5R.4

POSSIBLE ALGORITHMS FOR THE DUAL-FREQUENCY PRECIPITATION RADAR (DPR) ON THE GPM CORE SATELLITE

Toshio Iguchi*
NICT, Tokyo, Japan

1. INTRODUCTION

This paper concerns precipitation retrieval algorithms for a spaceborne or airborne dual-frequency radar, in particular for the Dual-frequency Precipitation Radar (DPR) onboard the Global Precipitation Measurement (GPM) core satellite. From the dual-frequency radar data we can in principle estimate two parameters of the drop size distribution (DSD) at each range bin. In practice, however, estimates from such algorithms are not reliable in many cases unless we are given other information such as the attenuation to the first gate or to the farthest range gate that mitigates the effect of uncertainties in radar calibration and attenuation due to unobservable gases and particles, i.e., water vapor and cloud water.

This paper first reviews the general principles of dual-frequency algorithm and then examines some possible alternative algorithms that do not require additional information but can estimate reliable rain parameters by abandoning the possibility of estimating two independent DSD parameters at each range bin. Some possible methods are derived by assuming a certain plausible relationship between DSD parameters or between some rain parameters.

2. DUAL-FREQUENCY ALGORITHMS

We assume that the beams of two channels are matched and that the rain is uniform in the lateral directions with respect to the range direction within the beams. We also assume that multiple scattering effects can be ignored, and that only the backscattering and absorption of radar waves by individual particles need to be considered. Then, the effective radar reflectivity factor $Z_{e\lambda}$ at wavelength λ, which characterizes the strength of backscattering from a unit scattering volume, can be expressed as the sum of backscattering cross sections of individual particles in the unit volume. We assume that the backscattering cross section $\sigma_{b\lambda}$ and the extinction cross section (or total radar cross section) $\sigma_{t\lambda}$ of each particle are determined only by its diameter D as in the case of scattering and absorption by a spherical particle. Under these assumptions, once the particles' phase state and temperature are given, we can calculate $\sigma_{b\lambda}$, $\sigma_{t\lambda}$ for any D by using Mie's formula. The only remaining unknown factor that relates the radar reflectivity factor $Z_{e\lambda}$, the specific attenuation k_λ, and the rainfall rate R is the drop size distribution funtion $N(D)$.

Once $N(D)$ is estimated from $Z_{e\lambda}$ and k_λ, we can calculate R. However, $N(D)$ itself has an infinite degree of freedom and cannot be determined uniquely only by giving $Z_{e\lambda}$ and k_λ. Our general strategy is that we first choose a DSD model function $N_m(D)$ that is characterized by two parameters, θ_1 and θ_2. $N_m(D;\theta_1,\theta_2)$ should represent the natural variations of $N(D)$ at variety of space and time reasonably well by adjusting θ_1 and θ_2. Since the dual-frequency radar echoes provide us with two independent pieces of information at each range bin, we would like to retrieve the two DSD model parameters (θ_1 and θ_2) there and calculate the rainfall rate R from $N_m(D;\theta_1,\theta_2)$.

2.1 *Differential Equations for the DSD Function with Two Parameters*

In our retrieval problem, we deal with a one-dimensional problem. We want to estimate $\theta_1(r)$ and $\theta_2(r)$ from measured radar reflectivities $Z_{m1}(r)$ and $Z_{m2}(r)$, and then calculate $R(r)$ from $\theta_1(r)$ and $\theta_2(r)$ by using the model function $N_m(D;\theta_1,\theta_2)$. Suppose our model function $N_m(D;\theta_1,\theta_2)$ approximates the true DSD function closely so that we can express $Z_{e\lambda}$ and k_λ by using the backscattering cross section $\sigma_{b\lambda}$ and the total cross section $\sigma_{t\lambda}$ as the follows.

$$Z_{e\lambda}(\theta_1,\theta_2) = c_{Z\lambda}\int \sigma_{b\lambda}(D)N_m(D;\theta_1,\theta_2)\,dD \qquad (1)$$

and

$$k_\lambda(\theta_1,\theta_2) = c_k\int \sigma_{t\lambda}(D)N_m(D;\theta_1,\theta_2)\,dD. \qquad (2)$$

Here, $c_{Z\lambda}$ is defined by $c_{Z\lambda} = \lambda^4/(\pi^5|K_\lambda|^2)$ with $K_\lambda = (m_\lambda^2-1)/(m_\lambda^2+2)$ where m_λ is the complex refractive index of the particle for electromagnetic waves with wavelength λ. π is the ratio of the circumference to the diameter of a circle. c_k is a proportional constant that changes with the unit of k_λ. If the unit of k_λ is neper per unit length, $c_k = 1$. However, if for example, the unit is dB per unit length, $c_k = 10\log_{10} e$.

