11B.6 Precipitation Canting Angle Distribution Estimation from Covariance Matrix Analysis of CSU-CHILL Radar Data

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1. INTRODUCTION

It is only recently that full polarimetric covariance matrix measurement capability has been implemented on research weather radars, in particular on the CSU-CHILL (Brunkow et al. 2000) radar which is the focus of this paper. A two-transmitter/two receiver system is used on the CSU-CHILL to measure the three real and three complex correlation terms of the covariance matrix in the H/V basis. The covariance matrix must be calibrated, in particular, for the differential gain and phase offsets between the two receivers; details of these and related calibration and polarization error issues may be found in Hubbert and Bringi (2001).

While it is known that raindrops form a highly oriented medium (Beard and Jameson 1983), rain models used in polarimetric rain rate algorithms often assume that the mean canting angle is 0^o with standard deviation in the range $0-10^o$. While the mean canting angle is expected to be very close to 0^o, the standard deviation could vary with drop size, turbulence, etc. For example, the K_{dp} in the presence of a canting angle distribution with σ_β is easily related to the K_{dp} for an equi-oriented medium as (Bringi and Chandrasekar, 2001),

$$K_{dp} = (K_{dp})_o exp(-2\sigma_\beta^2) \quad (1)$$

where $(K_{dp})_o$ is the specific differential phase for the equi-oriented case. Similarly at long-wavelengths, the LDR is dependent on σ_β^2 via (Bringi and Chandrasekar, 2001),

$$L \approx \frac{1}{4}\left[1 - exp(-8\sigma_\beta^2)\right]\left[(1-\bar{r}_Z)^2 + var(r_Z)\right] \quad (2)$$

where L is the linear depolarization ratio (in linear scale), $\bar{r}_Z$ is the reflectivity-weighted mean axis ratio and $var(r_Z)$ is the variance of axis ratios. It is clear from (2) that LDR is a product of both "orientation" effects as well as "shape" effects and separation of the two is not possible using LDR data alone. In addition, if there is a mean canting angle, then the expression for LDR is more complicated as compared with (2).

With recent measurement of the full covariance matrix, it is now possible to use the theory of optimal polarizations for estimating separately the mean and variance of the canting angle distributions without the complication produced by "shape" effects. In addition, once the covariance matrix is measured in the H/V basis, it can be transformed to other bases (e.g. circular). While direct measurement in the circular basis has some advantages, the estimation of the mean and variance of the canting angle was only previously accomplished by the "slow" method of rotating the linear polarization basis (Hendry et al. 1987). The method use here is based on the theoretical formulation of Tragl et al. (1991) in combination with the ideas of Hendry et al. (1987). The covariance matrix in the H/V-basis ($\mathbf{\Sigma_o}$) can be transformed to any orthogonal basis by using the unitary transformation matrix $\mathbf{T}$ as,

$$\mathbf{\Sigma}(\rho) = \mathbf{T}(\rho)\mathbf{\Sigma_o}\mathbf{T}^{t*}(\rho) \quad (3)$$

where ρ is the complex polarization ratio. The transformed matrix $\mathbf{\Sigma}(\rho)$ can be expressed as,

$$\mathbf{\Sigma}(\rho) = \begin{bmatrix} P_{co}^A(\rho) & \sqrt{2}R_x^A(\rho) & R_{co}(\rho) \\ - & 2P_x(\rho) & \sqrt{2}R_x^B(\rho) \\ - & - & P_{co}^B(\rho) \end{bmatrix} \quad (4)$$

The real elements P_{co}^A and P_{co}^B along the diagonal are the copolar backscattered powers while P_x is the cross-polar power (e.g. if ρ corresponds to horizontal polarization, then $P_{co}^A \equiv P_{hh}$, $P_{co}^B \equiv P_{vv}$ and $P_x \equiv P_{hv}$ or P_{vh}). Also, the three complex covariances would correspond to $R_x^A \equiv R_{hh,vh}$, $R_{co} \equiv R_{hh,vv}$ and $R_x^B \equiv R_{vv,hv}$. The optimal cross-polar power states (i.e., the transmitted polarization state ρ that results in a minimum in the cross-polar return power) is given by the condition,

$$\frac{\partial P_x(\rho)}{\partial \rho^*} = \frac{1}{(1+\rho\rho^*)}\{[R_x^B(\rho)]^* - R_x^A(\rho)\} = 0 \quad (5)$$

