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FOOTPRINT MODELING FOR FLUX MEASUREMENTS AT MULTIPLE LEVELS FROM A VERY TALL TOWER OVER A MIXED FOREST

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1. INTRODUCTION

The interpretation of eddy-covariance flux measurements over a heterogeneous surface depends largely on the footprint over which fluxes are sampled. The location and size of this footprint for the measurements varies with time. In this regard, footprint modeling is a powerful tool for designing experiments and interpreting measured fluxes. There are a number of studies of flux footprints for measurements within the surface layer (Horst and Weil, 1992; Schmid, 2002; Schuepp et al., 1990). Models for the footprints have been successfully used to interpret long-term flux measurements within the surface layer (Amiro, 1998; Stoughton et al., 2000). Fluxes are also measured above the surface layer using aircraft (e.g., Davis et al., 1997; Mahrt, 1998; Oncley et al., 1997) and tall-tower platforms (Davis et al., 2003) in order to cover large horizontal distances and observe vertical structure within the atmospheric boundary layer (ABL). Unlike in the surface layer, flux footprints for measurements above the surface layer have not been extensively studied, although numerical investigations in the convective boundary layer (CBL) include a stochastic model (Weil and Horst, 1992), large eddy simulation (Leclerc et al., 1997), and a second-order closure model (Wang and Davis, 2002). None of these methods can be used in practice for long-term calculations. Due to complicated diffusion processes in the CBL, it is rather difficult to describe the footprint in terms of an explicit analytical expression.

In this study, we introduce an empirical model for the footprint above the surface layer in the CBL. The model is derived by combining an analytical footprint solution under an ideal convective boundary layer with the results from a Lagrangian stochastic model driven by more realistic atmospheric variables. As an application, footprints for eddy-covariance CO_2 fluxes measured at 30m, 122m, and 396m at the WLEF tower over a mixed forest are simulated under unstable conditions.

2. MODEL

The scalar flux footprint, $f(x,y,z_m)$, is equal to the vertical flux downwind of a unit surface point source (Horst and Weil, 1992), i.e.,

$$f(x,y,z_m)=\frac{F_m(x,y,z_m)}{Q} , \qquad (1)$$

where x and y are the horizontal distances of the measurement point from the source; z_m is the measurement height; $F_m(x, y, z_m)$ is the vertical flux measured at position (x, y, z_m) in the Eulerian field due to a surface point source (sink) with an emission rate Q at the origin.

2.1 An Analytical Expression Under Ideal Conditions

To obtain a simple analytical expression for the footprint above the surface layer, dispersion is considered in an ideal CBL with horizontally homogeneous conditions, constant horizontal wind speed, and constant vertical velocity skewness and variance. In addition, the Lagrangian time scale is assumed to be infinite as usually adopted in probability density function (PDF) dispersion models (Luhar, 2002; Misra, 1982; Weil et al., 1997). This assumption makes use of the observation that in convective turbulence the Lagrangian time scale is so large that some passive particles tend to remain close to their initial trajectory for a considerable distance. Using the above assumptions, an expression for the cross-wind integrated footprint function can be derived using formulas given by van Dop(1985; Weil, 1988) and Weil(1988),

$$f^y(x,z_m)=\begin{cases}\dfrac{1}{x}\sum\limits_{j=-1}^{+1}\sum\limits_{k=-\infty}^{\infty} w_{kj}(x,z_m)\,p_w(w_{k\ j}), & x>0\\ 0, & otherwise\end{cases} , \qquad (2)$$

where $w_{k,j}$ is the initial vertical velocity of a particle passing (x, z_m) and can be written as

$$w_{kj}(x,z_m)=\frac{(2kh+jz_m-z_s)U}{x} , \qquad (3)$$

where z_m is the measurement height; x is the distance from the source; z_s is the height of the source; U is the horizontal wind speed; and h is the CBL depth; k is any integer where $|k|$ is the number of reflections from the top of CBL; j is an integer equal to either 1 or -1, considering reflection on the top and bottom boundaries; p_w is the probability density function of w_{kj} at the source height. The PDF is usually taken to be the sum of two Gaussian distributions, which provides a good match to observations; i.e.,

$$p_w(w)=\frac{\lambda_1}{\sqrt{2\pi}\sigma_1}\exp\left[-\frac{(w-\overline{w}_1)^2}{2\sigma_1^2}\right]+\frac{\lambda_2}{\sqrt{2\pi}\sigma_2}\exp\left[-\frac{(w-\overline{w}_2)^2}{2\sigma_2^2}\right] , \qquad (4)$$

where $\lambda_1, \lambda_2, \sigma_1, \sigma_2, \overline{w}_1, \overline{w}_2$ are found by equating the zeroth through third moments of the assumed distribution with those specified and by assuming that $\sigma_1/\overline{w}_1=\sigma_2/\overline{w}_2=R$, a constant. Details on how to determine the six parameters can be found in the literature (e.g., Weil, 1990). Figure 1 shows the footprint calculated