Then, the two-way attenuation factor to range r can be written as

$$A_\lambda(r) = \exp(-\frac{2}{c_k}\int_0^r k_\lambda(s)\,ds). \qquad (3)$$

and the measured apparent radar reflectivity factor $Z_{m\lambda}$ is related to the effective radar reflectivity factor through

$$Z_{m\lambda}(r) = A_\lambda(r)Z_{e\lambda}(r). \qquad (4)$$

* *Corresponding author address:* Toshio Iguchi, National Institute of Information and Communications Technology, Tokyo 184-8795, Japan; e-mail: iguchi@nict.go.jp

Let us define $X_{m\lambda} = \ln(Z_{m\lambda})$ and $X_{e\lambda} = \ln(Z_{e\lambda})$ for brevity. We also choose the unit of k_λ as neper per unit length so that $c_k = 1$ to remove unnecessary constant from equations. Taking the logarithm of (4) and differentiate it with respect to r, we obtain

$$\frac{dX_{m\lambda}}{dr} = -2k_\lambda + \frac{dX_{e\lambda}}{dr}. \tag{5}$$

Or expressing it in terms of θ_1 and θ_2, we obtain

$$\frac{dX_{m\lambda}}{dr} = -2k_\lambda(\theta_1, \theta_2) + \frac{\partial X_{e\lambda}}{\partial\theta_1}\frac{d\theta_1}{dr} + \frac{\partial X_{e\lambda}}{\partial\theta_2}\frac{d\theta_2}{dr}. \tag{6}$$

If we have radar reflectivity measurements at two frequency channels $\lambda = 1, 2$, then we have two equations of the form (6), and we get a simultaneous pair of differential equations for θ_1 and θ_2.

$$\begin{pmatrix} \dfrac{d\theta_1(r)}{dr} \\ \dfrac{d\theta_2(r)}{dr} \end{pmatrix} = \mathcal{A}^{-1} \begin{pmatrix} \dfrac{dX_{m1}(r)}{dr} + 2k_1(\theta_1(r), \theta_2(r)) \\ \dfrac{dX_{m2}(r)}{dr} + 2k_2(\theta_1(r), \theta_2(r)) \end{pmatrix} \tag{7}$$

where

$$\mathcal{A} = \begin{pmatrix} \dfrac{\partial X_{e1}}{\partial\theta_1} & \dfrac{\partial X_{e1}}{\partial\theta_2} \\ \dfrac{\partial X_{e2}}{\partial\theta_1} & \dfrac{\partial X_{e2}}{\partial\theta_2}. \end{pmatrix} \tag{8}$$

Selecting a different set of DSD parameters θ_1' and θ_2' will give exactly the same form of equations. In other words, the selection of DSD parameters is not essential in the formulation as long as the model function covers the actual variations of DSD. It is the selection of the model function, and not the selection of the parameters, that determines the performance of the retrieval algorithm. We therefore choose a set of parameters that simplifies the appearance of equations.

The equations become singular at a point where the determinant of $\mathcal{A}$ is 0: $|\mathcal{A}| = 0$. Note that the change of parameters does not remove this point.

Without loosing generality, therefore, we can choose one of the parameters, which we denote by N_0 hereafter, as a scale factor that is proportional to the total number of particles or the magnitude of the distribution. If the DSD is graphically represented, N_0 represents the vertical height of the distribution. The other parameter we choose should represent the horizontal spread of the distribution. We denote it by D_0. It could be the mass-weighted diameter, or could be the mass-weighted median diameter D_0. There is no essential difference among them mathematically once the model DSD function is selected.

For example, if we choose a Γ-distribution model, the DSD can be expressed as

$$N_m(D) = N_0 D^\mu \exp(-\Lambda D). \tag{9}$$

Here μ is called a shape parameter. Λ is related to the mass-weighted median diameter D_0 by $\Lambda = (\mu + 3.67)/D_0$, or to the mass-weighted mean diameter D_m by $\Lambda = (\mu + 4.0)/D_m$.

In our treatment, however, N_0 and D_0 are not as restrictive as they are in the above Γ-distribution model. As long as the model DSD function $N_m(D)$ can be expressed in the form as

$$N_m(D) = N_0 f(D : D_0). \tag{10}$$

our formulation is valid. Here, $f(D : D_0)$ is an arbitrary function of D.

Since both $Z_{e\lambda}$ and k_λ are proportional to N_0, we can extract this proportional constant in $Z_{e\lambda}$ and k_λ by defining $I_{b\lambda}$ and $I_{t\lambda}$ as follows:

$$Z_{e\lambda} = N_0 I_{b\lambda}(D_0) \tag{11}$$

$$k_\lambda = c_k N_0 I_{t\lambda}(D_0) \tag{12}$$

where

$$I_{b\lambda}(D_0) \stackrel{\text{def}}{=} c_{Z\lambda} \int \sigma_{b\lambda}(D) f(D : D_0)\, dD \tag{13}$$

$$I_{t\lambda}(D_0) \stackrel{\text{def}}{=} \int \sigma_{t\lambda}(D) f(D : D_0)\, dD \tag{14}$$

We consider N_0 and D_0 are functions of range r.

$$\begin{aligned} Z_{m\lambda}(r) =& N_0(r) I_{b\lambda}(D_0(r)) \\ & \times \exp(-2 \int_0^r N_0(s) I_{t\lambda}(D_0(s))\, ds) \end{aligned} \tag{15}$$

