Suitable Background Error Covariances for Radar Reflectivity Direct Assimilation in the Rapid Refresh Forecast System (RRFS)

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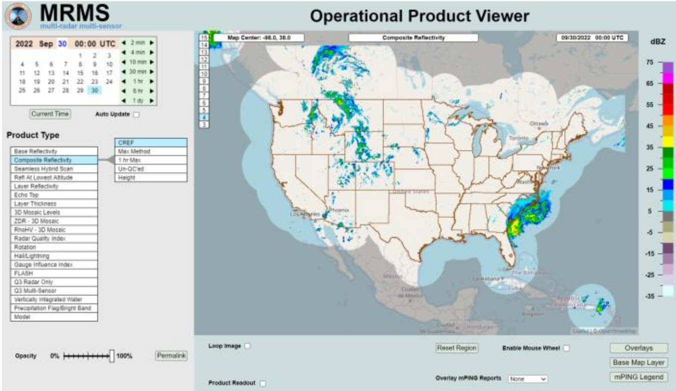
Outline

- Motivation to improve static background error covariance for radar reflectivity direct assimilation
- Introduction of two methods
  1. Ensemble-based Tangent Linear Model (ETLM)
  2. Convective-scale Static Background Error Covariance (CSB)
- Cycling tests and discussion

How to Assimilate Radar Reflectivity

The radar reflectivity observation has a lot of potential to improve precipitation forecasts. However, it is not necessarily used effectively in the operational data assimilation systems.

- **Cloud Analysis** (e.g., Albers et al. 1996)
  - After data assimilation without reflectivity, hydrometeors and thermodynamical variables are adjusted based on reflectivity.
- **1D+3DVar** (e.g., Caumont et al. 2010)
  - Atmospheric variables retrieved from reflectivity with 1DVar are assimilated with 3DVar
- **Direct assimilation** (e.g., Dowell et al., 2004)
  - Reflectivity is directly assimilated through ensemble covariances of variables estimated with ensemble forecasts



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Difficulties of direct assimilation of hydrometeor observations:
- Nonlinearity of observation operator
- Short correlation length and large bias
- How to create static B (static background error covariance)

→ **How should we defeat these difficulties?**

Radar Reflectivity Direct Assimilation

Wang and Wang (2017, *MWR*)

Cost function of Hybrid 3DEnVar:

$$J(\delta\mathbf{x}_s, \mathbf{a}_1, \dots, \mathbf{a}_K) = \frac{1}{2}\beta_s(\delta\mathbf{x}_s)^T\mathbf{B}^{-1}(\delta\mathbf{x}_s) + \frac{1}{2}\sum_{k=1}^{K}[\beta_e(\mathbf{a}_k)^T\mathbf{A}^{-1}(\mathbf{a}_k)] + \frac{1}{2}(\mathbf{H}\delta\mathbf{x} - \mathbf{d})^T\mathbf{R}^{-1}(\mathbf{H}\delta\mathbf{x} - \mathbf{d})$$

$$\delta\mathbf{x} = \delta\mathbf{x}_s + \sum_{k=1}^{K}(\mathbf{a}_k \circ \mathbf{x}_k^e)$$

Linearized observation operator
(if original operator is non-linear,
the cost is not efficiently minimized)

Here, radar reflectivity is added and analyzed together as:

$$\delta\mathbf{x}^{(\mathrm{dBZ})} = \sum_{k=1}^{K}\left(\mathbf{a}_k \circ \mathbf{x}_k^{e(\mathrm{dBZ})}\right)$$

Analysis increment of reflectivity

Ensemble perturbations of reflectivity

$\rightarrow \mathbf{H}^{(\mathrm{dBZ})} = \mathbf{I}$

Observation operator is linear (identity matrix)

- Analysis increment is computed based on the cross-variable covariance to $\mathbf{x}_k^{e(\mathrm{dBZ})}$
- However, **optimal localization scale of $\mathbf{x}_k^{e(\mathrm{dBZ})}$ is smaller** than atmospheric variables

Variable-Dependent Localization (VDL)

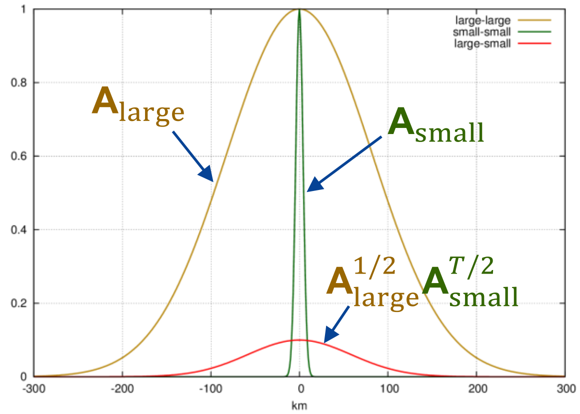
Wang and Wang (2023, *JAMES*)
Yokota et al. (2024, submitted)

Control vector:

$$\mathbf{a}_k \rightarrow \begin{bmatrix} \mathbf{a}_k^{(\mathrm{atmos})} \\ \mathbf{a}_k^{(\mathrm{hydro})} \end{bmatrix}$$

Localization:

$$\mathbf{A} \rightarrow \begin{bmatrix} \mathbf{A}_{\mathrm{large}} & c\mathbf{A}_{\mathrm{large}}^{1/2}\mathbf{A}_{\mathrm{small}}^{1/2} \\ c\mathbf{A}_{\mathrm{small}}^{1/2}\mathbf{A}_{\mathrm{large}}^{1/2} & \mathbf{A}_{\mathrm{small}} \end{bmatrix} \quad (0 \le c \le 1)$$



