9.14 MONIN-OBUKHOV SIMILARITY FUNCTIONS OF THE STRUCTURE PARAMETER OF TEMPERATURE AND TKE DISSIPATION RATE IN THE STABLE BOUNDARY

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1. INTRODUCTION

Point source scintillometers have proven to be a good alternative method to obtain fluxes of heat and momentum in the stable boundary layer (SBL) (De Bruin et al., 2002 and Hartogensis et al, 2002). The main advantage over the traditional eddy-covariance method is that turbulent fluxes can be obtained over short averaging intervals (~1 minute and less) and close to the surface (less than 1 m), which are necessary conditions for measuring the often non-stationary and shallow SBL.

The basic turbulent variables measured with scintillometers are the dissipation rate of turbulence kinetic energy, e, and the structure parameter of temperature C_T^2. To determine the turbulent fluxes, traditionally use is made of the universal relationships between the dimensionless e, and C_T^2 and the dimensionless height z = z/L, where z denotes height and L the Monin-Obukhov length. These functions exist by virtue of the Monin-Obukhov similarity theory (MOST).

Little e and C_T^2 data have been presented in the literature for z > 1. It is the objective of this study to present experimental e and C_T^2 data for a stability range 0 < z < ~10, from which we will derive new MOST functions. These data were gathered in the context of the CASES-99 project (Poulos et al., 2002). We will compare our findings with previously reported MOST functions for e and C_T^2.

We will also introduce a direct scaling approach that circumvents the iteration procedure needed to calculate fluxes using MOST.

2. THEORY

2.1 Monin-Obukhov scaling of e and C_T^2

According to MOST the following relations define the scaling relationships of e and C_T^2 in the atmospheric surface layer:

$$\frac{kze}{u_*^3} = f_e(z) \qquad (1)$$

and

$$\frac{C_T{}^2 z^{\frac{2}{3}}}{q_*^2} = f_T(z), \qquad (2)$$

where z is the measurement height, k the von Kármán constant (here taken as 0.4), q_* the temperature scale, u_* the friction velocity, z = z/L is a dimensionless height parameter with L is the Monin-Obukhov length and f_e and f_T are universal MOST functions. Here, we will confine ourselves to stable conditions, i.e. L > 0.

In this study we will consider for f_e.;

$$f_e = \left[1 + 2.3(z)^{0.6}\right]^{\frac{3}{2}} \qquad (3a)$$

proposed by Wyngaard and Cote (1971) and adapted by Andreas (1989) to account for k = 0.4 instead of 0.35;

$$f_e = \left[1 + 4z + 16(z)^2\right]^{\frac{1}{2}} \qquad (3b)$$

proposed by Thiermann and Grassl (1992) and;

$$f_e = c_{e1} + c_{e2} z \qquad (3c)$$

proposed by Wyngaard (1973). Several authors used Eq. (3c) with different values for the constants c_{e1} and c_{e2}; Wyngaard (1973) gave c_{e1} = 1 and c_{e2} = 5 and recently, Pahlow et al. (2001) obtained c_{e1} = 0.61 and c_{e2} = 5. From Frenzen and Vogel (2001) we used a f_e formulation which they fitted to their data-set using an expression linking f_e. to the non-dimensional wind shear and arrived at:

$$f_e = 0.85 + 4.26z + 2.58z^2 \qquad (3d)$$

Note that for $c_{e1} \neq 1$, there is no balance between dissipation and production rates of TKE at neutral conditions.

For f_T we will consider:

$$f_T = c_{T1}\left[1 + c_{T2} z^{\frac{2}{3}}\right] \qquad (4a)$$

after Wyngaard et al. (1971) with c_{T1} = 4.9 and c_{T2} = 2.4. We will use c_{T2} = 2.2 after Andreas