Taking the logarithms of (15) and differentiating it with respect to r, we obtain

$$\begin{aligned} \frac{d}{dr}\ln(Z_{m\lambda}(r)) =& \frac{d\ln N_0}{dr} + \frac{d\ln(I_{b\lambda}(D_0))}{dD_0}\frac{dD_0(r)}{dr} \\ & - 2\exp(\ln N_0) I_{t\lambda}(D_0(r)). \end{aligned} \tag{16}$$

Now, suppose we have measurements of $Z_m(r)$ at two wavelengths, λ_1 and λ_2. We use suffixes 1 and 2 for corresponding variables. Then, by combining (16) for λ_1 and λ_2, we obtain coupled differential equations for D_0 and N_0:

$$\begin{aligned} \frac{dD_0(r)}{dr} =& \frac{1}{b_1 - b_2}\Big[\frac{d\ln(Z_{m1})}{dr} - \frac{d\ln(Z_{m2})}{dr} \\ & + aN_0\{I_{t1}(D_0) - I_{t2}(D_0)\}\Big] \end{aligned} \tag{17}$$

and

$$\begin{aligned} \frac{1}{N_0}\frac{dN_0(r)}{dr} =& \frac{1}{b_2 - b_1}\Big[b_2\frac{d\ln(Z_{m1})}{dr} - b_1\frac{d\ln(Z_{m2})}{dr} \\ & + aN_0\{b_2 I_{t1}(D_0) - b_1 I_{t2}(D_0)\}\Big] \end{aligned} \tag{18}$$

where

$$b_1 = \frac{d}{dD_0}\ln(I_{b1}(D_0)) \quad \text{and} \quad b_2 = \frac{d}{dD_0}\ln(I_{b2}(D_0)) \tag{19}$$

These coupled differential equations for $N_0(r)$ and $D_0(r)$ (Iguchi and Meneghini, 1995) are the specific expression of the more general equations (7) when we choose N_0 and D_0 as the two parameters θ_1 and θ_2, respectively.

2.2 *Differential Equations of Meneghini's Method*

Define the path-integrated attenuation $S_\lambda(r)$ to range r by

$$S_\lambda(r) \stackrel{\text{def}}{=} 2\int_0^r N_0(s) I_{t\lambda}(D_0(s))\, ds \tag{20}$$

Then,

$$\begin{aligned}\frac{1}{2}\frac{dS_\lambda(r)}{dr} &= \frac{k_\lambda(r)}{c_k} = N_0(r) I_{t\lambda}(D_0(r)) \\ &= \frac{Z_{e\lambda}(r)}{I_{b\lambda}(D_0(r))} I_{t\lambda}(D_0(r)) = Z_{e\lambda}(r) I_{tb\lambda}(D_0(r))\end{aligned} \tag{21}$$

where $I_{tb\lambda}(D_0)$ is defined as

$$I_{tb\lambda}(D_0) \stackrel{\text{def}}{=} \frac{I_{t\lambda}(D_0)}{I_{b\lambda}(D_0)} \tag{22}$$

Since $X_{m\lambda}$ is defined by $X_{m\lambda} \stackrel{\text{def}}{=} \ln Z_{m\lambda}$,

$$Z_{e\lambda}(r) = Z_{m\lambda}(r)\exp(S_\lambda(r)) = \exp(X_{m\lambda}(r) + S_\lambda(r)) \tag{23}$$

Substitution of this equation into (21) gives

$$\frac{dS_\lambda}{dr} = 2\exp(X_{m\lambda} + S_\lambda) I_{tb\lambda}(D_0(r)) \tag{24}$$

Since the ratio

$$\frac{Z_{e1}}{Z_{e2}} = H(D_0) \stackrel{\text{def}}{=} \frac{I_{b1}(D_0)}{I_{b2}(D_0)} \tag{25}$$

is a function of D_0 only and independent of N_0, D_0 can be expressed as a function of Z_{e1}/Z_{e2}.

$$D_0 = H^{-1}(Z_{e1}/Z_{e2}) \tag{26}$$

Substitution of (23) into this equation gives

$$\begin{aligned}D_0 &= H^{-1}(\exp((X_{m1}(r) + S_1(r)) - (X_{m2}(r) + S_2(r)))) \\ &= J(X_{m1}(r) + S_1(r) - X_{m2}(r) - S_2(r))\end{aligned} \tag{27}$$

where $J(x) \stackrel{\text{def}}{=} H^{-1}(\exp(x))$. Substituting this D_0 into (24), we finally obtain a coupled pair of equations for S_1 and S_2:

$$\frac{dS_1}{dr} = 2\exp(X_{m1}+S_1) I_{tb1}(J(X_{m1}+S_1-X_{m2}-S_2)) \tag{28}$$

$$\frac{dS_2}{dr} = 2\exp(X_{m2}+S_2) I_{tb2}(J(X_{m1}+S_1-X_{m2}-S_2)) \tag{29}$$

If you rewrite these equations by using N_0 and D_0 as intermediate auxiliary variables, they become

$$\frac{dS_1(r)}{dr} = 2N_0(r) I_{t1}(D_0(r)) \tag{30}$$

$$\frac{dS_2(r)}{dr} = 2N_0(r) I_{t2}(D_0(r)) \tag{31}$$

$$N_0(r) = \frac{Z_{m1}(r)}{I_{b1}(D_0(r))}\exp(S_1(r)) \tag{32}$$

$$D_0(r) = H\left(\frac{I_{b1}(D_0(r))}{I_{b2}(D_0(r))}\right) \tag{33}$$

$$\frac{I_{b1}(D_0(r))}{I_{b2}(D_0(r))} = \frac{Z_{m1}(r)}{Z_{m2}(r)}\exp(S_1(r) - S_2(r)) \tag{34}$$