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from the ideal model (eq. (2)) with a skewness of 0.5 and R=1 varying with the horizontal dimensionless distance (X=*hx/Uw**) for the heights of 0.1, 0.3, 0.5, and 0.7h.

The effects of stability, roughness, and vertical variations in turbulence in the vertical direction are not included in the ideal model. With more realistic wind, temperature, and turbulence profiles, calculation of the footprint function requires numerical evaluation of the vertical velocities and is not convenient in practice, particularly for long-term calculation. Consequently, we propose an empirical formula by adjusting equation (2) to approximate the more realistic solution of the footprint from a stochastic model.

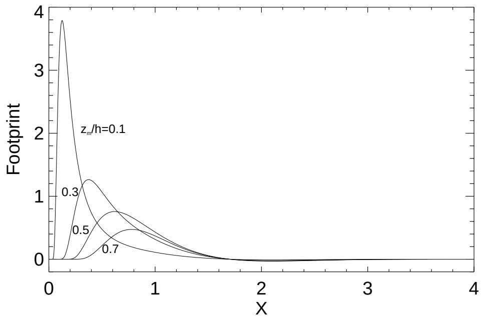

Fig.1 Cross-wind integrated flux footprint, f^y, calculated from the ideal model (equation (2)) with a vertical velocity skewness of 0.5 and R=1 vs. the horizontal dimensionless distance (X) for the heights of 0.1, 0.3, 0.5, and 0.7h.

2.2 Adjusted Model

Two parameters are introduced to adjust the main features of the footprint from the ideal model to those from the stochastic model. One, denoted by *β*, is used to adjust the predicted half-width and the maximal value of the footprint. The other, *γ*, is used to adjust the position of the maximum of the footprint by translating the footprint function in the along-wind direction. A necessary condition should be satisfied is that the cumulative footprints of the adjusted and non-adjusted models be equal at the same height, i.e.,

$$\int_0^{+\infty} f_a^y(x,z_m)dx=\int_0^{+\infty} f^y(x,z_m)dx=1-\frac{z_m}{h}\ , \qquad (5)$$

where $f_a^y(x,z_m)$ is the adjusted footprint function, which therefore can be written as,

$$f_a^y(x,z_m)=\begin{cases}\sum\limits_{j=-1}^{+1}\sum\limits_{k=-\infty}^{\infty}\beta\dfrac{w_{kj}(\beta x+\gamma,z_m)p_w[w_{kj}(\beta x+\gamma,z_m)]}{\beta x+\gamma}, & \mathit{for}\quad \min[\beta x+\gamma,x]>0\\ 0, & \mathit{otherwise}\end{cases} \qquad (6)$$

where *β* is a function of stability and measurement height, z_m. *β* can be calculated using,

$$\beta=\alpha\frac{\Delta X_{ideal}}{\Delta X_{random}}+(1-\alpha)\frac{f_{\max,random}}{f_{\max,\ ideal}}\ , \qquad (7)$$

where *α* is a coefficient determining the relative adjustment weights. The parameter *γ* is calculated using,

$$\gamma=X_{\max,\ ideal}-\beta X_{\max,\ random}\ , \qquad (8)$$

where ΔX_{ideal}, $f_{\max,ideal}$, and $X_{\max,\ ideal}$ are the half-width at maximum, the maximum value and the position of the maximum of the cross-wind integrated footprint as determined from the ideal footprint model, respectively, which can be fitted by simple curves. The subscript '*random'* represents stochastic model variables. These characteristics of the footprint from the numerical model can be fitted as functions of stability and measurement height (not shown here). It also can be readily shown that the adjusted function (6) satisfies the asymptotic constraint (5) in most cases within the usual range of atmospheric stability.

