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1. OVERVIEW

In this work we explore the existence and potential application of non-Rayleigh signal statistics to studies of rain. (Details not presented here may be found in Jameson 2008). Drop clustering and statistical heterogeneity are intrinsic properties of most rain. These, in turn, lead to variability in the intrinsic radar reflectivity factor, Z , and rainfall rate, R . For a radar having a stationary antenna, this intrinsic variability is invisible because the backscattered radar signals are described completely by the fluctuations prescribed by Rayleigh statistics; that is, the measured amplitudes are simply the net result of random complex amplitudes from all the illuminated scatters. However, once the antenna begins moving, this is no longer true because the variability in the intrinsic Z augments the Rayleigh signal fluctuations as discussed in Jameson and Kostinski (1996). This means, then, that, in principle, we can use these augmentations to probe the variability of the intrinsic Z itself, i.e., we can use deviations from Rayleigh signal fluctuations to explore the variability of the rain occurring over scales smaller than a beam dimension.

Obviously, radars usually measure a mean backscattered intensity, I , related to the intrinsic mean Z through the radar constant. But the mean intensity alone is insufficient for quantifying non-Rayleigh signal fluctuations which act to broaden the pdf of I beyond that expected from normal Rayleigh fluctuations. Measurement of the higher moments of the pdf of I are, therefore, required. Since the difficulty (i.e., the number of independent samples required for statistically meaningful estimates) increases rapidly as the power of the moment increases, we focus here on using the variance of the pdf of I . (There are other important physical reasons as well for using the variance.) Specifically, the square of the relative dispersion, $\sigma^2(I)/\bar{I}^2 = \sigma^2(Z)/\bar{Z}^2$ is considered.

It is shown that in conditions of raindrop clustering and statistical heterogeneity, we can measure the intrinsic relative dispersion of Z . It is also demonstrated that when there is no clustering the drop size distributions do play the dominant important role in determining the intrinsic relative dispersion of Z . (However, the relative dispersions are then all about zero anyway.) But when there is clustering the variability caused by drop size distributions are not

important at least over the dimensions of most radar volumes. The square of the intrinsic relative dispersion of Z is then the square of the raindrop clustering index (essentially $\sigma^2(n)/\bar{n}^2$ when \bar{n} is large) appropriately weighted by the square of the fractional contribution each component of the heterogeneous rain makes to the mean overall reflectivity factor. That is,

$$\frac{\sigma^2(Z)}{E^2(Z)} \approx \sum_i \left(\frac{\sigma_i(n)}{E_i(n)} \right)^2 \times \left(\frac{w_i E_i(Z)}{\langle E(Z) \rangle} \right)^2 \quad (1)$$

where the summation is over all of the statistically homogeneous components of the statistically heterogeneous rain (see Jameson, 2007), E is the expectation value and w are the weights corresponding to each component.

The statistics of the square of the relative dispersion are presented and confidence limits are derived numerically, for want of a closed-form expression, as a function of the number of independent samples. These confidence limits are necessary in order to exclude normal Rayleigh amplitude fluctuations. It is found that when there are only a relatively few statistically independent samples, most values of the square of relative dispersion of I (and Z) are less than unity. Moreover, as the number of independent samples increases, the normal Rayleigh fluctuations are confined to an ever increasingly narrow region about unity thus enhancing the detectability of non-Rayleigh signal fluctuations.

Some preliminary observations in rain are presented. Pulse to pulse time-series I and Q observations over several range bins and azimuths of data were analyzed by computing the intensities and then by combining observations both in range and azimuth. The number of independent samples, and the 99% confidence limits are calculated.

Measurable and significant dispersions of Z and, therefore, R were found (see Figs. 1 and 2). These observations suggest that often just where the rainfall is likely to be most intense, the 'average' is also likely to be the most variable and its meteorological interpretation the most ambiguous. One interpretation is that these contours represent an 'uncertainty' in the mean value. This is true, but one must understand that this 'uncertainty' in this case is not because of statistical signal fluctuations. Rather, it is a reflection of the intrinsic uncertainty and, thus, ambiguity in the very meaning of an average value at those locations because of sub-beam scale variability.

