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P2.5

Variational Retrieval of Raindrop Size Distribution from Polarimetric Radar Data In Presence of Attenuation

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1. Motivation

- Raindrop size distribution (DSD) is fundamental for rain microphysics.
- The use of polarimetric radar data (PRD) can be optimized through the variational approach to mitigate the effect of measurement errors.
- Attenuation correction is the primary issue for rain retrieval from C-band and X-band radar data.

2. Methodology

❑ Variational approach

- Cost function

$$J(\mathbf{x}) = J_b(\mathbf{x}) + w_{Z_H} J_{Z_H}(\mathbf{x}) + w_{Z_{DR}} J_{Z_{DR}}(\mathbf{x}) + w_{K_{DP}} J_{K_{DP}}(\mathbf{x})$$

where, $J_b(\mathbf{x}) = \frac{1}{2}(\mathbf{x}-\mathbf{x}_b)^T \mathbf{B}^{-1}(\mathbf{x}-\mathbf{x}_b)$

$$J_{Z_H}(\mathbf{x}) = \frac{1}{2}\left[H_{Z_H}(\mathbf{x}) - \mathbf{y}_{Z_H}\right]^T \mathbf{R}_{Z_H}^{-1}\left[H_{Z_H}(\mathbf{x}) - \mathbf{y}_{Z_H}\right]$$

$$J_{Z_{DR}}(\mathbf{x}) = \frac{1}{2}\left[H_{Z_{DR}}(\mathbf{x}) - \mathbf{y}_{Z_{DR}}\right]^T \mathbf{R}_{Z_{DR}}^{-1}\left[H_{Z_{DR}}(\mathbf{x}) - \mathbf{y}_{Z_{DR}}\right]$$

$$J_{K_{DP}}(\mathbf{x}) = \frac{1}{2}\left[H_{K_{DP}}(\mathbf{x}) - \mathbf{y}_{K_{DP}}\right]^T \mathbf{R}_{K_{DP}}^{-1}\left[H_{K_{DP}}(\mathbf{x}) - \mathbf{y}_{K_{DP}}\right]$$

Use transformation: $\mathbf{v} = \mathbf{D}^{-1}\delta\mathbf{x}$, so the cost function becomes

$$J(\mathbf{v}) = \frac{1}{2}\mathbf{v}^T\mathbf{v} +$$
$$\frac{1}{2}w_{Z_H}\left[H_{Z_H}(\mathbf{x}_b + \mathbf{D}\mathbf{v}) - \mathbf{y}_{Z_H}\right]^T \mathbf{R}_{Z_H}^{-1}\left[H_{Z_H}(\mathbf{x}_b + \mathbf{D}\mathbf{v}) - \mathbf{y}_{Z_H}\right] +$$
$$\frac{1}{2}w_{Z_{DR}}\left[H_{Z_{DR}}(\mathbf{x}_b + \mathbf{D}\mathbf{v}) - \mathbf{y}_{Z_{DR}}\right]^T \mathbf{R}_{Z_{DR}}^{-1}\left[H_{Z_{DR}}(\mathbf{x}_b + \mathbf{D}\mathbf{v}) - \mathbf{y}_{Z_{DR}}\right] +$$
$$\frac{1}{2}w_{K_{DP}}\left[H_{K_{DP}}(\mathbf{x}_b + \mathbf{D}\mathbf{v}) - \mathbf{y}_{K_{DP}}\right]^T \mathbf{R}_{K_{DP}}^{-1}\left[H_{K_{DP}}(\mathbf{x}_b + \mathbf{D}\mathbf{v}) - \mathbf{y}_{K_{DP}}\right]$$

- Minimization of cost function:
- Search the gradient of cost function

$$\nabla_v J = \mathbf{v} + w_{Z_H}\mathbf{D}^T\mathbf{H}_{Z_H}^T\mathbf{R}_{Z_H}^{-1}\left(\mathbf{H}_{Z_H}^T\mathbf{D}\mathbf{v} - \mathbf{d}_{Z_H}\right) + + w_{Z_{DR}}\mathbf{D}^T\mathbf{H}_{Z_{DR}}^T\mathbf{R}_{Z_{DR}}^{-1} \times \left(\mathbf{H}_{Z_{DR}}^T\mathbf{D}\mathbf{v} - \mathbf{d}_{Z_{DR}}\right) + w_{K_{DP}}\mathbf{D}^T\mathbf{H}_{K_{DP}}^T\mathbf{R}_{K_{DP}}^{-1}\left(\mathbf{H}_{K_{DP}}^T\mathbf{D}\mathbf{v} - \mathbf{d}_{K_{DP}}\right)$$