Analysis increment:

$$\begin{cases} \delta\mathbf{x}^{(\mathrm{atmos})} = \delta\mathbf{x}_s^{(\mathrm{atmos})} + \sum_{k=1}^{K} \left[\mathbf{a}_k^{(\mathrm{atmos})} \circ \mathbf{x}_k^{e(\mathrm{atmos})}\right] & \text{for atmospheric variables (horizontal wind, temperature, humidity, and surface pressure)} \\ \delta\mathbf{x}^{(\mathrm{hydro})} = \sum_{k=1}^{K} \left[\mathbf{a}_k^{(\mathrm{hydro})} \circ \mathbf{x}_k^{e(\mathrm{hydro})}\right] & \text{for hydrometeor variables (radar reflectivity, hydrometeors, and vertical wind)} \end{cases}$$

- Different localization scales are applied for atmospheric & hydrometeor variables
- However, $\mathbf{x}_k^{e(\mathrm{hydro})}$ is sometimes underestimated (zero in some places)
  - In such places, reflectivity is not assimilated efficiently
- In addition, $\delta\mathbf{x}_s^{(\mathrm{atmos})}$ is not affected by radar reflectivity assimilation directly
- **Static B should be improved for reflectivity assimilation**

Outline

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    1. Ensemble-based Tangent Linear Model (ETLM)
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- Cycling tests and discussion

Ensemble-Based Tangent Linear Model (ETLM)

B: background error covariance
R: observation error covariance
H: linearized observation operator
d: innovation vector
$\delta\mathbf{x}_t$: analysis increment at time t

Cost function of 3DVar: $J(\delta\mathbf{x}_0) = \frac{1}{2}(\delta\mathbf{x}_0)^T\mathbf{B}^{-1}(\delta\mathbf{x}_0) + \frac{1}{2}(\mathbf{H}\delta\mathbf{x}_0 - \mathbf{d})^T\mathbf{R}^{-1}(\mathbf{H}\delta\mathbf{x}_0 - \mathbf{d})$

If ETLM is introduced:

$$J(\delta\mathbf{x}_0) = \frac{1}{2}(\delta\mathbf{x}_0)^T\mathbf{B}^{-1}(\delta\mathbf{x}_0) + \frac{1}{2}(\mathbf{H}\mathbf{M}\delta\mathbf{x}_0 - \mathbf{d})^T\mathbf{R}^{-1}(\mathbf{H}\mathbf{M}\delta\mathbf{x}_0 - \mathbf{d})$$

$$= \frac{1}{2}(\delta\mathbf{x}_t)^T(\mathbf{M}\mathbf{B}\mathbf{M}^T)^{-1}(\delta\mathbf{x}_t) + \frac{1}{2}(\mathbf{H}\delta\mathbf{x}_t - \mathbf{d})^T\mathbf{R}^{-1}(\mathbf{H}\delta\mathbf{x}_t - \mathbf{d}) \qquad (\delta\mathbf{x}_t = \mathbf{M}\delta\mathbf{x}_0)$$

Here, $\mathbf{M} = (\mathbf{X}_t\mathbf{X}_0^T)(\mathbf{X}_0\mathbf{X}_0^T)^{-1}$

ensemble perturbations at time t

$= \frac{1}{\sqrt{N-1}}\left[\delta\mathbf{x}_{t,1}^f \quad \cdots \quad \delta\mathbf{x}_{t,N}^f\right]$ (N: ensemble size)

Localization + Simplification $\longrightarrow$

$$\boxed{\left(\mathbf{C}^{1/2} \circ \mathbf{X}_t\mathbf{X}_0^T\right)\left(\mathbf{I} \circ \mathbf{X}_0\mathbf{X}_0^T\right)^{-1}}$$

identity matrix (to be able to compute inverse matrix)
(minimum $\mathbf{X}_0\mathbf{X}_0^T$ is set to [horizontal mean] x 0.01 not to make **M** too large)

normalized localization matrix (realized by recursive filter)

- Cross-variable covariance is included in $\mathbf{C}^{1/2} \circ \mathbf{X}_t\mathbf{X}_0^T$ (excluded from $\mathbf{I} \circ \mathbf{X}_0\mathbf{X}_0^T$)
  - ➢Control variables are correlated based on **ensemble correlation** with each other
  - ➢**Reflectivity assimilation can change atmospheric variables**

Convective-Scale Static B (CSB)

Wang and Wang (2021, *MWR*)

B: background error covariance
R: observation error covariance
H: linearized observation operator
$\mathbf{d}$: innovation vector
$\delta\mathbf{x}_t$: analysis increment at time t

Cost function of 3DVar: $J(\delta\mathbf{x}_0) = \frac{1}{2}(\delta\mathbf{x}_0)^T\mathbf{B}^{-1}(\delta\mathbf{x}_0) + \frac{1}{2}(\mathbf{H}\delta\mathbf{x}_0 - \mathbf{d})^T\mathbf{R}^{-1}(\mathbf{H}\delta\mathbf{x}_0 - \mathbf{d})$