If you convert the first two differential equations into difference equations and solve them stepwise in backwards in range starting with a given pair of initial conditions $S_1(r_b) = S_1^0$ and $S_2(r_b) = S_2^0$, you will get Meneghini's algorithm (Meneghini et al., 1992). In this sense, Meneghini's algorithm is equivalent to a numerical method to solve the coupled differential equations (28) and (29) by a simple one-sided difference approximation of differentials.

Note that the pair of differential equations of N_0 and D_0 and those of S_1 and S_2 are mathematically equivalent.

If we want to solve the equations in section 2.1 numerically, it is more stable to solve the equations backwards in r. Meneghini's equations share the same property as these equations. They are generally stable if they are solved backwards in r. Mardiana's iteration method (Mardiana et al., 2004) is actually a method to find the solutions that satisfy the initial conditions that are given at the lower end $(r = r_l)$ of the interval $[r_l, r_u]$. He gives (arbitrarily chosen) tentative initial conditions at $r = r_u$ instead of $r = r_l$, and solve the equation backwards numerically. Then he calculates the difference between the solutions and the original conditions given at $r = r_l$. From this difference, he calculates the correction factors to the initial conditions at $r = r_u$ and repeats the calculation processes until the solutions satisfy the original initial conditions. The original initial conditions he uses are that the attenuations at $r = r_l$ are zero at both frequency channels.

In short, the difference between Meneghini's backward recursion method and Mardiana's iteration method is whether the initial conditions are given at $r = r_u$ or at $r = r_l$. Both of them use one-sided difference approximation to calculate the differentials, which is called Euler's method, to solve equations (28) and (29). To solve these equations numerically for a given set of data $Z_{m\lambda}(r_i)$ $(r_i = r_1, ..., r_n\,,\ \lambda = 1, 2)$, we can use a different numerical technique such as Runge-Kutta method that gives more accurate and stable solutions than Euler's method (Press et al., 1992).

2.3 *Adding Other Conditions*

In actual situation, the radar signal suffers from attenuation due to water vapor and clouds that are not detectable by radar itself. If we want to retrieve the rain profile below a bright band, for example, we need to estimate the attenuation due to the bright band as well. Therefore, we cannot adopt the initial condition that the attenuation is zero or a known amount at $r = r_l$. We need to treat it as an unknown parameter in the retrieval algorithm. When the surface reference technique is applicable, the condition is given at $r = r_u$ and the equation can be solved backwards in the raining interval. However, if the surface reference is unavailable or unreliable, we need to use a method that can estimate the attenuation in other means.

To do that, we need to abandon the possibility of estimating two DSD parameters at each range bin (Iguchi et al., 2001, Marzoug and Amayenc, 1994). There may be many possible ways to do so, but here we discuss the cases in which N_0 and D_0 are related in some ways. For example, giving a condition that the normalized intercept parameter N_0^* is constant in a two-parameter DSD model is equivalent to giving a functional relationship between N_0 and D_0 (See Appendix). The condition can be generalized more. For example, the condition that N_0^* is a linear function of range r could be used.

If N_0 and D_0 are related and N_0 is expressed as a function of D_0, then equation (16) has only one DSD parameter D_0 as the unknown variable, and we have two simultaneous equations of the form

$$\begin{aligned} g_1(D_0)\frac{dD_0(r)}{dr} &= f_1(r) + h_1(D_0), \\ g_2(D_0)\frac{dD_0(r)}{dr} &= f_2(r) + h_2(D_0). \end{aligned} \tag{35}$$

If we eliminate dD_0/dr from these equations, we obtain

$$g_2(D_0)(f_1(r) + h_1(D_0)) = g_1(D_0)(f_2(r) + h_2(D_0)). \tag{36}$$

This equation shows that $D_0(r)$ can be obtained once a N_0–D_0 relation is specified. A remarkable characteristic of this equation is that the D_0 profile can be obtained from locally measured data without knowing the attenuation to the first range gate, because (36) is not a differential equation but an ordinary equation and because $f_\lambda(r)$ includes only the derivative of $\ln Z_{m\lambda}(r)$ with respect to r which is independent of calibration. In practice, however, the measurement error in $Z_{m\lambda}(r)$ and its fluctuations prevent us from solving equation (36) with realistic values of D_0. However, if D_0 does not change very much over the path $[r_1, r_2]$, we can approximate $\ln dZ_{m\lambda}/dr$ by a finite difference $(\ln Z_{m\lambda}(r_1) - \ln Z_{m\lambda}(r_2))/(r_1 - r_2)$. Then, the fluctuations in $f_\lambda(r)$ can be suppressed to a large extent and we should be able to find a reasonable solution for D_0. Of course, other techniques to reduce fluctuating noise such as regression and smoothing can be used in estimating the slopes. We call this method the dual-frequency-with-a-single-parameter method or the DFS method for short in this paper.

