

## DEMONSTRATION OF OPTIMUM PROCESSING OF OVERSAMPLED SIGNALS IN RANGE TO IMPROVE DOPPLER SPECTRAL MOMENT ESTIMATES

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### 1. INTRODUCTION

Rapid acquisition of volumetric radar data has significant scientific and practical ramifications. For example, observations at minute intervals are required to understand the details of vortex formation and demise near the ground. Even faster rates of volumetric data are needed to determine the presence of transverse winds (Shapiro et al. 2001). Fast update rates would also yield more timely warnings of impending severe weather phenomena such as tornadoes and strong winds.

Surveillance weather radars have a mechanical control of beam position and dwell a relatively long time (~ 50 ms) to obtain sufficient number of independent echo samples for accurate estimates of Doppler spectral moments. Thus, the volume update times are dictated by two limitations: (1) the inertia of the mechanically steered antenna, and (2) the correlation time of weather signals. Phased array radars promise to increase the speed of volume coverage. Reports about simultaneous use of phased array radar for tactical application and weather observation indicate that a radial of data can be obtained from only two transmitted pulses as opposed to over 40 on the WSR-88D. Closer examination of this success reveals that it is not the rapid beam swinging that allowed such fast updates, rather it is the pulse compression. Pulse compression requires wide transmitter bandwidth that is not available to the weather radar community. Hence the dilemma of how to increase the data acquisition and fully utilize the rapid beam steering afforded on phased array radars. An increase by a factor of two is possible by "beam multiplexing", where the data from different directions are collected in a repetitive time-shared pattern. This, however, falls short of the desired capability.

A very different scheme to process weather radar data has been proposed (Torres and Zrnic 2001, Zrnic and Torres 2001). It capitalizes on the known behavior of weather signals in range. An in-depth analysis of the scheme is documented elsewhere (Torres 2001), whereas herein we present its experimental verification on the NOAA research and development WSR-88D.

### 2. THEORY

Briefly, if the returned echoes are sampled more frequently in range than once per pulse duration, it is possible to reduce the variances of estimates. Simple

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averaging of  $L$  oversampled estimates can reduce the variance by a factor  $L_e$ , called the effective number of independent samples, which is computed from

$$L_e^{-1} = \sum_{l=-(L-1)}^{L-1} \frac{L-|l|}{L^2} \rho(l), \quad (1)$$

where  $\rho(l)$  is the correlation coefficient (Doviak and Zrnic 1993). As example consider a rectangular transmitted pulse and a receiver with infinite bandwidth so that the magnitude of the correlation coefficient of oversampled complex signals is

$$\rho(l) = \left(1 - \frac{|l|}{L}\right). \quad (2)$$

The correlation coefficient for oversampled powers is  $\rho^2(l)$  so that in the limit of very large  $L$  one finds that  $L_e = 2$ . Therefore, a reduction of at most two in variance is possible by straight averaging the oversampled power estimates.

There is a different scheme to process the samples and achieve a significantly smaller variance of estimates. It entails decorrelating the oversampled signals in range and then processing these in the usual manner. That is, average the powers of the uncorrelated (in range) signals to obtain the reflectivity and estimate the autocovariances in sample time and average these to retrieve the mean velocity and spectrum width.

Decorrelating (i.e., whitening) the samples in range is possible if the reflectivity is uniform on a distance of two pulse depths. Then, the correlation coefficient of oversampled signals along range-time is a function of the pulse shape and receiver impulse response (Doviak and Zrnic 1993). It can be measured on data or by passing the transmitted pulse (attenuated) through the receiver and recording the response. Several well known techniques can whiten the samples and in the sequel we follow Torres and Zrnic (2001).

Define a Toeplitz symmetric correlation coefficient matrix  $\mathbf{C}$  and decompose it into a product of a matrix  $\mathbf{H}$  and its transpose

$$\mathbf{C} = \mathbf{H} \mathbf{H}^t \quad (3)$$

The whitening transformation matrix is  $\mathbf{W} = \mathbf{H}^{-1}$ . If the matrix is applied to the range samples  $V(i, n)$ , indexed with  $i$  from 0 to  $L-1$ , and at a fixed range location  $n$ , it produces  $L$  uncorrelated random variables. Then, the  $l$ -th variable is given by

$$X(l, n) = \sum_{j=0}^{L-1} w_{l,j} V(j, n), \quad (4)$$

where  $w_{l,j}$  are the entries of the whitening matrix  $\mathbf{W}$ .