After the adjustment, the analytical solution is generally in good agreement with the stochastic model. Figure 2 shows the comparison of the footprints obtained from the stochastic model and the adjusted model for two heights, *z*=0.21*h* and *z*=0.41*h*, in the case of *L*=-0.03*h*. In the calculation, the coefficient *α* is taken as 0.5, indicating equal adjustment weights of the width and the maximum of the footprint.

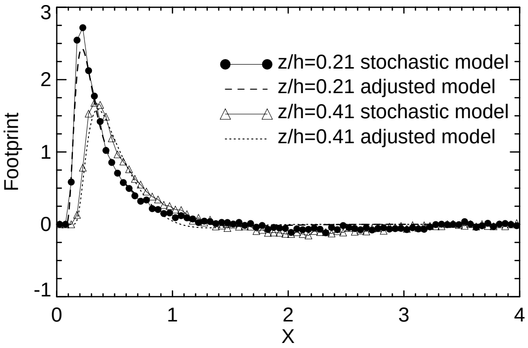

Fig. 2 Comparison of the footprints calculated from the adjusted model and from the stochastic model for heights of 0.21 and 0.41***h*** in the case of ***L***/***h***=-0.03 and a typical roughness length for forest, 5×10^{-4} ***h***.

The performance of the adjusted model is poor far away the source, i.e. *X*>1 or 2, due to oversimplified physics. The negative footprint and its location are not well simulated in the adjusted model, particularly for strongly unstable conditions or at high measurement levels. Nevertheless, the model still can be used to estimate the footprint in the lower levels of the CBL because the portion of the footprint in the range of X>1 is small.

3. APPLICATION

The adjusted model is applied to interpret CO_2 fluxes measured at three levels of a 447-m tall tower (WLEF) over a mixed forested area in northern Wisconsin. One level is 30m, which is usually within the surface layer during the daytime. The other two levels are 122 and 396m, which are usually above the surface layer.

Detailed descriptions of the flux measurements are given by Berger et al., (2001) and Davis et al., (2003). Upland forest and wetland (largely forested as well) vegetation dominate in the tower area (figure 3). However, there is a grass-covered area of about 150-m radius centered at the tower base. The grassy area is included in the flux footprint area in some cases, possibly resulting in measurements unrepresentative of the predominant vegetation types.

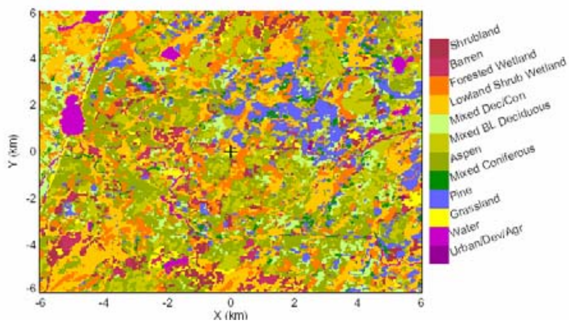

Fig. 3 Vegetation map centered at the WLEF tower. The cross represents the location of the tower. The data is from the state of Wisconsin's Department of Natural Resources, http://www.dnr.state.wi.us/maps/gis/datalandcover.html.

As an example, we calculate the footprint during the day on June 4, 1998 (day 155) and overlay it on a vegetation cover map to assess the weightings of the flux from each vegetation type. The weather on that day is fair and the CBL is well developed in the midday hours. The M-O length is estimated using the meteorological variables and fluxes measured at the tower. The depth of the mixed layer is obtained from a 915-MHz boundary layer profiling radar (Yi et al., 2001). The roughness length is assumed to be 0.9m. The zero-plane displacement height is assumed to be about two-thirds of the canopy height which is about 25m.

The flux footprint, $f(x,y,z_m)$, can be written as the product of the cross-wind integrated footprint, $f^y(x,z_m)$ and the crosswind mixing ratio distribution function, $D_y(x,y)$ (Horst and Weil, 1992), i.e.,

$$f(x,y,z_m)=f^y(x,z_m)D_y(x,y) \tag{9}$$

Dispersion in the lateral direction (*y*) is assumed to be a Gaussian distribution, i.e.,

$$D_y(x,y)=\frac{1}{\sqrt{2\pi}\sigma_y}\exp(-\frac{y^2}{2\sigma_y^2}) , \tag{10}$$

where σ_y is the standard deviation of the plume in the lateral direction, which can be derived empirically. For the surface layer, we adopt the relationship $\sigma_y=\lambda x/(1+0.0001x)^{1/2}$; λ is taken as $-1.3/L+0.1$ for strongly unstable conditions (Amiro, 1998). Above the surface layer, an empirical formula derived from observations (Briggs, 1988), $\sigma_y=0.6w_*x/U$, is used. Errors in the parameterization of the lateral dispersion can affect the results. As an exercise, the above empirical formulas are used in the calculation. The model developed by Horst and Weil(1992) is used to estimate the crosswind integrated footprint for the surface layer, and the adjusted footprint model developed in this paper is used for the measurements above the surface layer.