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While the above may be solved using standard numerical methods, Tragl et al. (1991) have developed an elegant formulation which reduces the solution of (5) to a known extremization problem involving real linear algebra. In brief, the cross-polar power function, $P_x(\rho)$ in (4), can be expressed as (in mixed vector/matrix notation),

$$P_x = \frac{1}{2}\vec{v}^t \cdot \mathbf{A_o} \cdot \vec{v}; \quad \vec{v}^t \cdot \vec{v} = 1 \tag{6}$$

where the matrix $\mathbf{A_o}$ is obtained as,

$$\mathbf{A_o} = \mathbf{Q}\mathbf{\Sigma_o}\mathbf{Q}^{t*} \tag{7}$$

with $\mathbf{Q}$ a known unitary matrix. The solutions $\vec{v}$ which extremize the quadratic form in (6) are given by,

$$Re[\mathbf{A_o}] \cdot \vec{v} = \lambda\vec{v} \tag{8}$$

with λ the eigenvalues of the matrix $Re[\mathbf{A_o}]$. It can be shown that there exists three solutions that extremize the cross-polar power function which may be analytically determined once $\mathbf{\Sigma_o}$ in the (H/V)-basis is known (i.e., via measurements). The three solutions correspond to a maximum, a minimum and a saddle point of P_{cx}. The tilt angle of the minimum solution is taken as the mean orientation angle of the medium ($\bar{\beta}$). Once $\bar{\beta}$ is known, the $\mathbf{\Sigma_o}$ matrix is transformed using (3) to a linear polarization-basis whose tilt angle is 45^o away from it. The difference in LDR's in these two bases (i.e., the optimal basis and the one 45^o away) is known to be a function of the variance in canting angle only (Hendry et al. 1987) and independent of any "shape" effects. Hubbert and Bringi (1996) show, via simulations of raindrop size distributions, that this LDR difference (comparable to the difference in maximum cross polar power to minimum cross polar power in the rotating linear basis data of Hendry et al. 1987) is directly related to the standard deviation (σ_β) and nearly independent of the axis ratio distribution. Their model results are used to infer σ_β from the computed LDR difference.

2. DATA ANALYSIS

The data reported herein were collected with the CSU-CHILL radar during STEPS (Severe Thunderstorm Electrification and Precipitation Study) conducted in eastern Colorado during the summer of 2000. Covariance matrix data were available at each resolution volume during a heavy rain event on 11 June 2001. Fig. 1 shows Z_h and Z_{dr} versus height (agl) through the core of the storm cell using RHI scan data. Fig. 2 shows corresponding vertical profiles of the mean canting angle ($\bar{\beta}$) and the co-to-cross correlation coefficient ($\rho_{hh,vh}$). Below 2 km, the $\bar{\beta}$ is very close to 0^o while from 2-6 km it increases from 0^o to around 6^o on average. This increase appears correlated with a small increase in $\rho_{hh,vh}$. It is known that $\rho_{hh,vh}$ is related to $sin(2\bar{\beta})$ as a first approximation (see exact relation in Bringi and Hubbert 2001), and thus the inferred increase in $\bar{\beta}$ may be a result of the increase in $\rho_{hh,vh}$ with height. Fig. 3 shows σ_β and K_{dp} with height. Below 2.5 km (agl) the K_{dp} increases nearly linearly with decreasing height in the rain layer below the melting level. Also, σ_β decreases from 25^o (near the melting level) to 10^o near the surface indicating the increasing stability of the raindrop's orientation as the ice particles melt to form oblate, oriented raindrops. Fig. 4 shows a histogram of $\bar{\beta}$ from PPI scans through the core of the storm cell at low elevation angles (well below the melting level and in the rain layer). The mode is close to 0.5^o with extremes of the histogram less than 8^o in magnitude. We believe that the mode of $\bar{\beta}$ at 0.5^o is most probably related to a system offset, i.e., the transmitted H/V basis is rotated by 0.5^o (see Hubbert and Bringi 2001). The histogram of σ_β is shown in Fig. 5 with mode near 10^o, generally validating the assumption used in several rain models for deriving polarimetric rain rate algorithms. Finally Fig. 6 shows σ_β versus Z_{dr} as a scatterplot (each data point is from one range resolution volume). The decrease of σ_β with Z_{dr} can be related to the increased stability of large raindrops versus small ones; large raindrops are formed by melting of large ice particles and the presence of a melting ice core tends to stabilize the orientation of the particle. We believe these are the first data to show the decrease of σ_β with increasing Z_{dr} in convective rain, and such a behavior may need to be included in rain models used for deriving polarimetric-based rain rate algorithms.