2.4 *DAD Method*

The method that uses the difference of attenuation differences (DAD) between two frequencies over a certain range has been used as a standard dual frequency rain retrieval method. This method usually is applied to a system in which one radar frequency is relatively low so that the attenuation at the higher frequency channel is the dominant part of the attenuation difference between the two channels.

It is obvious from equations (11) and (23) that

$$\ln Z_{m\lambda}(r) = \ln N_0(r) + \ln I_{b\lambda}(D_0(r)) - S_\lambda(r). \tag{37}$$

From this equation, we immediately obtain

$$\begin{aligned} &[\ln Z_{m1}(r_1) - \ln Z_{m1}(r_2)] - [\ln Z_{m2}(r_1) - \ln Z_{m2}(r_2)] \\ &\quad = \ln\left(\frac{I_{b1}(D_0(r_1))I_{b2}(D_0(r_2))}{I_{b1}(D_0(r_2))I_{b2}(D_0(r_1))}\right) \\ &\quad\quad - [S_1(r_1) - S_1(r_2)] + [S_2(r_1) - S_2(r_2)]. \end{aligned} \tag{38}$$

Therefore, if

$$\frac{I_{b1}(D_0(r_1))I_{b2}(D_0(r_2))}{I_{b1}(D_0(r_2))I_{b2}(D_0(r_1))} = 1, \tag{39}$$

the lefthand side of equation (38) represents the difference of attenuation difference. If the specific attenuations k_1 and k_2 are expressed in the unit of dB/km and they are related to rainfall rate R through power laws, $k_1 = a_1 R^{b_1}$ and $k_2 = a_2 R^{b_2}$, then

$$\begin{aligned} &[\mathrm{dB}Z_{m1}(r_1) - \mathrm{dB}Z_{m1}(r_2)] - [\mathrm{dB}Z_{m2}(r_1) - \mathrm{dB}Z_{m2}(r_2)] \\ &= 2\int_{r_1}^{r_2} (a_1 R^{b_1} - a_2 R^{b_2})\, dr \\ &\approx 2(r_2 - r_1)(a_1 \overline{R}^{b_1} - a_2 \overline{R}^{b_2}) \end{aligned} \tag{40}$$

where $\overline{R}$ is the average rainfall rate over the path. This approximation is valid as long as its relative fluctuation remains small and b_1 is close to 1. If channel 1 is in the Ka-band and channel 2 is in a lower frequency band, $k_1 \gg k_2$ and $b_1 \approx 1$. Therefore, (40) can be approximated by

$$2(a_1 - a_2)(r_1 - r_2)\overline{R}^{b_1} \tag{41}$$

If we denote the lefthand side of (40) by DAD (difference of attenuation differences), we can calculate the path-averaged rainfall rate from (41) as

$$\overline{R} = \left(\frac{\mathrm{DAD}}{2(a_1 - a_2)(r_1 - r_2)}\right)^{1/b_1} \tag{42}$$

Since $a_2 \ll a_1$ and b_2 is not very different from b_1, the relative error associated with the last approximation is rather small.

It is often believed that this method is applicable only when the Rayleigh approximation for scattering is valid at both frequencies. However, this is not necessarily so. The validity of this method is wider than the Rayleigh scattering case. The condition (39) is of course satisfied if scattering is the Rayleigh scattering in which case, $I_{b1} = I_{b2}$. However, the condition (39) does not require the Rayleigh scattering. The condition can be met if $D_0(r_1) = D_0(r_2)$.

Even if this condition is not met, the ratio $I_{b1}(D_0)/I_{b2}(D_0)$ does not change by more than a factor of two in the usual rain rate in which the dual-frequency method is applicable in a vertical observation with the Ka- and Ku-band combination. Consequently, unless the rainfall rate changes substantially between the two ranges r_1 and r_2, the deviations of this ratio from unity at two range gates generally cancel to result in a value close to unity. As a result, as long as we use a valid k–R relationship, this method gives a rather robust estimate of rainfall rate.

Note that this case can be obtained more generally from equation (36). If $g_1(D_0) = g_2(D_0)$ in (36), then $f_1(r) - f_2(r) = h_2(D_0) - h_1(D_0))$ or in terms of the original variables

$$\begin{aligned}&\frac{d}{dr}\ln(Z_{m1}(r)) - \frac{d}{dr}\ln(Z_{m2}(r)) = \\ &-2N_0(D_0(r))(I_{t1}(D_0(r)) - I_{t2}(D_0(r)))\end{aligned} \tag{43}$$

Integration of this equation from r_1 to r_2 will give equation (38) without the logarithmic term ($\ln(\ )$) on the righthand side. The condition $g_1(D_0) = g_2(D_0)$ is equivalent to (39), and is satisfied in the Rayleigh scattering case because $I_{b1} = I_{b2}$.

Note that specifying the k–R relationship for a given DSD model function with two parameters is equivalent to specifying the functional relationship between these parameters. Therefore, under the assumption of using a two-parameter DSD model, the formula (36) can be regarded as a generalization of the DAD method.