If CSB is introduced: $J(\delta\mathbf{x}_0^{CS}) = \frac{1}{2}(\delta\mathbf{x}_0^{CS})^T\mathbf{B}_{CS}^{-1}(\delta\mathbf{x}_0^{CS}) + \frac{1}{2}(\mathbf{H}\delta\mathbf{x}_0^{CS} - \mathbf{d})^T\mathbf{R}^{-1}(\mathbf{H}\delta\mathbf{x}_0^{CS} - \mathbf{d})$

$$\underbrace{\begin{bmatrix} \boldsymbol{\psi}_{sf} \\ \boldsymbol{\phi}_{vp} \\ \mathbf{t} \\ \mathbf{p}_s \\ \mathbf{q}_{rh} \end{bmatrix}}_{\delta\mathbf{x}_0}
\begin{array}{l} \text{(stream function)} \\ \text{(velocity potential)} \\ \text{(temperature)} \\ \text{(surface pressure)} \\ \text{(relative humidity)} \end{array}
\Rightarrow
\underbrace{\begin{bmatrix} \mathbf{u} \\ \mathbf{v} \\ \mathbf{t} \\ \mathbf{p}_s \\ \mathbf{q}_{rh} \\ \mathbf{w} \\ \mathbf{q}_l \\ \mathbf{q}_r \\ \mathbf{q}_s \\ \mathbf{q}_i \\ \mathbf{q}_g \\ \mathbf{q}_{dbz} \end{bmatrix}}_{\delta\mathbf{x}_0^{CS}}
\begin{array}{l} \text{(zonal wind)} \\ \text{(meridional wind)} \\ \text{(temperature)} \\ \text{(surface pressure)} \\ \text{(relative humidity)} \\ \text{(vertical wind)} \\ \text{(cloud water)} \\ \text{(rainwater)} \\ \text{(snow)} \\ \text{(cloud ice)} \\ \text{(graupel)} \\ \text{(reflectivity)} \end{array}$$