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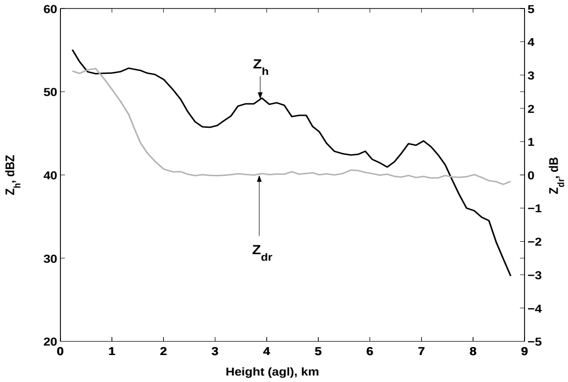

Figure 1: Vertical profile of Z_h and Z_{dr}.

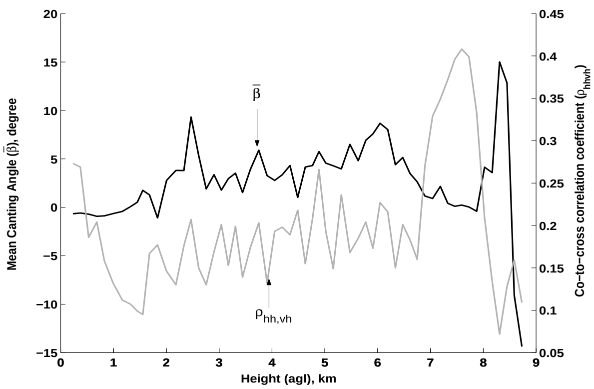

Figure 2: Vertical profile of $\bar{\beta}$ and $\rho_{hh,vh}$.

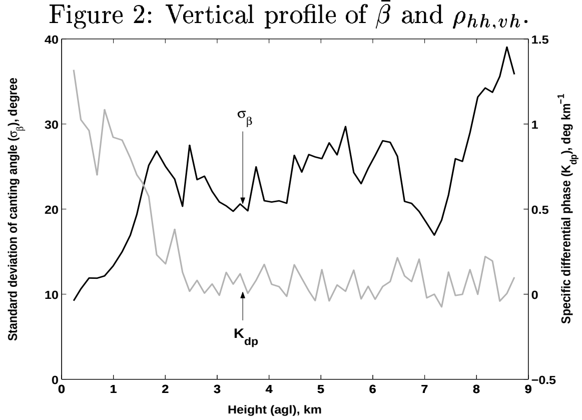

Figure 3: Vertical profile of σ_β and K_{dp}.

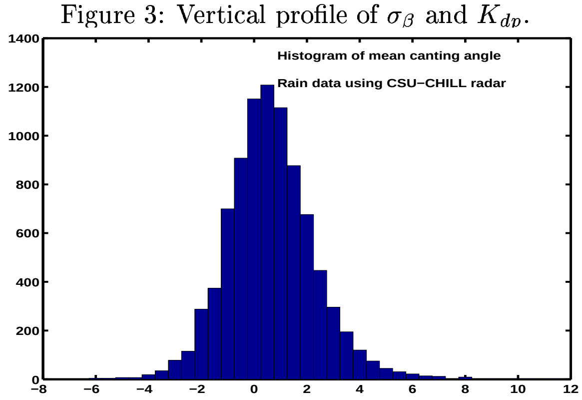

Figure 4: Histogram of $\bar{\beta}$.

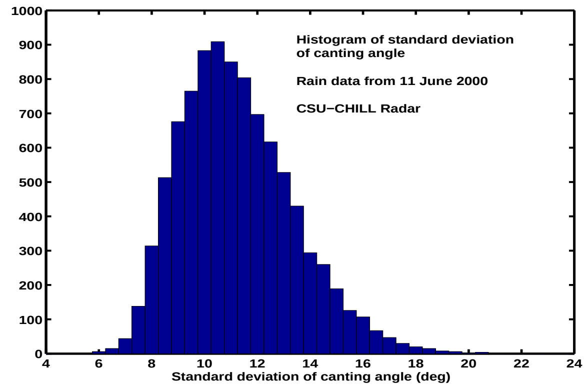

Figure 5: Histogram of σ_β.

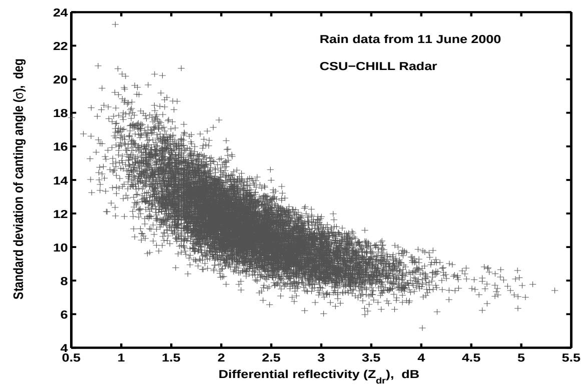

Figure 6: Scatterplot of σ_β vs. Z_{dr}.

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