2.5 *Dual-Frequency Hitschfeld-Bordan Method*

Assuming a relationship between N_0 and D_0 in a two-parameter DSD model is equivalent to defining a relationship between any two of DSD integral parameters such as k_λ, $Z_{e\lambda}$ and R. Suppose that power laws hold between k_1 and Z_{e1} and between k_2 and Z_{e2}.

$$k_1 = \alpha_1 Z_{e1}^{\beta_1}, \quad k_2 = \alpha_2 Z_{e2}^{\beta_2} \tag{44}$$

Assume further that A_{b1} and A_{b2} at $r = r_b$ are not known.

$$\begin{aligned}Z_{m1}(r_b) &= A_{b1} Z_{e1}(r_b), \\ Z_{m2}(r_b) &= A_{b2} Z_{e2}(r_b)\end{aligned} \tag{45}$$

The Hitschfeld-Bordan solutions (Hitschfeld and Bordan, 1954, Iguchi et al., 1994) for $Z_{e\lambda}(r)$ in $r_b \le r \le r_e$ become

$$\begin{aligned}Z_{e1}(r; A_{b1}) &= \frac{Z_{m1}(r)}{[A_{b1}^{\beta_1} - q\beta_1\alpha_1 \int_{r_b}^{r} Z_{m1}^{\beta_1}(s)\,ds]^{1/\beta_1}} \\ Z_{e2}(r; A_{b2}) &= \frac{Z_{m2}(r)}{[A_{b2}^{\beta_2} - q\beta_2\alpha_2 \int_{r_b}^{r} Z_{m2}^{\beta_2}(s)\,ds]^{1/\beta_2}}\end{aligned} \tag{46}$$

where $q = 2/c_k$. The rainfall rate estimated by the two methods must be the same at all points, i.e., $R_1(r) = R_2(r)$ for all r. In practice, this condition cannot be satisfied because of noise in Z_{mi} and other error sources such as the deviation of the assumed power-law k_λ–$Z_{e\lambda}$ relations from the true k_λ–$Z_{e\lambda}$ relationships. We try to find the best combination of A_{b1} and A_{b2} by imposing the condition that the difference between the rainfall rates estimated from Z_{e1} and Z_{e2} must be as small as possible because they should represent the estimates of the common quantity R. In practice, we adjust A_{b1} and A_{b2} in such a way that the solutions minimize

$$\int \left(\frac{R_1(s; A_{b1}) - R_2(s; A_{b2})}{R_1(s; A_{b1}) + R_2(s; A_{b2})}\right)^2 ds \tag{47}$$

where R_1 and R_2 are calculated from the solutions Z_{e1} and Z_{e2}. If power law relations hold between them, i.e., if $R_1 = a_1 Z_{e1}^{b_1}$ and $R_2 = a_2 Z_{e2}^{b_2}$, then R_1 and R_2 are given by

$$\begin{aligned}R_1(r; A_{b1}) &= a_1 \frac{Z_{m1}^{b_1}(r)}{[A_{b1}^{\beta_1} - q\beta_1\alpha_1 \int_{r_b}^{r} Z_{m1}^{\beta_1}(s)\,ds]^{b_1/\beta_1}} \\ R_2(r; A_{b2}) &= a_2 \frac{Z_{m2}^{b_2}(r)}{[A_{b2}^{\beta_2} - q\beta_2\alpha_2 \int_{r_b}^{r} Z_{m2}^{\beta_2}(s)\,ds]^{b_2/\beta_2}}\end{aligned} \tag{48}$$

Note that two conditions such as the equality of R_1 and R_2 at two different ranges are enough to uniquely determine A_{b1} and A_{b2}. Therefore, to remove the overdeterminedness, we impose the minimization condition on the integral expressed by (47).

An important advantage of this method lies in the fact that it can estimate the attenuations A_{b1} and A_{b2}. This means we can estimate rainfall profiles even when there is significant unknown attenuation to the first range gate, or when the radar calibration factor is not well known. The method works without surface reference. A drawback is that the accuracy of the estimates is limited by the closeness of the assumed one-parameter DSD model to the true DSD.

It is worth noting here that we cannot estimate a DSD parameter in addition to A_{b1} and A_{b2} in this formulation. For example, if all k-Z_e and Z_e–R relations are fixed after normalization by N_0^*, i.e., the coefficients that appear in power laws among k/N_0^*, Z_e/N_0^*, and R/N_0^* do not change when the actual DSD changes, adjustment of N_0^* correspond to the adjustment of a_i and α_i by factors ϵ_{a_i} and ϵ_{α_i} in such a way that $a_i' = \epsilon_{a_i} a_i$ and $\alpha_i' = \epsilon_{\alpha_i}\alpha$ where $\epsilon_{a_i} = \epsilon_0^{1-b_i}$ and $\epsilon_{\alpha_i} = \epsilon_0^{1-\beta_i}$. Then we can prove that the effect of ϵ_0 and A_{b1} cannot be separated only by the conditions imposed. In other words, we cannot introduce in equations (48) another adjustable parameter ϵ_0 that changes the k–Z_e relationship and find the best set of $(A_{b1}, A_{b2}, \epsilon_0)$ by minimizing the integral (47) because ϵ_0 and A_{b1} are practically degenerated.