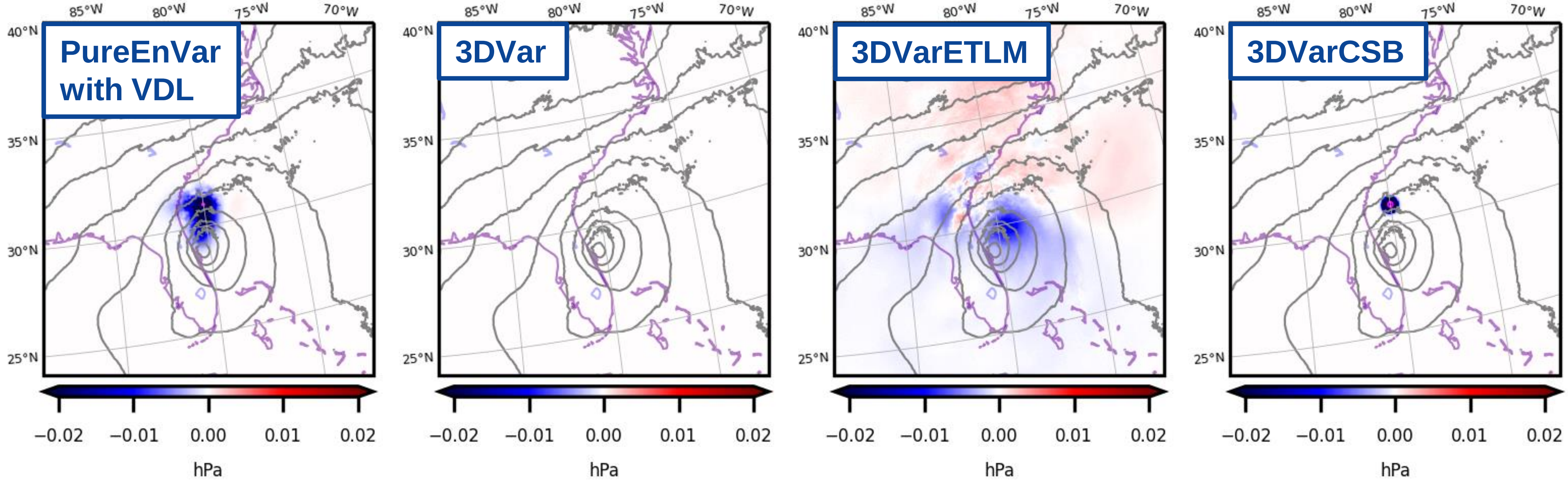
$$\underbrace{\begin{bmatrix} \mathbf{f}_\psi & 0 & 0 & 0 & 0 \\ \mathbf{r}_\phi\mathbf{f}_\psi & \mathbf{f}_\phi & 0 & 0 & 0 \\ \mathbf{r}_t\mathbf{f}_\psi & 0 & \mathbf{f}_t & 0 & 0 \\ \mathbf{r}_p\mathbf{f}_\psi & 0 & 0 & \mathbf{f}_p & 0 \\ 0 & 0 & 0 & 0 & \mathbf{f}_q \end{bmatrix}}_{\mathbf{B}^{1/2}}
\Rightarrow
\underbrace{\begin{bmatrix}
\mathbf{f}_u & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\mathbf{r}_{vu}\mathbf{f}_u & \mathbf{f}_v & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\mathbf{r}_{tu}\mathbf{f}_u & \mathbf{r}_{tv}\mathbf{f}_v & \mathbf{f}_t & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\mathbf{r}_{pu}\mathbf{f}_u & \mathbf{r}_{pv}\mathbf{f}_v & \mathbf{r}_{pt}\mathbf{f}_t & \mathbf{f}_p & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\mathbf{r}_{qu}\mathbf{f}_u & \mathbf{r}_{qv}\mathbf{f}_v & \mathbf{r}_{qt}\mathbf{f}_t & \mathbf{r}_{qp}\mathbf{f}_p & \mathbf{f}_q & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\mathbf{r}_{wu}\mathbf{f}_u & \mathbf{r}_{wv}\mathbf{f}_v & \mathbf{r}_{wt}\mathbf{f}_t & \mathbf{r}_{wp}\mathbf{f}_p & \mathbf{r}_{wq}\mathbf{f}_q & \mathbf{f}_w & 0 & 0 & 0 & 0 & 0 & 0 \\
\mathbf{r}_{lu}\mathbf{f}_u & \mathbf{r}_{lv}\mathbf{f}_v & \mathbf{r}_{lt}\mathbf{f}_t & \mathbf{r}_{lp}\mathbf{f}_p & \mathbf{r}_{lq}\mathbf{f}_q & \mathbf{r}_{lw}\mathbf{f}_w & \mathbf{f}_l & 0 & 0 & 0 & 0 & 0 \\
\mathbf{r}_{ru}\mathbf{f}_u & \mathbf{r}_{rv}\mathbf{f}_v & \mathbf{r}_{rt}\mathbf{f}_t & \mathbf{r}_{rp}\mathbf{f}_p & \mathbf{r}_{rq}\mathbf{f}_q & \mathbf{r}_{rw}\mathbf{f}_w & \mathbf{r}_{rl}\mathbf{f}_l & \mathbf{f}_r & 0 & 0 & 0 & 0 \\
\mathbf{r}_{su}\mathbf{f}_u & \mathbf{r}_{sv}\mathbf{f}_v & \mathbf{r}_{st}\mathbf{f}_t & \mathbf{r}_{sp}\mathbf{f}_p & \mathbf{r}_{sq}\mathbf{f}_q & \mathbf{r}_{sw}\mathbf{f}_w & \mathbf{r}_{sl}\mathbf{f}_l & \mathbf{r}_{sr}\mathbf{f}_r & \mathbf{f}_s & 0 & 0 & 0 \\
\mathbf{r}_{iu}\mathbf{f}_u & \mathbf{r}_{iv}\mathbf{f}_v & \mathbf{r}_{it}\mathbf{f}_t & \mathbf{r}_{ip}\mathbf{f}_p & \mathbf{r}_{iq}\mathbf{f}_q & \mathbf{r}_{iw}\mathbf{f}_w & \mathbf{r}_{il}\mathbf{f}_l & \mathbf{r}_{ir}\mathbf{f}_r & \mathbf{r}_{is}\mathbf{f}_s & \mathbf{f}_i & 0 & 0 \\
\mathbf{r}_{gu}\mathbf{f}_u & \mathbf{r}_{gv}\mathbf{f}_v & \mathbf{r}_{gt}\mathbf{f}_t & \mathbf{r}_{gp}\mathbf{f}_p & \mathbf{r}_{gq}\mathbf{f}_q & \mathbf{r}_{gw}\mathbf{f}_w & \mathbf{r}_{gl}\mathbf{f}_l & \mathbf{r}_{gr}\mathbf{f}_r & \mathbf{r}_{gs}\mathbf{f}_s & \mathbf{r}_{gi}\mathbf{f}_i & \mathbf{f}_g & 0 \\
\mathbf{r}_{zu}\mathbf{f}_u & \mathbf{r}_{zv}\mathbf{f}_v & \mathbf{r}_{zt}\mathbf{f}_t & \mathbf{r}_{zp}\mathbf{f}_p & \mathbf{r}_{zq}\mathbf{f}_q & \mathbf{r}_{zw}\mathbf{f}_w & \mathbf{r}_{zl}\mathbf{f}_l & \mathbf{r}_{zr}\mathbf{f}_r & \mathbf{r}_{zs}\mathbf{f}_s & \mathbf{r}_{zi}\mathbf{f}_i & \mathbf{r}_{zg}\mathbf{f}_g & \mathbf{f}_z
\end{bmatrix}}_{\mathbf{B}_{CS}^{1/2}}$$

$\mathbf{f}_x$: recursive filter
$\mathbf{r}_x$: balance operators

- Cross-variable covariance is included in $\mathbf{B}_{CS}$
  - ➢ Control variables are correlated based on **balance operators** with each other
  - ➢ **Reflectivity assimilation can change atmospheric & hydrometeor variables**

Single-Reflectivity Assimilation Test

Analysis increment (color) and first guess (contours) of sea-level pressure (hPa) and analysis increment of reflectivity (magenta, 30dBZ) in reflectivity assimilation at 1-km height (innovation: 50dBZ, obs error: 1dBZ)



Flow-dependent analysis within localization

No increment

Flow-dependent and broader, but no change for hydrometeors

Successful hydrometeor analysis, but narrow for surface pressure

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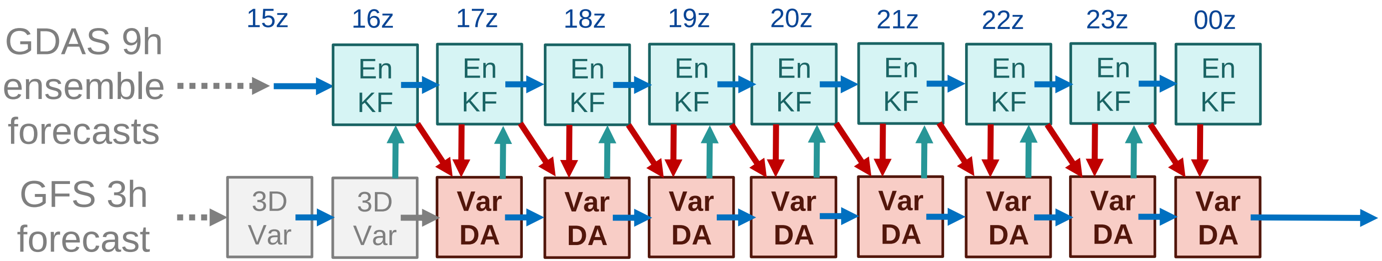
Cycling Tests

