Discrete Gust Model for Launch Vehicle Assessments

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Introduction

Analysis of spacecraft vehicle responses to atmospheric wind gusts during flight is important in the establishment of vehicle design structural requirements and operational capability. Typically, wind gust models can be either a spectral type determined by a random process having a wide range of wavelengths, or a discrete type having a single gust of predetermined magnitude and shape. Classical discrete models used by NASA during the Apollo and Space Shuttle Programs included a 9 m/sec quasi-square-wave gust with variable wavelength from 60 to 300 m (NASA, 2000). A later study derived discrete gust from a military standard (MIL-STD) document that used a "1-cosine" shape (Adelfang and Smith, 1998). The MIL-STD document contains a curve of non-dimensional gust magnitude as a function of non-dimensional gust half-wavelength based on the Dryden spectral model, but fails to list the equation necessary to reproduce the curve (DoD, 1990). Therefore, previous studies could only estimate a value of gust magnitude from the curve, or attempt to fit a function to it (Adelfang and Smith, 1998). Furthermore, the MIL-STD curve is based on a 1% risk gust magnitude, so there was no way to determine gust magnitudes for other risk levels. The development of the MIL-STD curve is provided herein, and the necessary information to calculate discrete gust magnitudes as a function of both gust half-wavelength and the desired probability level of exceeding a specified gust magnitude are provided.

Background

For analyses of spacecraft vehicles, wind gusts can be treated as either random (spectral turbulence) or discrete. For random gusts, typical spectral models include the Von Karman and Dryden turbulence models. The Von Karman model has widely been considered the more "realistic" model when it comes to defining turbulence spectra. However, due to the computational complexity of the Von Karman model, the Dryden model is typically used in aerospace vehicle analyses. The longitudinal, lateral, and vertical Dryden spectra are:

$$\Phi_u(\Omega) = \sigma_u^2 \frac{2L_u}{\pi} \frac{1}{1+(L_u\Omega)^2}$$

$$\Phi_v(\Omega) = \sigma_v^2 \frac{L_v}{\pi} \frac{1+3(L_v\Omega)^2}{\left[1+(L_v\Omega)^2\right]^2} \qquad (1)$$

$$\Phi_w(\Omega) = \sigma_w^2 \frac{L_w}{\pi} \frac{1+3(L_w\Omega)^2}{\left[1+(L_w\Omega)^2\right]^2}$$

where Ω is spatial frequency (wavenumber), σ is the turbulence standard deviation, L is the turbulence scale length, and the subscripts u, v, and w denote the longitudinal, lateral, and vertical components, respectively (NASA, 2000). The longitudinal component of turbulence is parallel to the steady-state wind vector, while the lateral and vertical components are perpendicular to it. The non-dimensional Dryden spectra are shown in Figure 1.

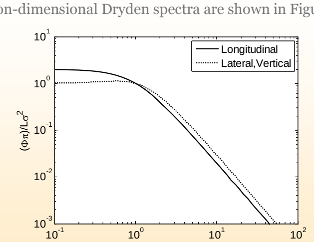

Figure 1. Non-dimensional Dryden spectra for the longitudinal, lateral, and vertical components of turbulence.

An important detail needed in the development is that the autocorrelation (lag correlation) is determined by taking the inverse Fourier transform of the Dryden spectra. The autocorrelation describes the correlation between gusts separated by a distance and are given by:

$$R_u(d) = e^{\frac{-d}{L_u}}$$

$$R_v(d) = e^{\frac{-d}{L_v}}\left(1-\frac{d}{2L_v}\right) \qquad (2)$$

$$R_w(d) = e^{\frac{-d}{L_w}}\left(1-\frac{d}{2L_w}\right)$$

where d is the lag distance. The autocorrelations as a function of non-dimensional values d/L are given in Figure 2.

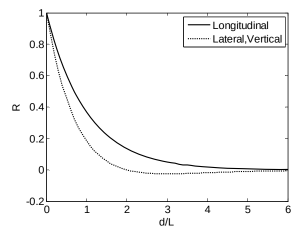

Figure 2. Autocorrelation as a function of non-dimensional values d/L.

The discrete gust provides a "spike-type" input whose magnitude is based on information from the spectral model. Along with the gust magnitude, the gust gradient over a specified temporal or spatial interval is also important. The classical shape of the discrete gust is that of a "1-cosine" shape given in Equation 3 and shown in Figure 3 as a function of gust width (Chalk et al, 1969).

$$V = \begin{cases} 0 & x < 0 \\ \frac{V_m}{2}\left[1-\cos\left(\frac{\pi x}{d_m}\right)\right] & 0 \le x \le 2d_m \\ 0 & x > 2d_m \end{cases} \qquad (3)$$

Here, V is the gust magnitude at distance x, and V_m is the gust magnitude at d_m, the gust half-width. Several values of d_m can be chosen to tune the gust width to excite desired vehicle responses. The next section describes the development of the methodology to determine appropriate discrete gust magnitudes for given gust half-widths.