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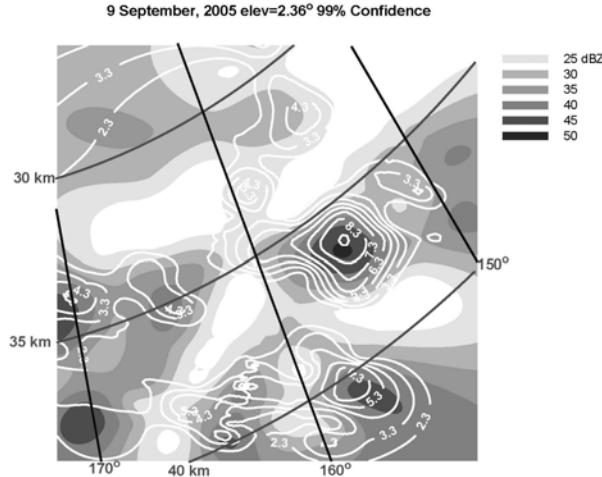


FIG. 1: Shaded contour plots of the radar reflectivity factor, Z , in units of dBZ with overlays of contours of the squares of the relative dispersions of Z at the 99% confidence level as functions of the radar range and azimuth.

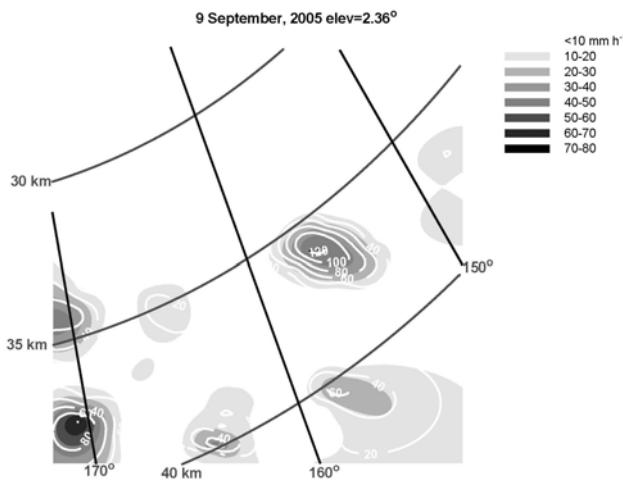


FIG. 2: Range-azimuth shaded contour plots of the mean rainfall rate, \bar{R} , estimated from the mean radar reflectivity factor, Z , using the Sekhon-Srivastava (1971) Z - R relation. Line contours of the standard deviation, $\sigma_{\bar{R}}$, are plotted as well. These latter quantities were calculated using the square of the relative dispersion of Z .

The second and alternative interpretation is that the standard deviation means that a wide range of mean values are occurring simultaneously so that no one average value applies uniformly to the entire domain. After a moments consideration this is, perhaps, not a big surprise. However, in almost all studies radar rainfall measurements, the mean value is taken to be uniformly applicable to the entire radar volume, and it certainly reveals the inherent difficulty in any comparison of, say, a rain gage measurement to that estimated using a radar.

Perhaps more sobering, hidden biases and errors in the deduced mean value are possible, if not likely, where $\sigma_{\bar{R}}$ is most significant since we have no idea about the actual pdf of R at those locations. A large $\sigma_{\bar{R}}$ might suggest, for example, that the radar could be observing one value, say, more toward the tail of a skewed distribution of \bar{R} , while, in reality, a watershed may be experiencing a different \bar{R} more akin to the modal value. At the very least, investigators should be aware of this ambiguity and perhaps should even begin bounding their estimates by assigning standard deviations arising from raindrop clustering weighted by the statistical heterogeneity.

Nevertheless, these results suggest that routine observations of the relative dispersion of \bar{R} in excess of those arising from Rayleigh fluctuations could be quantitatively useful and could be readily achieved even using existing meteorological radars with the processing capability. Clearly from this analysis, the detectability and reliability of non-Rayleigh signal fluctuations can be improved the more statistically independent radar pulses one has available. Moreover, a greater number of statistically independent pulses increases the spatial resolution to reveal ever finer structure. Thus, a highly desirable improvement to future observations and applications of non-Rayleigh signal fluctuations is to increase the number of available statistically independent pulses using whitening, pulse encoding, chirping or any other accessible technique.

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2. REFERENCES

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