: Initial conditions
: FV3LAM-based forecasts
: Error covariance / Re-centering



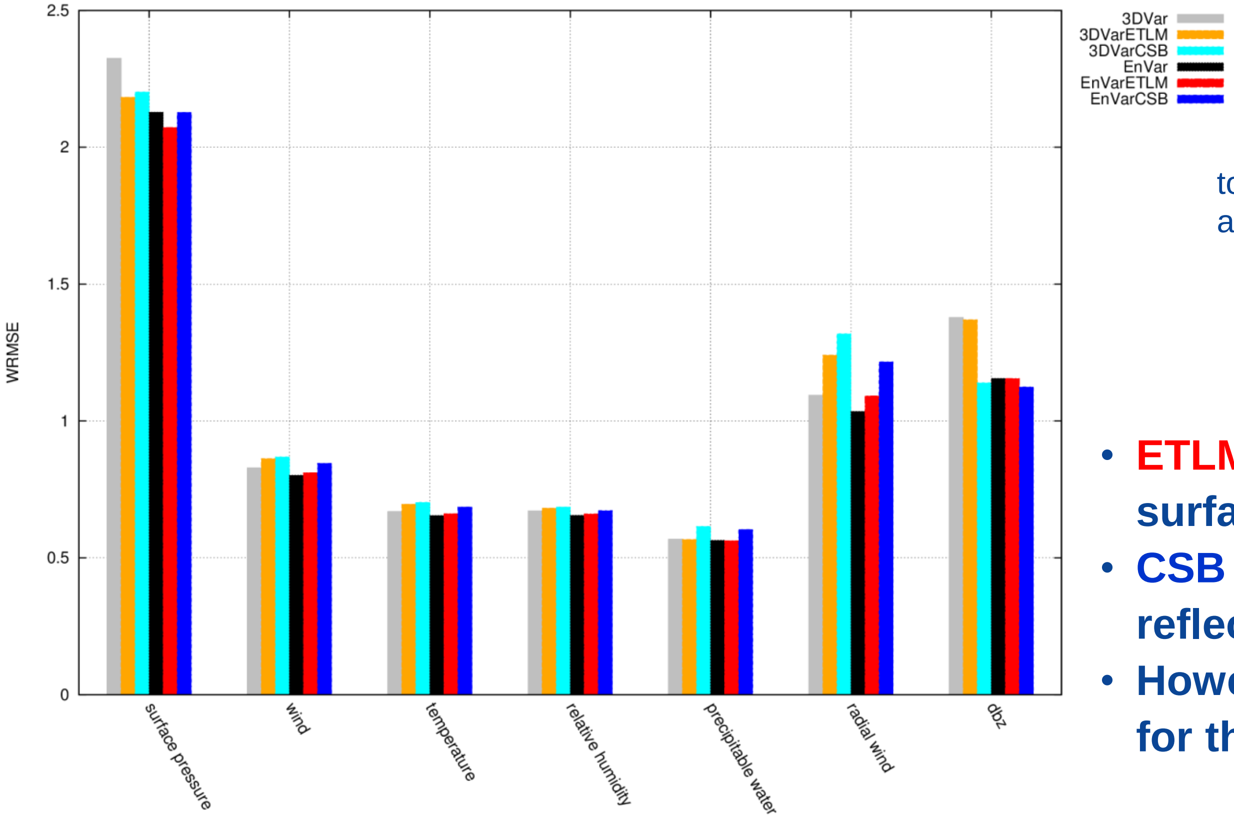
- Period: September 29, 2022
- Region: CONUS
- Ensemble size: 30

VarDA name	Weight of (static B, ensemble B)	ETLM?	CSB?
3DVar	(1.0, 0.0)	-	-
3DVarETLM	(1.0, 0.0)	Yes	-
3DVarCSB	(1.0, 0.0)	-	Yes
EnVar	(0.5, 0.5)	-	-
EnVarETLM	(0.5, 0.5)	Yes	-
EnVarCSB	(0.5, 0.5)	-	Yes

- Assimilated observations:
  - surface pressure, wind, temperature, relative humidity, precipitable water vapor, radar radial wind, and **radar reflectivity**
- Localization for ensemble B ($e^{-20/3}$ scale):
  - 300 km (horizontally) for atmospheric variables
  - 15 km (horizontally) for hydrometeor variables
    - Cross-variable covariances: x0.05 (=15/300)
  - 1.1 lnp (vertically)
- Localization for ETLM ($e^{-20/3}$ scale)
  - 300 km (only horizontally)

Weighted RMSE of First Guess

Weighted RMSE (WRMSE) for observations assimilated in **20220929 18-24z**



$$\mathrm{WRMSE} = \sqrt{\frac{2}{N_o}\sum_{i=1}^{N_t} J_i^{Init}}$$

number of analyses (N_t)

total number of observations assimilated in all analyses (N_o)