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(1989) to account for k = 0.4 instead of k = 0.35 used by Wyngaard. Thiermann and Grassl (1992) found

$$f_T = 6.34\left[1 + 7z + 20z^2\right]^{\frac{1}{3}} \qquad (4b)$$

2.2 Direct scaling of e and C_T^2

Our main motivation for this study was to find suitable MOST functions for C_T^2 and e to obtain fluxes of heat and momentum using scintillometer measurements of C_T^2 and e. Calculating these fluxes requires a numerical iteration of the f_e and f_T functions. To be able to calculate the fluxes directly, without iteration, we introduce the dimensionless length scale, Z, derived from C_T^2 and e

$$\mathrm{Z} = \frac{gkz}{T}\frac{T_C}{U_e^2} \qquad (5)$$

in which $T_C = \sqrt{C_T{}^2 z^{2/3}}$ and $U_e = \sqrt[3]{kze}$ are a temperature and wind speed scale. Next, we looked for a relationship between Z and z and found the best fit for $z = 0.55\mathrm{Z}^{1.15}$. By substituting this empirical expression in the f_e and f_T functions, one can directly calculate q_* and u_*, and from these the kinematic sensible heat flux, $\overline{w'T'} = u_* q_*$.

2.3 e and C_T^2 from raw time series

C_T^2 is a scaling parameter of the temperature spectrum in the inertial range of turbulence and is defined as (e.g. Stull, 1988):

$$C_T{}^2 = \frac{D_T}{r^{2/3}} = \frac{\overline{\left[T(x) - T(x+r)\right]^2}}{r^{2/3}}, \qquad (6)$$

where D_T denotes the structure function, $T(x)$ is the temperature at position x, $T(x+r)$ the temperature at position $x+r$, where r should lie within the inertial range of turbulent length scales. We calculated 10-minute C_T^2 values from the 20 Hz sonic data using Taylor's frozen turbulence hypothesis to estimate a time lag that approximates best a space separation, r, of 1 m. We corrected for path averaging of the sonic temperature measurements after Hill (1991). The C_T^2 calculation and correction procedure are described in more detail by Hartogensis et al. (2002).

Like C_T^2, e is also a scaling parameter of spectra in the inertial range, in this case of turbulent kinetic energy (TKE). For the longitudinal wind component, u, the inertial range of the spectrum, S_u, is described by

$$S_u(k) = a\, e^{2/3} k^{-5/3}, \qquad (7)$$

where S_u is the spectral energy density, a is the Kolmogorov constant, and k is the spatial wave-number expressed in cycles per unit length. We adopted a = 0.55, which is mid-range the values found in literature.

To obtain 10-minute values of e from 20Hz sonic data the following procedure was followed:

First, the wind vector was rotated with the planar fit routine (Wilczak et al., 2001), and aligned to the mean wind direction.

Second, 10-minute spectra of the longitudinal wind velocity, u, were calculated with the ARMASA toolbox, developed at the University of Delft, the Netherlands (Broersen, 2002). ARMASA determines an optimal auto-regression (AR), moving-average (MA) time series model for a given data-set from which can be determined directly. The principle advantages of spectra determined from ARMA models over conventional Fourier transforms are that the signal is not treated as a windowed periodogram where the first data-point is treated as a neighbor of the last data-point in the record, and no arbitrary smoothing of the spectrum is needed. ARMASA is written for MATLAB and is freely available at www.tn.tudelft.nl/mmr.

Third, we calculated e using Eq. (7) for all points of the spectrum.

Fourth, we performed a quality check on the spectrum and the calculated e -values to determine whether an inertial range was present in the spectrum. Moving point by point though the data, we determined the slope of the spectrum and the RMS of e for blocks of 25 % of all the spectral points. An average e was determined for all blocks for which the spectral slope deviated less than 20 % of the theoretical –5/3 slope, and the RMS of e was less than 30 % of its block-average value. When none of the blocks fulfilled these criteria, the e-value was rejected for that 10-minute period. We performed our analyses with e determined with ARMASA and traditional Fourier transforms and found less scatter using ARMASA.

Only stable conditions (z > 0) between 7:00 PM and 7:00 AM are considered in this study. The data were 'cleaned' based on the following criteria: z > 0.0001, $\overline{w'T'}$ > -0.0001, and u_* > 0.01.

3. RESULTS

3.1 Experiment

We will use data gathered during CASES-99. The CASES-99 stable boundary layer experiment took place during October 1999 at a grassland site in Kansas, USA (Poulos et al., 2002). We operated a CSAT3 sonic anemometer from Campbell Scientific Inc., Logan, USA at 2.65 m. Raw 20 Hz data were stored on a laptop and processed afterwards with the latest version of the EC-pack flux-software package, developed by Wageningen University. The source code and documentation of the software can be found at http://www.met.wau.nl/projects/jep/index.html.

