# 17C.1 THE ROLE OF INERTIAL STABILITY IN THE RAPID DEVELOPMENT OF THE TROPICAL CYCLONE WARM CORE

Jonathan L. Vigh\*and Wayne H. Schubert Colorado State University, Fort Collins, Colorado

## 1. INTRODUCTION

This extended abstract provides selected highlights of a study which seeks to isolate the conditions under which the tropical cyclone warm core can rapidly develop. While the main work will soon be submitted for publication (Vigh and Schubert 2008), this abstract contains some supplementary color figures which will not appear in the published version of this work.

## 2. THEORETICAL ARGUMENT

The theoretical argument is based on the balanced vortex model and, in particular, on the associated transverse circulation equation and the geopotential tendency equation. These are second order partial differential equations containing the diabatic forcing and three spatially varying coefficients: the static stability A, the baroclinity B, and the inertial stability C. Thus, the transverse circulation and the temperature tendency in a tropical vortex depend not only on the diabatic forcing, but also on the spatial distribution of A, B, and C. Experience shows that the large radial variations of C are typically the most important effect. Under certain simplifying assumptions on the vertical structure of the diabatic forcing and on the spatial variability of A, B, and C, the transverse circulation equation and the geopotential tendency equation can be solved via separation of variables. The resulting radial structure equations retain the dynamically-important radial variation of C and can be solved in terms of Green's functions. These analytical solutions illustrate how the vortex response to a delta function in the diabatic heating depends on whether this heating lies in the low inertial stability region outside the radius of maximum wind or in the high inertial stability region inside the radius of maximum wind. The results suggest that rapid intensification is not possible for storms in which the diabatic heating has become completely locked out of the region inside the radius of maximum wind. Thus, rapid intensification is favored for storms in which at least some of the eyewall convection occurs inside the radius of maximum wind.

## 3. RESULTS FROM ANALYTIC MODEL



Figure 1: Isolines of  $r\psi$  and temperature tendency  $T_t$ in the (r, z)-plane for the resting atmosphere case. The radial axis is labeled in km and the vertical axis in the dimensionless vertical coordinate  $z/z_T$ . The radius of diabatic heating is  $r_h = 25$  km (as indicated by the vertical dashed line).

Fig. 1 shows contours of  $r\psi$  and  $T_t$  in the (r, z)plane for the case of a resting atmosphere with a Dirac delta function in diabatic heating placed at r = 25 km. Approximately 99.88% of the upward mass flux is compensated by downward mass flux outside r = 25 km and only 0.12% is compensated by downward mass flux inside r = 25 km. As can be seen in the right panel, there is very little variation of the temperature tendency on a fixed isobaric surface. In other words, the Dirac delta function in the diabatic heating leads to a transverse circulation that raises the temperature on a given isobaric surface nearly uniformly over a large area. The production of very weak horizontal temperature gradients and corresponding weak vertical shears of the azimuthal wind is consistent with the well-known

<sup>\*</sup>Corresponding author address: Jonathan Vigh, Department of Atmospheric Science, Colorado State University, Fort Collins, CO 80523-1375; e-mail: vigh@atmos.colostate.edu

result that diabatic heating on a horizontal scale smaller than the Rossby length is a very inefficient way to produce rotational flow.



Figure 2: Isolines of  $r\psi(r, z)$  and temperature tendency  $T_t(r, z)$  for the four Rankine-like vortices shown in the left column. The radius of maximum wind is  $r_c = 20$  km and the radius of diabatic heating is  $r_h = 25$  km (as indicated by the vertical dashed line).

Next consider the case with in which the core of high inertial stability extends to  $r_c = 20$  km, while the Dirac delta function of heating remains at  $r_h = 25$  km. This is typical of cases in which the diabatic heating lies outside the radius of maximum wind. The second and third columns of Fig. 2 show isolines of  $r\psi(r, z)$  and  $T_t(r, z)$  for the four vortices displayed in the left column. These vortices all have a maximum wind at  $r_c = 20$  km, but with  $v(r_c) = 10, 20, 30, 40$  m s<sup>-1</sup>. The main conclusion to be drawn from Fig. 2 is that diabatic heating in the low inertial stability region outside the radius of maximum wind produces a temperature tendency that is nearly uniform horizontally and similar to that found for a resting atmosphere. In other words, diabatic heating outside the radius of maximum wind is very inefficient at producing rotational flow, no matter how small the Rossby length inside the radius of maximum wind.



Figure 3: Same as Fig. 2 except  $r_c = 30$  km, so that the diabatic heating occurs inside the radius of maximum wind. Note the change in isoline intervals from those used in Fig. 2.