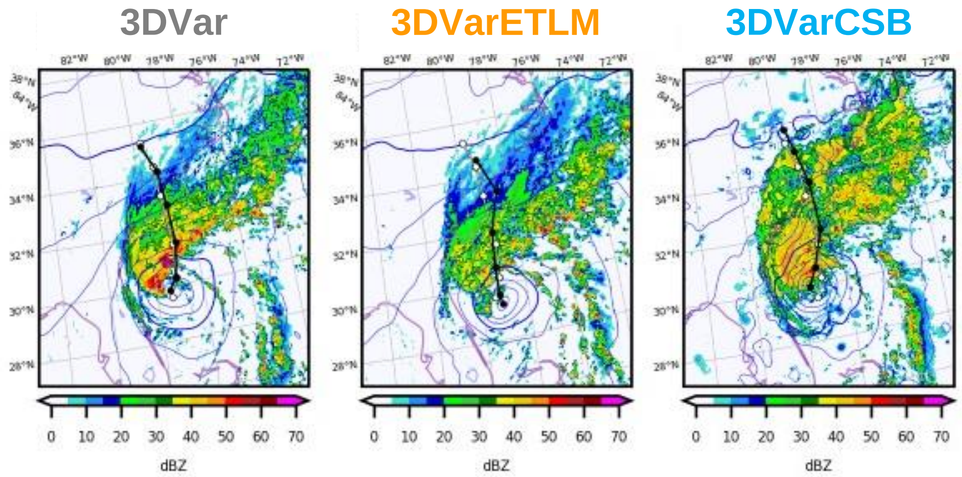
initial cost at *i*-th analysis (J_i^{Init})

(Duc and Saito 2018, *QJRMS*)

- **ETLM makes smaller WRMSE for surface pressure (and reflectivity)**
- **CSB makes smaller WRMSE for reflectivity (and surface pressure)**
- **However, both make larger WRMSE for the other observations**

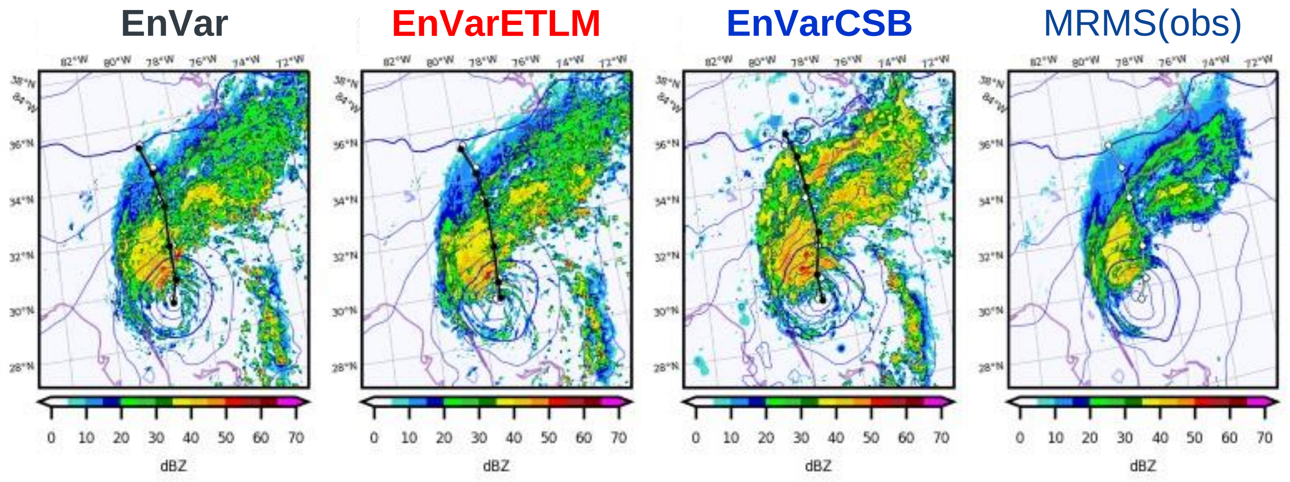
Hurricane Ian (2022) Analysis

Analysis of composite reflectivity (color, dBZ) and sea-level pressure (contours, every 4 hPa) at **20220930 00z**



Black: Hurricane track forecast from these analyses

White: Best track



- **Both ETLM and CSB improve reflectivity distribution**
- **ETLM tends to decrease reflectivity**
- **CSB tends to increase reflectivity**

Summary and Future Works

- **Static background error covariance (static B) was improved by ETLM and CSB for radar reflectivity assimilation.**

- **Ensemble-based tangent linear model (ETLM)**
  - Impact of radar reflectivity assimilation is broad and flow-dependent for atmospheric variables, but no impact for hydrometeors
  - ETLM tends to decrease precipitation and makes RMSE smaller mainly for surface pressure, but larger for the other observations

- **Convective-scale static B (CSB)**
  - Hydrometeor is successfully analyzed, but correlation length is too short for atmospheric variables
  - CSB tends to increase precipitation and makes RMSE smaller mainly for reflectivity, but larger for the other observations

- **Future works**
  - Simultaneous application of conventional B, ETLM, and CSB
  - Development of multiscale static B