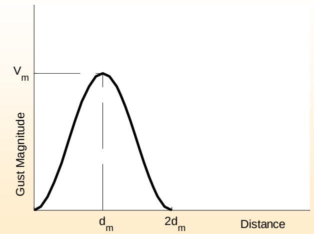

Figure 3. Graphical depiction of the 1-cosine discrete gust.

Development of Discrete Gust Magnitudes

Generally, random gusts about a mean wind are considered to be normally distributed. This is true for each of the gust components (longitudinal, lateral, and vertical). Therefore, the random gusts, V, will have a probability density function (pdf) of the form:

$$f(V) = \frac{1}{\sqrt{2\pi\sigma_V^2}}\exp\left(-\frac{1}{2}\frac{(V-\mu_V)^2}{\sigma_V^2}\right) \qquad (4)$$

where μ_V and σ_V are the mean and standard deviation of the gusts. An initial gust, V_1, will be related to a gust some distance away, V_2, by the conditional probability:

$$f(V_2\,|\,V_1) = \frac{f(V_1,V_2)}{f(V_1)} \qquad (5)$$

where $f(V_1,V_2)$ is the joint pdf of the two gusts. The mean values of the gust, V_1 and V_2, are simply the value of the mean wind. The important factor in this development is the deviation of gusts around a mean, not the value of the mean itself. Therefore, these can be removed by setting them equal to zero. If the initial gust, V_1, is assumed to start at zero, then Equation 5 will reduce to:

$$f(V_2\,|\,V_1=0) = \frac{1}{\sqrt{2\pi\sigma^2(1-\rho^2)}}\exp\left(-\frac{1}{2}\frac{1}{1-\rho^2}\frac{V_2^2}{\sigma^2}\right) \qquad (6)$$

where ρ is the correlation between V_1 and V_2. If we let

$$e = \sigma\sqrt{1-\rho^2} \qquad (7)$$

and substitute into Equation 6, we get the common form of the normal pdf (see Equation 4) :

$$f(V_2\,|\,V_1=0) = \frac{1}{\sqrt{2\pi e^2}}\exp\left(-\frac{1}{2}\frac{V_2^2}{e^2}\right) \qquad (8)$$

The cumulative distribution function (cdf) then becomes:

$$F(V_2) = P(x \le V_2) = \frac{1}{\sqrt{2\pi e^2}}\int_{-\infty}^{V_2}\exp\left(-\frac{1}{2}\frac{V^2}{e^2}\right)dV \qquad (9)$$

Since Equation 9 can not be integrated in closed form, computer routines are relied upon to determine the normal cdf. To determine the gust magnitude value for V_2, computer routines to calculate the inverse of the normal cdf are needed. Here, one would input the probability level (P), the mean of the gusts (μ = 0), and the standard deviation (e). To determine e, substitute the appropriate value of R from Equation 2 for ρ in Equation 7. Appropriate turbulence standard deviations, σ, and length scales, L, are provided in Table 2-79b of NASA, 2000 for various altitudes and turbulent cases (light, moderate, and severe). Table 1 below lists the values from 1 to 10 kilometers for the severe case.

Table 1. Severe turbulence standard deviations and length scales for the longitudinal (U), lateral (V), and vertical (W) turbulent components.

Altitude (km)	Standard Deviation (m/sec)		Length Scale (m)	
	U	V, W	U	V, W
1	5.70	4.67	832	624
2	5.80	4.75	902	831
4	6.24	5.13	1040	972
6	7.16	5.69	1040	1010
8	7.59	5.98	1040	980
10	7.72	6.00	1230	1100

Example Calculation

As an example, a longitudinal gust magnitude for severe turbulence at the 10 km level for a 500 m gust half-width is computed. The turbulence standard deviation and length scale at this level are σ = 7.72 m/s and L = 1230 m. The risk of exceeding the gust magnitude is 1%. Therefore, the probability to input into the inverse normal computer routine is P=1−(0.01/2)=0.995. The resulting gust magnitude is 14.83 m/sec. Figure 4 shows the longitudinal gust magnitude for this example with gust half-widths in the range 1 to 10000 m.

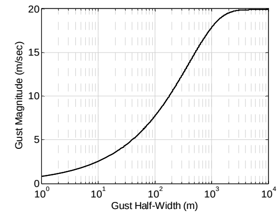

Figure 4. Longitudinal gust magnitude for varying gust half-width, σ = 7.72 m/sec and L = 1230 m.

Summary

A development method of the MIL-STD non-dimensional discrete gust magnitude curve has be presented. The development allows for slight reduction in gust magnitudes determined by other studies (Adelfang and Smith, 1998). Also, the new technique allows for selection of various risk levels of exceeding gust magnitudes (see Figure 5).

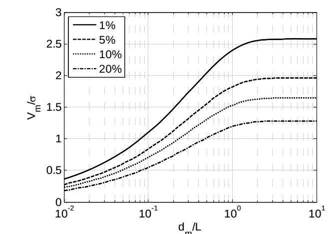

Figure 4. Non-dimensional gust half-width (d_m/L) vs. non-dimensional gust magnitude (V_m/σ) for various risk levels (MIL-STD is the 1% curve).

Acknowledgements

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