First, 5-minute fluxes were calculated, which were subsequently averaged to 10-minute values

3.2 Results Monin-Obukhov e and C_T^2 scaling

Figure 1 shows our data of the e-dimensionless group, the f_e scaling functions given by Eqs. (3a) to (3d), and two f_e curves that give a good fit to our data, namely a 'kink' function

$$f_e = \begin{cases} 0.8 + 2z & \text{for } z < 0.1 \\ \dfrac{\sqrt{z}}{\sqrt{0.1}} & \text{otherwise} \end{cases} \tag{8a}$$

and

$$f_e = 0.8 + 2.5z \tag{8b}$$

which is the Wyngaard (1973) form (Eq. 3c) with adjusted parameters c_{e1} and c_{e2}. In the limit $z \to \infty$ the formulations of Eqs. (8a) and (8b) differ; e becomes independent of z in Eq. (8b) (Pahlow et al, 2001), whereas in Eq. (8a) e remains a function of z.

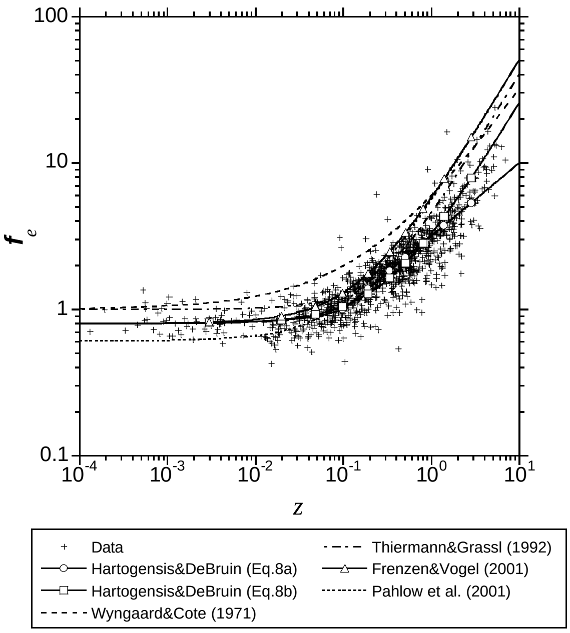

Figure 1: Dimensionless scaling group, f_e of the TKE dissipation rate, e, against the dimensionless stability parameter, z, for 10 minute-experimental values and relations found in literature.

When systematic measuring errors are assumed small, the imbalance between TKE production and dissipation found here implies that the pressure and flux-divergence terms in the TKE budget are not negligible. This result agrees, at least qualitatively, with the findings of Frenzen and Vogel (2001) Pahlow et al. (2001) and others. Unfortunately, the pressure and flux-divergence terms of the TKE budget are very difficult to measure. Recently, Cuxart et al. (2002) presented data of the full TKE budget for one CASES-99 nights, and found that for that night the pressure and flux divergence terms were indeed significant.

Figure 2 shows our data of the C_T^2-dimensionless group, the f_T scaling functions given by Eqs. (4a) and (4b), and two f_T curves that give a good fit to our data, namely a 'kink' function

$$f_T = \begin{cases} 5.5 & \text{for } z < 0.1 \\ 5.5\left(\dfrac{z}{0.1}\right)^{\frac{2}{5}} & \text{otherwise} \end{cases} \tag{9a}$$

and

$$f_T = 4.7\left[1 + 1.6z^{\frac{2}{3}}\right] \tag{9b}$$

which is the function proposed by Wyngaard et al. (1971) given in Eq. (4a) with different values for the constants c_{T1} and c_{T2}. Thiermann and Grassl (1992) imposed production-dissipation balance on the temperature fluctuation budget. Figure 2 shows that their f_T function gives higher values than our observatories, which suggests a production-dissipation imbalance of the temperature fluctuation budget in our data.

