# ANALYSIS OF A DUAL-WAVELENGTH RADAR TECHNIQUE FOR ESTIMATING LIQUID WATER CONTENT AND DROPLET SIZE

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### 1. INTRODUCTION

A dual-wavelength radar system comprised of Sand K<sub>a</sub>-bands is well suited for ground-based remote sensing of single and mixed-phase (ice and liquid) clouds. Transmitted radiation at Ka-band is measurably attenuated by liquid water; the range-differentiated difference between the reflectivities is proportional to the amount of liquid. In practice, factors such as non-Rayleigh scattering, the presence of ice crystals, measurement errors and sensitivity of the instruments have confounded evaluations of liquid water content (LWC) retrievals during field trials. In this paper a numerical analysis of a dual-wavelength radar technique for estimating LWC and particle size is presented. Estimation of LWC and the characteristic size of the liquid droplet spectrum in liquid cloud, mixture of liquid and ice cloud, and the influence of measurement error are discussed.

# 2. LIQUID WATER CONTENT AND RADAR ESTIMATED SIZE

Attenuation at  $K_a$ -band is composed of both absorption and scattering losses. For small droplets, the scattering loss is negligible, and hence attenuation is essentially the absorption loss. The absorption cross-section is proportional to the volume (or LWC) of the droplets and depends on the imaginary part of the refractive index of water. At  $-10^{\circ}$ C, the one-way attenuation of  $K_a$ -band radar signal by cloud droplets, A, is related to LWC as

$$LWC = 0.74 A. \tag{1}$$

where LWC is in g m $^{-3}$  and A is in dB km $^{-1}$  (as in Vivekanandan et al. 1999). As ambient temperature decreases (increases), the corresponding attenuation increases (decreases) as described by Gosset and Sauvageot (1992). For example, as the temperature decreases from 0 to  $-30^{\circ}$ C, attenuation increases by 200%, and thus temperature measurement would be a desired feature of a system designed to accurately retrieve LWC. The K $_{a}$ -band attenuation is estimated by comparing reflectivity gate-by-gate with that from the colocated S-band radar.

Using dual-wavelength radar measurements of reflectivity and attenuation, it is difficult to estimate more

than two independent parameters of the distribution without considerable application of assumptions about the cloud that may not always be appropriate. Since reflectivity and attenuation are available with the dual-wavelength system, it is desirable to choose a size characterization that takes advantage of these. Defining a characteristic size that can be directly retrieved should be a direct function of radar measurables such as attenuation and reflectivity (Vivekanandan et al. 2001).

For this wavelength pair, the Rayleigh scattering approximation is valid for droplets with diameters <~1 mm. If accurate reflectivity and attenuation measurements are available, their ratio (Z/A) may be used to define a new size characteristic, the radar estimated size (RES):

$$RES = \left\langle \left\langle \frac{D^6}{\sqrt{D^3}} \right\rangle \right\rangle \left\langle \frac{D^7}{\sqrt{D^3}} \right\rangle$$
 (2)

RES can be written using dual-wavelength radar measurements as

$$RES = \left(7.12 \times 10^{-4} \frac{z}{A}\right)^{1/3} \tag{3}$$

where *RES* is in mm, *Z* is in mm<sup>6</sup> m<sup>-3</sup> and *A* is in dB km<sup>-1</sup>. In a size distribution with both small and large particles (i.e. broad spectrum) RES value is primarily biased towards large particle size. Thus in a mixed-phase cloud with a few large ice particles, RES might not characterize the size distribution well. In general, for a liquid cloud with maximum particle size smaller than the radar wavelength, RES is greater than or equal to median volume diameter.

# 3. MODELING OF DUAL-WAVELENGTH RADAR OBSERVATIONS

The key component for modeling range-dependent dual-wavelength reflectivities is the droplet spectrum. The study uses actual spectra measured from the NASA Twin Otter research aircraft in the Ohio Valley for modeling dual-wavelength radar reflectivity range profiles. The measured spectra included a wide range of conditions: cloud droplets (maximum diameter  $\leq$  50  $\mu$  m), drizzle (20 - 500  $\mu$  m) and rain (> 500  $\mu$  m), with LWC of 0.01 - 0.5 g m $^3$ . The flight data were divided into 45 sec (  $\sim$  3 km) segments, and segments with ice crystals present (determined by visual inspection of optical imaging probe data) were removed from the data set. For each of the resulting droplet spectra LWC and RES were calculated. A model

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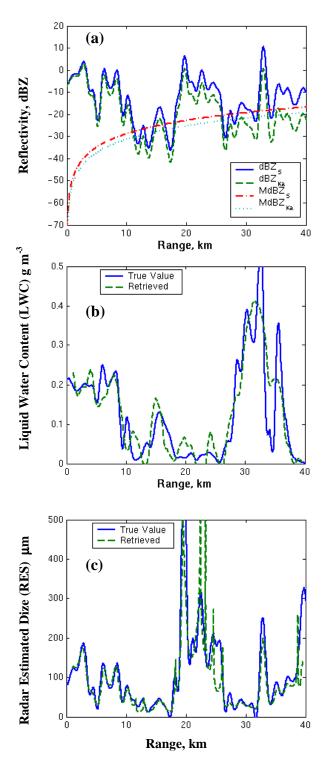


Figure 1: (a) Simulated range profiles of S- and  $K_a$ -band radar reflectivity for liquid cloud drop smaller than 0.5 mm . Minimum detectable reflectivity profiles for standard S- and  $K_a$  —band radars are also shown. (b) Comparison between retrieved and true LWC. (c) Comparison between retrieved and true RES.

range profile of LWC and RES and the corresponding calculated radar reflectivities at X- and  $K_a\text{-}band$  are shown in Figure 1. There are 600 range gates within a 40 km range interval and the width of each range gate is 66.7 m. The particle sizes are small compared to wavelength and hence intrinsic reflectivity at S and  $K_a\text{-}band$  is the same. A 1 dB random radar measurement error was introduced in the reflectivity simulation and a moving average of 10 gates was used to remove gate-to-gate fluctuation in reflectivity.  $K_a\text{-}band$  reflectivity is smaller than S-band reflectivity at farther ranges as a result of cumulative attenuation along the radar beam due to liquid water. A linear least squares fitting of reflectivity difference (S minus K-band) over 40 gates was used for estimating attenuation.