- Mitigate low SNR effect: add matrix **w** with element
  - Fine SNR: 1 for SNR>20dB, 1/2 for SNR>10dB
  - Low SNR: 1/4 for SNR>5 dB and 1/8 for SNR<5dB
- Background error model
  - Error covariance model: Huang (2000)

$$b_{ij} = \sigma_b^2 \exp\left[-\frac{1}{2}\left(\frac{r_{ij}}{r_L}\right)^2\right]$$

  - Recursive filter: Gao et al. (2004)

❑ Forward model

- DSD model: constrained-gamma distribution

$$N(D) = N_0 D^\mu \exp(-\Lambda D)$$

$$\mu = -0.0201\Lambda^2 + 0.902\Lambda - 1.718 \quad \text{(Cao et al. 2008)}$$

- Polarimetric radar measurements

$$Z_{H,V} = \frac{4\lambda^4}{\pi^4|K_w|^2}\int_0^\infty |f_{a,b}(\pi)|^2 N(D)dD, \quad Z_{DR}(dB) = \log_{10}\frac{Z_H}{Z_V}$$

$$K_{DP} = \frac{180\lambda}{\pi}\int_0^\infty \mathrm{Re}\left[f_a(0) - f_b(0)\right]N(D)dD$$

$$A_{H,V} = 4.343\times10^3\int_0^\infty \sigma_{ext}^{H,V}(D)N(D)dD, \quad A_{DP} = A_H - A_V$$

$$Z_H^a(n) = Z_H(n) - 2\sum_{i=1}^{n-1} A_H(i)\Delta r$$

$$Z_{DR}^a(n) = Z_{DR}(n) - 2\sum_{i=1}^{n-1} A_{DP}(i)\Delta r$$

(scattering theory, T-matrix method)

❑ Lookup table method

- Tables: $Z_H, Z_{DR}, K_{DP}, \frac{\partial Z_H}{\partial\Lambda}, \frac{\partial Z_{DR}}{\partial\Lambda}, \frac{\partial K_{DP}}{\partial\Lambda}, \frac{\partial Z_H}{\partial N_0^*}, \frac{\partial Z_{DR}}{\partial N_0^*}, \frac{\partial K_{DP}}{\partial N_0^*}$
- Efficient tool:
  - Simplify the coding of $\mathbf{H}^T$: discard the adjoint
  - Reduce computation cost

❑ Retrieval procedure

- State variable:
  $\mathbf{x} = [N_0^*, \Lambda]$ where $N_0^* = \log_{10}N_0$
- Input: Z_H, Z_{DR}, K_{DP}, SNR
- Minimization: Double-layer loop

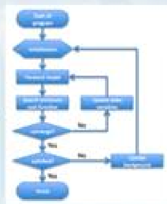


Fig. 1 (right) Flowchart of variational retrieval scheme

3. Analysis of Results

❑ Evaluation by Simulation

- 251×251 grids, 80 m resolution, 20km×20km region
- Measurement errors: 2 dB (Z_H), 0.2dB (Z_{DR}), 0.1 °km^{-1} (K_{DP})
- Analysis one: Simulation without DSD model error.
- Analysis two: Simulation with DSD model error.

Exponential DSD: $N(D) = N_0 D^\mu \exp(-\Lambda D)$

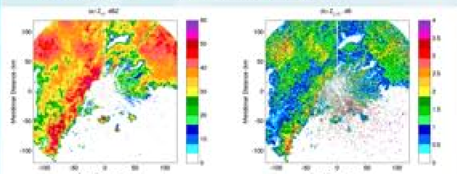
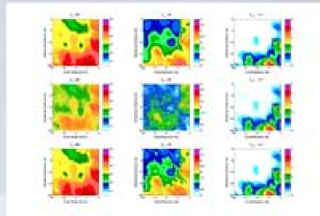


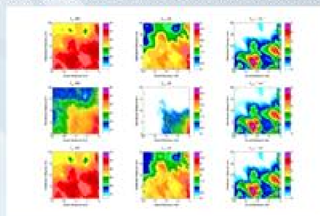
Fig. 2 (a) Z_H, (b) Z_{DR} measured by KOUN, elevation angle 0.5°, 1230 UTC, 8 May 2007. Two asterisks denote locations of KLWE (north asterisk) and KCYR (south asterisk). Two solid line boxes indicate the regions used for simulation.