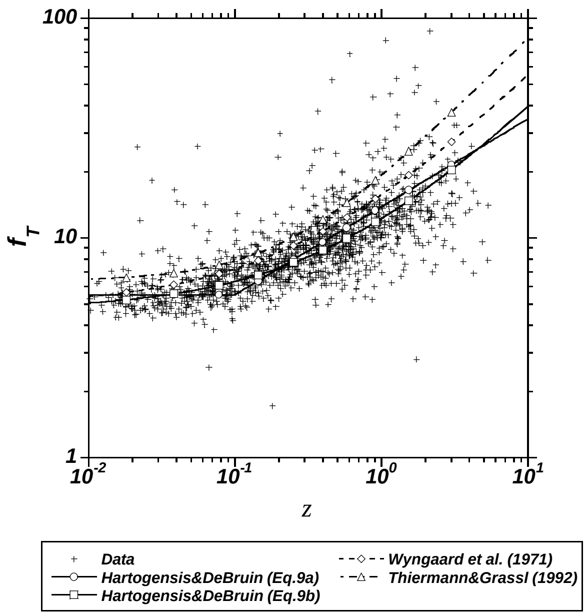

Figure 2: Dimensionless scaling group f_T of the structure parameter of temperature, C_T^2, against the dimensionless stability parameter, z, for 10 minute-experimental values and relations found in literature.

3.2 Results direct e and C_T^2 scaling

Figure 3 compares $-\overline{w'T'}$ calculated from e and C_T^2 with this simplified approach against $-\overline{w'T'}$ from e and C_T^2 calculated by means of iteration. For both approaches the f_e and f_T functions of Eqs. (8b) and (9b) are used. It is seen that the simplified approach can be used with little error.

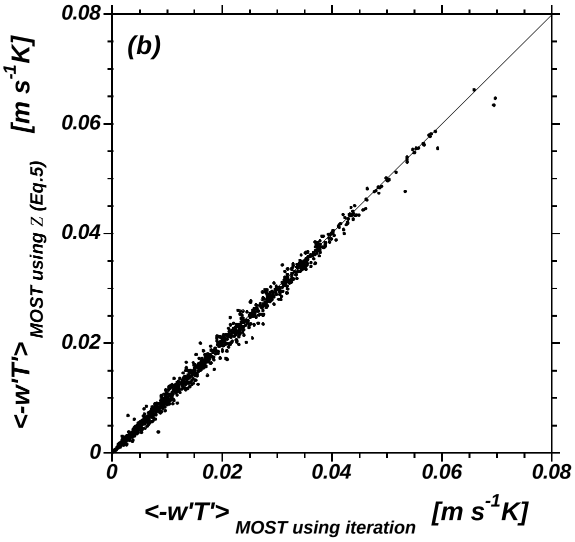

Figure 3: Comparison between the kinematic heat flux, $-\overline{w'T'}$, determined from the TKE dissipation rate, e, and structure parameter of temperature, C_T^2, calculated directly with the alternative dimensionless height parameter, Z, of Eq. (5), against the values calculated by means of numerical iteration of the MOST relationships.

4. CONCLUSIONS

In this study we analysed the MOST scaling functions f_e and f_T of the dissipation rate of TKE, e, and the structure parameter, C_T^2, for the stable atmospheric surface layer using data we gathered in the context of CASES-99. This data covers a relatively wide stability range, i.e. z up to ~10.

Our results differ somewhat from those reported in the literature. First of all, the stability range of our data-set (0 < $\underline{z}$ < ~10) is much larger than the authors above had available. Secondly, the way we determined the surface fluxes might have to do with the found differences. We included the cross-term stress in calculating u_* and we used a short flux-averaging interval to exclude non-turbulent contributions to the flux (Vickers and Mahrt, 2003). Also, we filtered our data based on inertial range behavior in the longitudinal wind speed

In determining e from the raw time series, we found that the ARMASA toolbox developed at the university of Delft (Boersen, 2002) is very suitable to obtain spectra from atmospheric turbulence time series. This approach has several advantages over the traditional Fourier transform method.

Since $f_e(0)$ is found to be about 0.8, there is no balance between the production and dissipation terms in the budget equations for TKE. Also, our results suggest a production-dissipation imbalance in the budget equation for temperature fluctuations. This has been found reported earlier by others.

The direct scaling approach, where we introduced a length and a temperature scale directly defined by the measured variables works well for this datas-set.

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