Now consider the case where the heating remains at  $r_h = 25$  km but the core of high inertial stability extends to  $r_c = 30$  km, which is typical of cases in which the diabatic heating lies inside the radius of maximum wind. In the second and third columns of Fig. 3 we show isolines of  $r\psi(r,z)$  and  $T_t(r,z)$  for the four vortices displayed in the left column. These vortices all have a maximum wind at  $r_c = 30$  km, but with  $v(r_c) = 10, 20, 30, 40$  m s<sup>-1</sup>. When diabatic heating occurs within the high inertial stability region that lies inside the radius of maximum wind, there is enhanced subsidence inside  $r = r_h$  and a tendency to rapidly form a warm core.

The dramatic role which inertial stability plays in controlling warm core development can be further illustrated by Fig. 4 which shows the variation of temperature tendency at the vortex center (r = 0as the location of the diabatic heating varies from  $r_h = 0$  to 40 km. The six panels show the temperature tendency response as the core of high inertial stability varies from  $r_c = 30, 25, 20, 15, 10$ , and 5 km. This progression of vortices can be thought of as illustrative of a tropical cyclone intensifying from a strong tropical storm to a Category 5 hurricane. Note that left ordinate shows maximum wind speed (which doesn't change from panel to panel), while the right ordinate shows the effective Rossby Radius (which decreases to smaller values as the core of high inertial stability shrinks in size from panel to panel). Taking an example from the top left-hand panel, a storm with radius of maximum winds at  $r_c = 30$  km and maximum wind speed of 15 m s<sup>-1</sup> has a Rossby Radius is 47.6 km. From the bottom right-hand panel, a storm with  $r_c = 5$  km and maximum wind speed of 75 m s<sup>-1</sup> has a remarkably small Rossby Radius of 1.7 km. In all panels, the central warming diminishes as the heating is moved further away from the center. Once the heating is removed outside the region of high inertial stability, there is practically no warming of the center.

Fig 5 is similar to Fig 4, except that the temperature tendency is shown at the location of the heating  $(r_h)$  rather than the center (r = 0). Although the plots look quite similar, the warming rates are greater at the location of heating than in the center of the vortex. Over time, this difference leads to the warm-ring structure discussed in Schubert et al. (2007). Fig. 6 displays the difference in temperature tendency at the location of heating vs. the temperature tendency at the center.

# 4. OBSERVATIONS OF INERTIAL STA-BILITY

It is of no mean interest to ask how well the results of the analytic model apply to actual tropical cyclones. Real storms may not be wellapproximated by such a simple vortex representation. The first author is currently developing a set of generalized codes to parse radial profiles from the multitude of raw flight level data available from the Hurricane Research Division's (HRD) web site. One goal is to compute radial profiles of inertial stability at the various stages of a storm's life cycle. Besides the role inertial stability plays in the rapidity of warm core development, the "wall" of high inertial stability may play a key role in the development of the eye wall as well. Another goal will be to compute the warming rates in the core of each storm as it intensifies.

In order to parse the raw data into useful radial legs, the data must first be transformed to stormrelative coordinates. As a start, this has been accomplished using the wind center tracks provided from HRD using the method of Willoughby and Chelmow (1982). Eventually the code should be able to compute optimized wind center tracks from

the raw flight level data. One somewhat novel feature of the current code is that an automated algorithm has been developed to determine the valid radial legs. Some previous workers laboriously parsed the radial legs "by hand", but as Fig. 7 shows, the automated algorithm appears to be quite accurate in discerning between valid legs and all other flight leg segments. In the past, a dataset of radial leg profiles was maintained by Ed Rahn and Hugh Willoughby. Unfortunately, this dataset only extends through 2002 and has not been updated with the numerous storms which have occurred in the years since. We hope to replicate that dataset and supplement it with storms up through the present. The NOAA and USAFR flight level data have a wealth of useful information waiting to be analyzed. Due to the onerous, or perhaps just time-consuming barrier of processing the raw flight level data, these data seem to be have been underutilized thus far.

#### dennis (2005)



Figure 7: Storm-relative trajectories for 21 flights into Hurricane Dennis (2005). "Good" radial legs are highlighted in red, while all other segments are indicated by blue.

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# Variation of T<sub>t</sub>(0) with Heating Location and Rossby Radius

Figure 4: Isolines of central (r = 0) temperature tendency response in six vortices with varying radial extent of the core of high inertial stability,  $r_c$ . Isolines show the temperature tendency response at the vortex center as the heating location varies from  $r_h = 0$  to 40 km.

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Variation of  $T_t(r_h)$  with Heating Location and Rossby Radius

Figure 5: Same as Fig. 4, but for the temperature tendency response at the radius of heating,  $r_h$ .



Figure 6: A difference plot showing the temperature tendency for the radius of heating (Fig.5) minus the temperature tendency at the center (Fig. 4).