Now there are  $L$  in-phase and quadrature-phase samples per pulse length (because  $V$  is a complex sum of  $I$  and  $Q$  so is  $X$ ). These samples can be processed in a conventional manner to obtain the autocovariances, which can then be averaged to reduce the variance of spectral moments by  $L$ ! It is instructive to compare the variances of estimates obtained by averaging in range autocovariances of oversampled signals with those obtained from averages of whitened signals. The ratio of the two variances is

$$\frac{\text{var}(E_{\text{correlated}})}{\text{var}(E_{\text{whitened}})} = \frac{2L}{L^2 + 1}, \quad (5)$$

where  $E_{\text{correlated}}$  stands for the estimate obtained by averaging  $L$  autocovariances from oversampled signal and  $E_{\text{whitened}}$  is the estimate obtained by averaging  $L$  autocovariances from whitened oversampled signal.

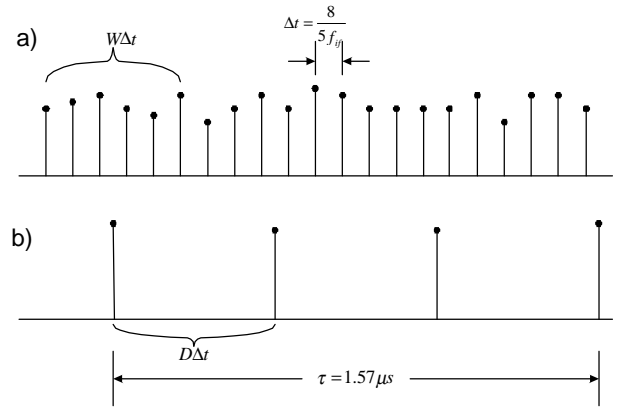
### 3. EXPERIMENTAL SET UP

For validation of the theoretically predicted variance reductions we have used an existing processor passively coupled to the research and development WSR-88D weather radar in Norman, OK. The processor is the RVP7 (from SIGMET); it incorporates a digital receiver at intermediate frequency (IF) and can record a limited amount of time series data. The processor has inherent capability to sample signals at higher rate than the reciprocal of the pulse width. The digital receiver was hooked before the existing matched filter so that the total bandwidth of the IF signal was 10 MHz.

The WSR-88D can easily switch between two pulse lengths, short equal to  $1.57 \mu\text{s}$  (250 m range resolution) and long equal to  $4.7 \mu\text{s}$  (750 m). The scheme was tested on both of these.

Constraints on the RVP7 around which we designed the experiment are as follows. The sampling frequency  $f_s$  of the IF signal is  $5/8 f_{if}$ . After sampling, digital down conversion is combined with an FIR filter. The filter has a variable number of taps  $W$  and the spacing of output digital samples is  $D/f_s$ , which is also variable (Fig. 1). The weights on the FIR filter can be programmed for a desired frequency response; we choose a uniform set. The digital sinusoid for down conversion starts with a same phase every  $D/f_s$  seconds apart and therefore has a variable phase relation (from sample to sample) with respect to the IF signal unless  $D$  is a multiple of five. It is not possible to set  $D$  to an odd number and the minimum even multiple of five for  $D$  is 20. Therefore, to avoid phase discontinuity among range samples, we have chosen  $D = 20$  (corresponding to 83.3 m) and the same value was used for the number of taps  $W$  (Fig. 1). With these settings the number of samples  $L$  within the short pulse is 3 and within the long pulse it is 9.

The processor can record time series data ( $I$  and  $Q$ ) from a limited number of range gates. In the experiment we recorded data from 101 consecutive range locations spaced at 83.3 m; 128 radials of such data constitute one record, and there is a gap of about 1.1 s between records.

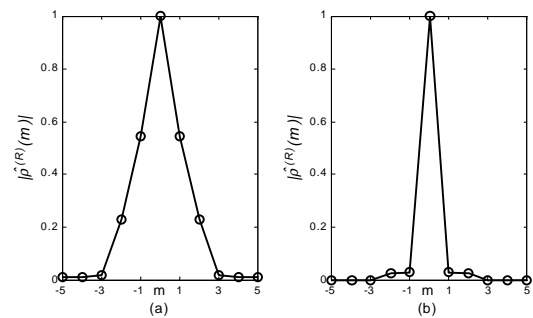


**Figure 1.** Signals in RVP7: (a) samples of IF signal and (b) base band  $I$  (or  $Q$ ) signal.

### 4. MEASUREMENTS AND ANALYSIS OF RESULTS

Several sets of data from weather events and ground clutter were recorded in both the short (regular) and the long pulse mode, and the whitening procedure was applied to these data. First, the correlation coefficient of the oversampled data was computed. Its magnitude (Fig. 2a) has a fairly triangular shape, which is expected from a rectangular pulse; the deviation from straight lines is likely due to the non-vertical leading/trailing edges of the pulse and the effects of the overall radar system filter. Data from all 101 range gates and ten records was used to obtain these graphs.