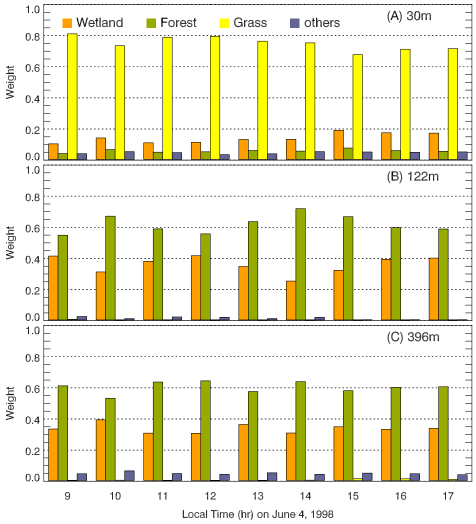

Fig. 4 Fractional weight by area of the flux footprint from wetland, forest, grass, and other vegetation to the total turbulent flux measured at the heights of (A) 30, (B) 122, and (C) 396m above the ground level as a function of time on June 4, 1998. The zero-plane displacement height is assumed to be 17m

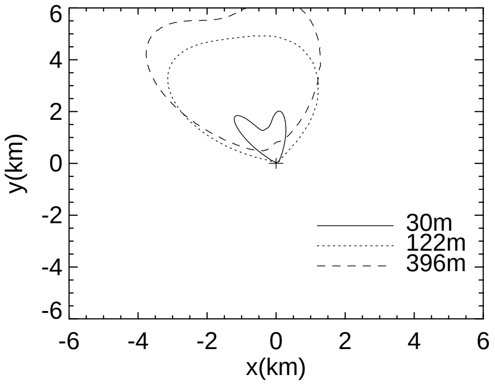

Fig. 5 The 90% source area boundaries, each of which corresponds to a footprint function contour, calculated using the averaged footprint from 0900 to 1700 on June 4, 1998 for the measurement heights of 30, 122, and 396m.

Figure 4 shows the fractional contributions of fluxes by different vegetation to the measured turbulent fluxes at three levels. These values are computed by integrating the footprint function over the areas covered by the corresponding vegetation. Footprint areas vary with atmospheric stability and wind direction, resulting in time-dependent contributions of each vegetation type to the total fluxes. At the 30m level, the fluxes from the grassy area at the base of the tower contribute about 60% of the measured fluxes depending on stability; while the flux sensors at 122m and 396m sample primarily from the areas covered by wetlands (forested and non-forested) and upland forest. Figure 5 presents the mean source areas for the measurements at the three levels. The three lines in the figure enclose 90% of the total integrated footprint averaged from 9am to 5pm LST for the three levels. The source area of the lower level measurement is closer to the tower base; while the source areas at higher levels are larger and do not include the grassy area at the tower base. The contributions of the wetlands and forest for the two high levels are similar and approximately independent of time in this case, indicating a relatively uniform distribution of wetlands and upland forest in the corresponding source area.

4. CONCLUSIONS

An empirical footprint model for flux measurements above the surface layer of the CBL is introduced. This model is derived using an idealized, analytic footprint solution for the CBL, and made more realistic by adjusting it to fit the footprints computed from a stochastic model that incorporates more realistic vertical structure in CBL turbulence, as well as a range of stability conditions. The model is used to compare the sampling areas of flux measurements at 122m and 396m above the ground at the 447-m tall WLEF tower. The footprints are compared to those for the measurements at 30m, usually within the surface layer.

Under strongly unstable conditions the lowest level is found to have significant flux contributions from the grassy area surrounding the tower base. The case study indicates that the sampling areas of the top two levels in the day consist mainly of wetland and upland forest areas while the lowest level measurements sample areas of grass, wetland, and upland forest; hence daytime measurements at the top two levels might better represent the daytime wetland and upland forest fluxes in this region.

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