Bias:
Z_H: 0.11 dB
Z_{DR}: 0.01 dB
K_{DP}: <0.001 °km^{-1}

Standard deviation (SD):
Z_H: 0.47 dB
Z_{DR}: 0.10 dB
K_{DP}: 0.06 °km^{-1}

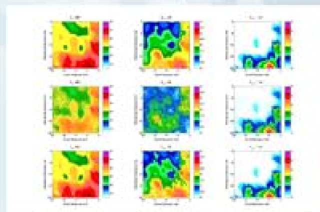
Fig. 3 Simulated PRD and retrieved results for KCYR. Three rows from top to bottom denote retrieval results, simulated PRD and truth fields. Three columns from left to right show Z_H, Z_{DR}, and K_{DP}. True DSDs are assumed to follow C-G DSD model.



Bias:
Z_H: 0.13 dB
Z_{DR}: 0.01 dB
K_{DP}: 0.006 °km^{-1}

SD:
Z_H: 0.40 dB
Z_{DR}: 0.09 dB
K_{DP}: 0.07 °km^{-1}

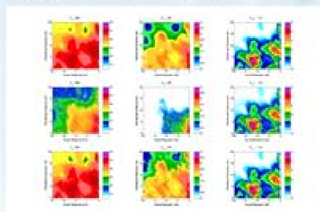
Fig. 4 The same as Fig. 3 but for KLWE



Bias:
Z_H: 0.09 dB
Z_{DR}: 0.01 dB
K_{DP}: 0.001 °km^{-1}

SD:
Z_H: 0.46 dB
Z_{DR}: 0.10 dB
K_{DP}: 0.06 °km^{-1}

Fig. 5 The same as Fig. 3 but true DSDs follow exponential DSD model.



Bias:
Z_H: 0.16 dB
Z_{DR}: 0.03 dB
K_{DP}: 0.08 °km^{-1}

SD:
Z_H: 0.46 dB
Z_{DR}: 0.11 dB
K_{DP}: 0.08 °km^{-1}

Fig. 6 The same as Fig. 5 but for KLWE

❑ Analysis of real data

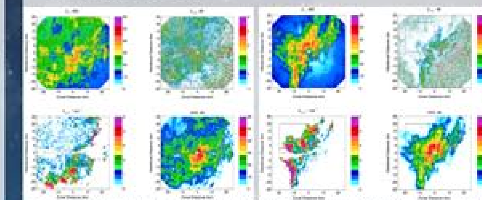


Fig. 7 KCYR (left four) and KLWE (right four) measurements: (a) Z_H, (b) Z_{DR}, (c) K_{DP}, and (d) SNR. Elevation angle 2°, 1230 UTC, 8 May 2007. The square boxes indicate analysis regions of the retrieval.

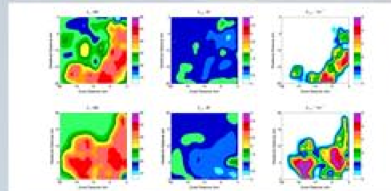


Fig. 8 Retrieved results based on constant background. KCYR (first row) and KLWE (second row). From left to right: (a) Z_H, (b) Z_{DR}, (c) K_{DP}

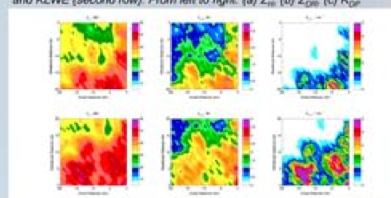


Fig. 9 The same as Fig. 8 but the background was based on the retrieval of S-band radar (KOUN) measurements.

4. Conclusions

❑ Combined DSD retrieval with the variational approach for the first time.

❑ Attenuation correction was automatically accomplished during the iterative optimization of PRD.

❑ The possible sources of errors:
- Major source: low quality of PRD
- Other sources: DSD model error; Error spatial structure; Forward model error.

❑ Potential improvements: considering contributions from snow and hail species. The classification is a challenge.

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