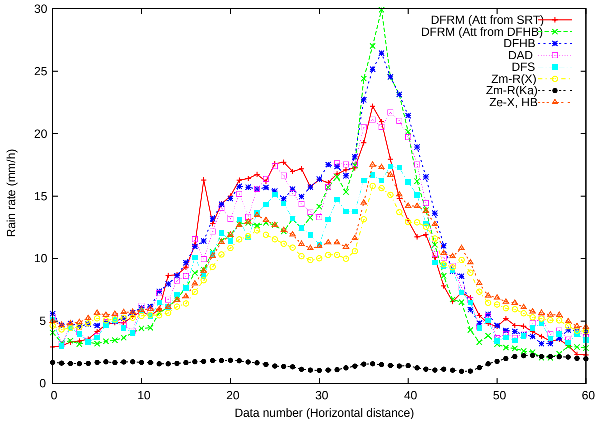

FIG. 1. Comparison of the path-averaged rain rate estimated from different methods. The abscissa shows the data number along the path of the flight. "DFRM (Att from SRT)" and "DFRM (Att from DFHB)" denote the rain estimates from Meneghini's dual-frequency method with the PIA from SRT and from DFHB, respectively. "Zm-R(X)" and "Zm-R(Ka)" are calculated by applying the Z_e–R power laws directly to measured $Z_{m,\mathrm{X}}$ and $Z_{m,\mathrm{Ka}}$ without any attenuation correction. "Ze-X, HB" denotes the Hitschfeld-Bordan method applied to the X-band channel with 0.5 dB attenuation assumed at the start of the processed interval.

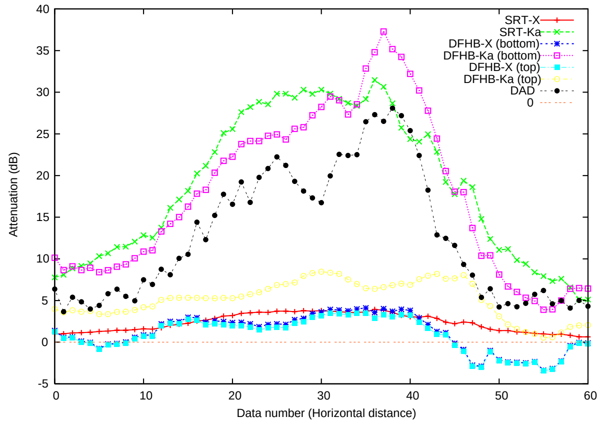

FIG. 2. Comparison of the path-integrated attenuation (PIA) to surface estimated from different methods. The dual-frequency Hitschfeld-Bordan method gives the attenuations in the X and Ka band channels at both top and bottom of the processed interval. The PIA estimates from the surface reference technique (SRT) include all attenuation from the radar to the surface, whereas the PIA estimates from "DAD" does not include the attenuation outside the processed interval of 3 km.

3. EXAMPLES

To test how each algorithm mentioned in this paper performs, we run these algorithm with an airborne dual-frequency radar data set obtained during the CaPE experiment. The combination of the frequencies is 10 GHz and 35.5 GHz. The aircraft was flying at the altitude of about 11 km. The rain observed was mostly stratiform with a clear bright band which appeared about 4.2 km above the surface. The radar echoes were sampled at every 30 m in range so that the surface echo appears around the range gate number of 370. In the current test of the algorithms, the data whose range bin numbers are between 250 and 350 were used. This range corresponds to a 3-km interval that starts at about 0.7 km below the bright band and ends at about 0.5 km above the surface. A gamma DSD model with $\mu = 3$ is assumed in the algorithms. In the DFS method, the assumption of a constant N_0^* is used.

Figure 1 shows the path-averaged rain rate estimated from different methods. Note that all dual-frequency methods give approximately similar estimates. The path-integrated attenuations (PIAs) estimated by the surface reference technique (SRT) and those by the DFHB method were used as the initial conditions in Meneghini's dual-frequency (DFRM) method. As expected, if the PIA estimates from the DFHB method are used, the rain estimates from DFRM and DFHB methods are close to each other, expecially when the attenuation is large. The rain estimates from the X-band channel and Ka-band channel by the DFHB method are virtually identical so that only the estimates from the X-band channel are shown in Fig. 1. This agreement is not surprising because the closeness of the two estimates is the condition for the solution. The DAD method and DFS method show zigzag variations compared with the rest of the estimates. These zigzag variations can probably be attributed to the fact that these methods use only the data at the end points. Note that the deviations of the estimates by all dual-frequency methods from the Z_m(X)–R estimates change their sign around data numbers 10 and 43. These changes cannot be explained by the attenuation correction alone, but by the changes in DSD parameters that are not included in the Z_e–R relationship.

Figure 2 shows the corresponding path-integrated attenuations estimated from DFHB, DAD and SRT methods. Note that the paths over which the attenuations are estimated from these methods are not the same. The SRT estimates include all attenuation from the radar to the surface. The DFHB attenuation estimates are calculated at the top and the bottom of the processed interval which correspond to about 3.5 km and 0.5 km above the sea surface, respectively. In spite of a small difference in the evaluation positions, the attenuation estimates at the bottom of the interval from the DFHB method agree fairly well with the attenuation estimates from the SRT. Apart from possible errors in the SRT estimates, the discrepancies between the DFHB and SRT attenuation estimates and the negative attenuation that appears in the X-band DFHB estimates can probably be attributed to the inappropriate DSD model assumed in the DFHB algorithm.