Correlation coefficient of samples was also computed after application of the whitening transform (4) to the signals that generated Fig. 2a. The result (Fig. 2b) is a noise-like peak at zero lag and insignificant values at other lags. Similar results were obtained with the long pulse.



**Figure 2.** The absolute value of the autocorrelation (a) before and (b) after whitening.

Mean power, mean velocity, and mean spectrum width were computed in the following three ways: (a) from regularly spaced range locations (at a pulse length of 250 m) two consecutive estimates (along range) were averaged; for short termed PS1; (b) by averaging  $L$  (3 or 9) autocovariances of oversampled signals (PS2); (c) by whitening the oversampled data and averaging the autocovariances (PS3). The dwell time to obtain the estimates was  $32 T_s$  ( $T_s$  is the pulse repetition time; that is 32 radials were used).

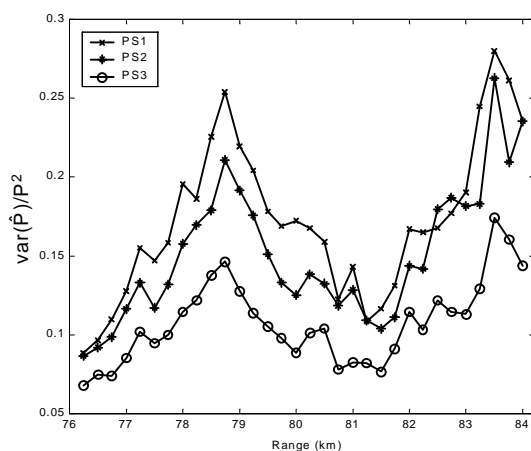
Short term (local) estimates of the mean values of autocovariances and corresponding spectral moments were obtained from 10 consecutive records of data. Variances obtained from local averages were then averaged and compared. Short times were necessary to mitigate the effects of radial changes (in the structure of the spectral moments) due to advection and evolution. Plots indicate radial displacement larger than 1 km in the reflectivity field during the time of 110 s (i.e., 100 records).

The variances of estimates, obtained with the whitening procedure are smallest, and agree with the theoretical prediction (Table 1).

**Table 1.** Variance ratios obtained with the oversampling factor  $L = 3$ .

Variance ratio	Estimated	Theoretical
$\frac{\text{var}(\hat{P}_{PS1})}{\text{var}(\hat{P}_{PS2})}$	1.125561	1.044347
$\frac{\text{var}(\hat{P}_{PS1})}{\text{var}(\hat{P}_{PS3})}$	1.564285	1.496384
$\frac{\text{var}(\hat{P}_{PS2})}{\text{var}(\hat{P}_{PS3})}$	1.438047	1.432806

Fig. 3 illustrates the variances of power as a function of range. The consistently best result from the whitening procedure is evident. Theoretical values of variances at each range location and for each procedure were also obtained (and will be shown at the meeting). To do this we estimated the sample time autocorrelation by fitting a Gaussian shape to the data. From the fitted autocorrelation estimates we have obtained the "theoretical" variances according to the formulas in Doviak and Zrníc (1993). The "theoretical" variances are in remarkable agreement with the measured ones.



**Figure 3.** Normalized variances of power estimates.

## 5. CONCLUSIONS

Definite demonstration of a novel procedure to process weather radar signals has been made. The validation was accomplished on actual time series data recorded on the NOAA research and development radar. A commercial processor capable to sample radar signals several times per pulse duration was utilized. Processing consists of decorrelating the oversampled signals in range, then applying standard procedures to the whitened samples. Variance reduction equal to the number of oversampled points was achieved as predicted by theory. Implication of this conceptual proof could be far reaching. The method has no requirements for transmitter bandwidth other than the usual ones for pulse Doppler radars and it allows traditional processing (i.e., matched filter) which is advantageous at low signal-to-noise ratios (SNR).

## 6. REFERENCES

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