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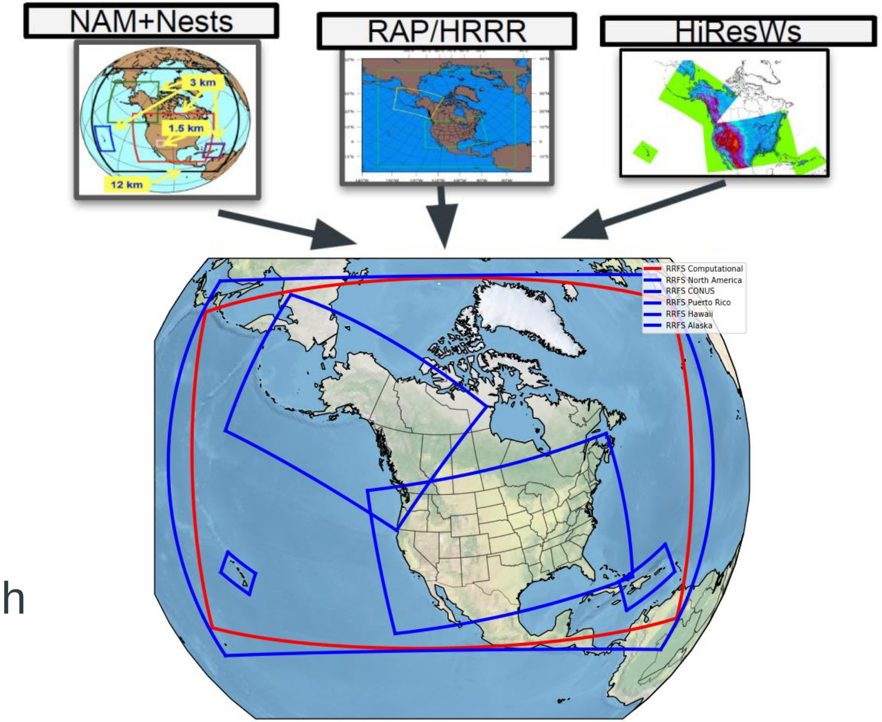
BACK UP

Rapid Refresh Forecast System (RRFS)

- **Next-generation convection-allowing operational forecast system in NCEP**
  - One of the UFS applications
  - Based on the FV3 limited area model (LAM) (Black et al. 2021, *JAMES*)
  - 3-km horizontal grid
  - 65 vertical levels
  - Hourly updated by hybrid 3DEnVar (with 30-member EnKF)
  - Deterministic forecasts to at least 18h every 1h
  - Deterministic & ensemble forecasts to 60h every 6h (6 members x 2 initial times)



RRFSv1 Compute Domain (red)

The impacts of ETLM and CSB for radar reflectivity assimilation are clarified in RRFS

Coding ETLM in GSI

Implement in subroutine “c2s” (and “c2s_ad”)

$$\mathbf{B}_{\mathrm{ens},t} = \mathbf{C}_{\mathrm{ens}} \circ \mathbf{X}_t \mathbf{X}_t^T$$

Localization for EnVar

$$\mathbf{X}_t = \frac{1}{\sqrt{N-1}} \left[\delta\mathbf{x}_{t,1}^f \quad \cdots \quad \delta\mathbf{x}_{t,N}^f \right]$$

$$\mathbf{B}_{\mathrm{ETLM}} = \mathbf{M}_{\mathrm{ens}} \mathbf{B}_{\mathrm{static}} \mathbf{M}_{\mathrm{ens}}^T$$

ETLM: $\mathbf{M}_{\mathrm{ens}} = \left(\mathbf{C}_{\mathrm{ETLM}}^{1/2} \circ \mathbf{X}_t \mathbf{X}_0^T \right) \left(\mathbf{I} \circ \mathbf{X}_0 \mathbf{X}_0^T \right)^{-1}$

Localization for ETLM

Identify matrix (diagonal)

```
call control2state(hat,mval,bias) !transform hat to mval (and bias)
if (l4dvar) then
   if (l_hyb_ens) then
      call ensctl2state(hat,mval(1),eval)
      mval(1)=eval(1)
   end if
   call model_tl(mval,val,llprt)  !TLM from mval to val
else
   if (l_hyb_ens) then
      if (l_etlm) then
         call etlm(mval(1),emval(1:nobs_bins),0) !ETLM from mval to emval
         call ensctl2state(hat,emval,eval)       !transform hat to eval and add emval
      else
         call ensctl2state(hat,mval(1),eval)     !transform hat to eval and add mval
      end if
      do ii=1,nobs_bins
         val(ii)=eval(ii)
      end do
   else
      do ii=1,nobs_bins
         val(ii)=mval(1)
      end do
   end if
end if
```