DAD in Fig. 2 refers to the attenuation difference between the two channels directly calculated from the measured $Z_{m,\mathrm{Ka}}$ and $Z_{m,\mathrm{X}}$ at the ends of the 3-km interval that does not include the bright band. The DADs calculated by using the attenuations from the DFHB method agree fairly well with the directly calculated DADs (not shown in the figure).

4. CONCLUSIONS

It is shown that if the attenuation to the farthest point in the interval of rain retrieval is available, we can use it as the initial condition to solve the differential equations for the two-parameter DSD model which gives a fairly accurate estimate of rainfall rate. In this case, we solve the equation backwards in range. If the total attenuation is not significant, the attenuation can be given at any point within the interval and we can still solve the equations forward without introducing too much error. In practice, however, there may be some unknown attenuation or uncertainty in the measured radar reflectivity factors. This paper proposes the dual-frequency Hitschfeld-Bordan (DFHB) method as a way to mitigate the difficulty. In this method we need to use a single parameter DSD model so that the accuracy of the retrieved rain rate may decrease if the actual DSD deviates substantially from the model function. Nevertheless, this method is free from the calibration error or unknown attenuation to the retrieving interval. This is a great advantage especially over land where the surface reference is not necessarily reliable. Compared with the traditional DAD (difference of the attenuation differences) method which only give the path-averaged rain rate, the DFHB method provides a rain profile that may change with range.

We need more simulations and sensitivity tests to find the applicable range of each method for variety of rain intensities and to evaluate the potential errors and biases. At the same time, we need to examine the effects of the factors ignored in this paper: The most important one may be the effect of inhomogeneity of rain distribution within the radar's instantaneous field of view.

Acknowledgments. This study was supported in part by the Japan Aerospace Exploration Agency (JAXA). Encouragement by Prof. Kenji Nakamura of Nagoya University is gratefully acknowledged. Special thanks go to Dr. Robert Meneghini for his valuable comments on this work.

APPENDIX

Constant N_0^* Implies $N_0 \propto D_0^{-\mu}$ in a Gamma DSD Model

N_0^* (Testud et al., 2001) is defined by

$$N_0^* = \frac{128}{3}\frac{M_3^5}{M_4^4} = \frac{128}{3}\frac{M_3}{D_m^4} \qquad (A1)$$

where

$$M_i = \int D^i N(D)\, dD, \quad D_m = \frac{M_4}{M_3}. \qquad (A2)$$

If we assume a gamma distribution, $D_0 = (\mu + 3.67)/\Lambda$, and

$$M_i = N_0 \frac{\Gamma(i+\mu+1)}{\Lambda^{i+\mu+1}}. \qquad (A3)$$

Therefore,

$$N_0^* = N_0 D_0^{\mu} \frac{128\,\Gamma(4+\mu)}{3\,(4+\mu)^4\,(3.67+\mu)^{\mu}} \qquad (A4)$$

If we can assume μ and N_0^* are constant, then this equation shows that N_0 is proportional to $D_0^{-\mu}$ and N_0 can be expressed as a power function of D_0.

REFERENCES

Hitschfeld, W. and J. Bordan, 1954: Errors inherent in the radar measurement of rainfall at attenuating wavelengths. *J. Meteor.*, **11**, 58–67.

Iguchi, T., and R. Meneghini, 1995: Differential equations for dual-frequency radar returns, *27th International Conf. on Radar Meteor.*, Vail, CO, **5B.5**, 190–192.

Iguchi, T., H Hanado, and T. Kozu, 2001: Accuracy of Rain Rate Retrieval from Dual-Frequency Precipitation Radar Data, *IGARSS 2001*, Sydney, Australia.

Mardiana, R., T. Iguchi, and N. Takahashi, 2004: A dual-frequency rain profiling algorithm without the use of the surface reference technique. *IEEE Trans. Geosci. Remote Sens.*, **42**, 2214-2225.

Marzoug, M., and J. P. Amayenc, 1994: A class of single- and dual-frequency algorithms for rain-rate profiling from a spaceborne radar. Part I: Principle and tests from numerical simulations, *J. of Atmos. Oceanic Technol.*, **11**, 1480-1506.

Meneghini, R., T. Kozu, H. Kumagai, and W. C. Boncyk, 1992: A study of rain estimation methods from space using dual-wavelength radar measurements at near-nadir incidence over ocean, *J. of Atmos. Oceanic Technol.*, **9**, 364–382.

Press, W.H., S.A. Teukolsky, W. T. Vetterling, and B.P. Flannery, 1992: Numerical Recipes in C, 2nd Edition, Cambridge University Press. 994 pp.

Testud. J., J. Oury, R.A. Black, P. Amayenc, and X. Dou, 2001: The concept of "normalized" distribution to describe raindrop spectra: a tool for cloud physics and cloud remote sensing. *J. Appl. Meteor.*, **40**, 1118-1140.