As described in the previous section attenuation and reflectivity were used for retrieving LWC and RES. Comparison between the estimated and actual values of LWC and RES are shown in Figure 1a and b. Radarbased estimates exhibit less spatial structure compared to the actual value but they agree well overall.

In the case of mixed-phase cloud such as liquid droplet and ice particles small compared to wavelength, total reflectivity is usually dominated by reflectivity of ice. Generally reflectivity due to small droplets is an order of magnitude smaller than the reflectivity due to ice because ice particle size is larger than liquid droplets. However, reflectivity difference between S and K<sub>a</sub> band is directly proportional to attenuation by liquid along the radar beam. Rayleigh reflectivity of ice is the same at S and Ka band and is eliminated in the reflectivity difference. Thus, LWC can be estimated in a mixed phase cloud with small ice particles as in the case of single-phase liquid cloud. The presence of larger ice particles effectively extends the range sensitivity of the radar. Since the reflectivity at S-band is the sum of ice and liquid droplet reflectivity, it is difficult to estimate RES in mixed-phase cloud.

Estimating LWC in the case of Mie scattering is limited. As the particle size become comparable to Kaband wavelength, the S-band reflectivity and Ka-band reflectivity difference is dominated by the difference in backscattering cross sections rather than by absorption. Figure 2a shows a range profile of reflectivity for a mixed phase cloud containing both small cloud droplets and Mie scattering ice particles. The Mie scattering is simulated using a Gaussian random process. A mean value of 16 dBZ, correlation length of 3 km and standard error of 8 dB are used for Mie reflectivity simulation at Sband. The intrinsic Mie reflectivity due to large ice particles and reflectivity due to cloud droplet are summed for obtaining total reflectivity at S-band. Kaband Mie reflectivity is derived by scaling the S-band ice reflectivity (85 % of ice reflectivity at S-band), summing the reflectivity due to cloud droplet and then subtracting appropriate two-way attenuation due to LWC. The Mie scattering signal was added at three range intervals: 4 and 8; 17 and 22; and 25 and 29 km. The reflectivity difference is shown in Figure 2b. The regions with Mie scattering show local enhancement in reflectivity difference. A direct curve fitting of reflectivity difference

might lead to a false estimate of attenuation and hence LWC. Recognizing the presence of a Mie scattering signal by the local maximum in reflectivity difference, local minima are joined as the possible reflectivity difference due to LWC (Tuttle and Reinhart 1982). The slope of the line joining the local minima is proportional

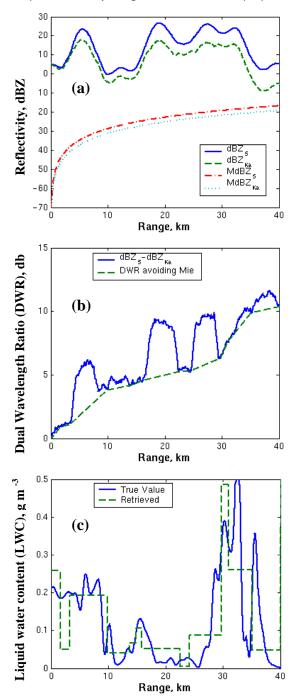


Figure 2: (a) Simulated range profiles of S- and K<sub>a</sub>-band radar reflectivity for mixed phase cloud with Mie scattering ice particles. (b) Reflectivity difference between S and K<sub>a</sub>-band reflectivity. (c) Comparison between retrieved and true LWC.

to attenuation and thus LWC. Figure 2c shows the comparison between true LWC and retrieved LWC from radar measurements. Only a mean LWC is derived in the regions with a Mie scattering signal. As in any mixed phase region, retrieval of RES is difficult because reflectivity of LWC from small droplets is not well measured.

#### 4. DISCUSSION

Analysis of the simulation results suggests in the case of cloud and drizzle conditions, dual-wavelength radar observations are capable of retrieving LWC and RES. Mixed-phase radar measurements were simulated using a one-dimensional Gaussian distribution with a specified mean, standard deviation and correlation length for ice reflectivity. Small ice crystals (<~1 mm diameter) do not affect the LWC retrieval but the RES estimate is biased upward by their contribution to total reflectivity. In the case of non-Rayleigh scattering from larger ice crystals or raindrops, the difference in reflectivity between the two wavelengths is no longer monotonically increasing as is the case for pure Rayleigh scattering; in these cases, the local minimum reflectivity differences were used to estimate attenuation. As a result, spatial resolution of the LWC estimate is compromised in the mixed-phase regions. The effect of radar measurement error on attenuation estimation was also investigated using various range-averaging lengths but the results were not included in this paper. Based on these analyses, an optimum design and data processing scheme for a dualwavelength system will be investigated.

## 5. ACKNOWLEDGMENTS

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