the resonances have a bigger impact on the spherical models than on the spheroidal one. This is due to the fact that the oblate model better represents the original aggregate thickness, which mostly determines the backscattering cross-section at vertical incidence. Finally, the large variability in the bulk model results (e.g., different models differ by up to 20 dB for 8 mm size particles at Ka band and up to 25 dB for 4 mm size particles at W band) show the biggest limitation in the bulk model approach: even when the target to be approximated is known (e.g., the random aggregate realizations), the bulk models have several degrees of freedom that can produce virtually any result if properly tuned.

4. CONCLUSIONS

Scattering models of complex ice hydrometeors must be given careful consideration to produce meaningful results. Electromagnetic scattering techniques such as DDA or FDTD allow for the computations of scattering properties from complex shaped particles, but their applicability is limited to relatively small sizes due to computational issues. On the other hand, the highly simplified approach of the bulk models ends up having too many “knobs” to turn and can erroneous results.

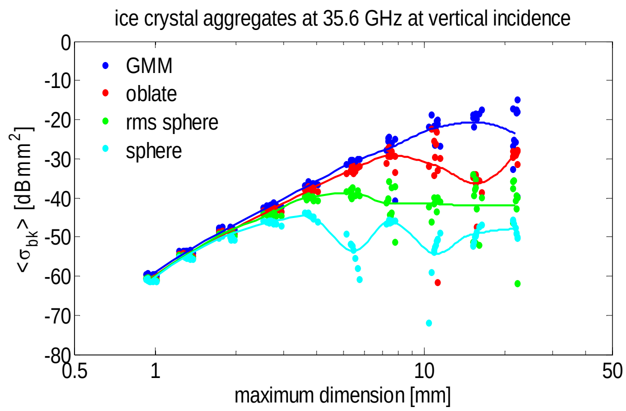

Figure 5: Backscattering cross-sections for aggregates at Ka band (35.6 GHz). The aggregates used for these results are similar to the ones developed by Botta et al., 2011.

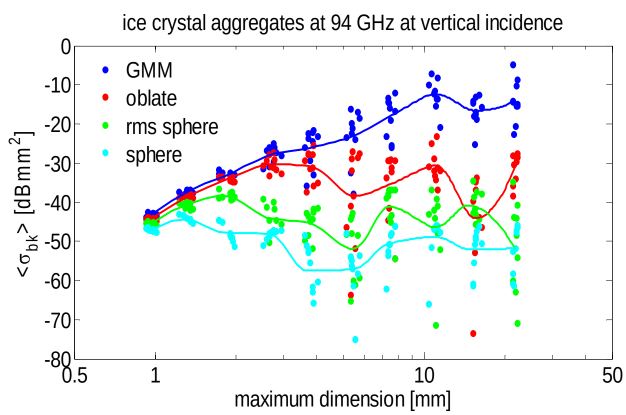

Figure 6: Same as figure 5 but for W band (94 GHz).

Computation of electromagnetic scattering from ice particles modeled as clusters of spheres and using the GMM method appears to be a versatile and viable approach. Further development and validation of the new technique are necessary to evaluate the impact of the many parameters involved in the model definition.

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