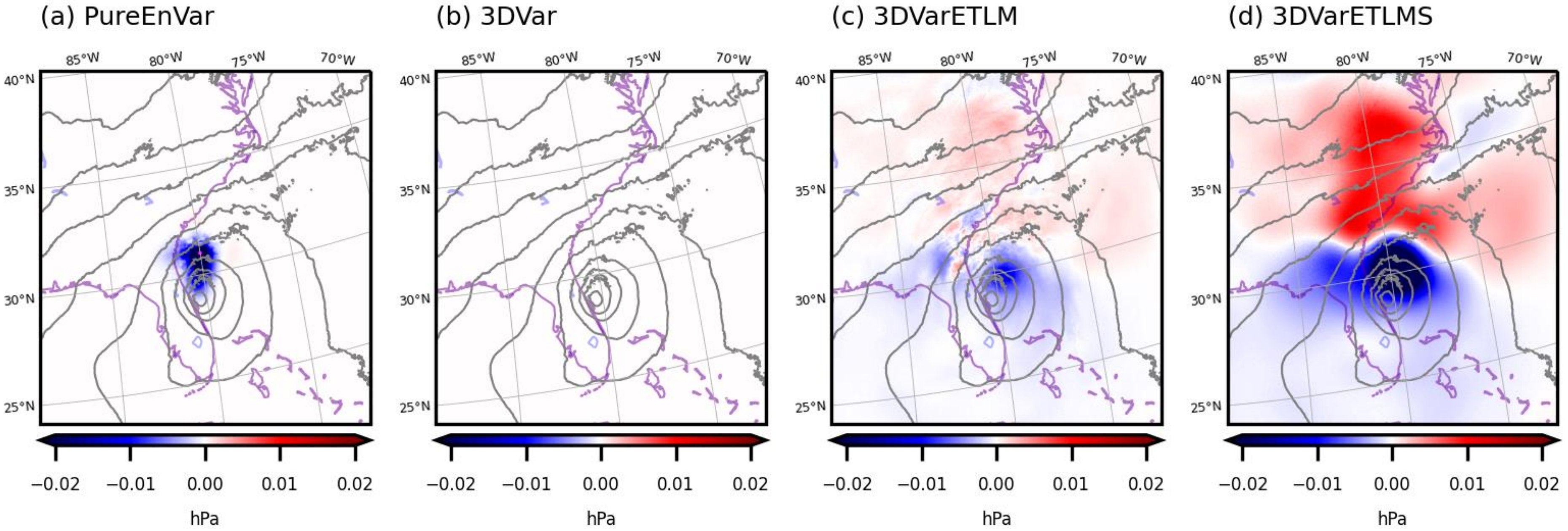
$\mathbf{B}_{\mathrm{static}}^{1/2} \rightarrow \mathbf{M}\mathbf{B}_{\mathrm{static}}^{1/2}$

$\mathbf{B}_{\mathrm{static}}^{1/2} \rightarrow \mathbf{M}_{\mathrm{ens}} \mathbf{B}_{\mathrm{static}}^{1/2}$

Single-Reflectivity Assimilation Test with ETLM

Analysis increment (color) and first guess (contours) of surface pressure (hPa)
in assimilation of radar reflectivity at 1-km height (innovation: 50 dBZ, observation error: 1 dBZ)

Using smoothed ensemble



Flow-dependent analysis within localization

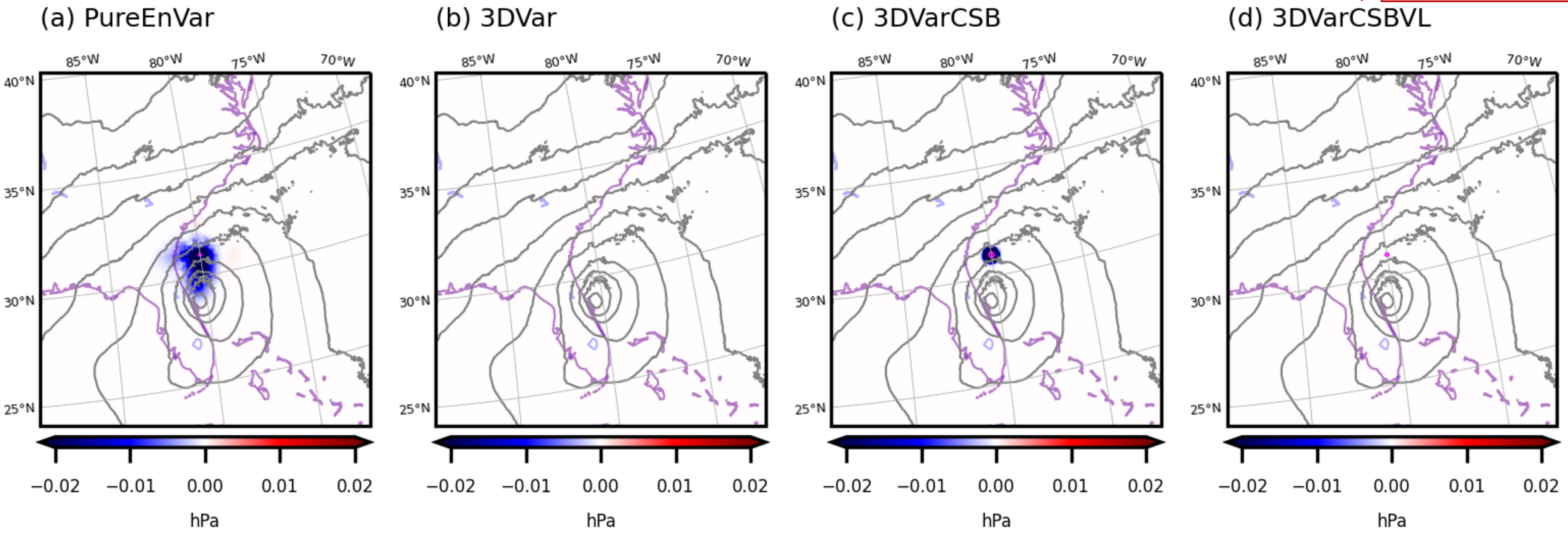
No increment

Flow-dependent and broader than PureEnVar, but no change for hydrometeors

Single-Reflectivity Assimilation Test with CSB

Analysis increment (color) and first guess (contours) of surface pressure (hPa)
in assimilation of radar reflectivity at 1-km height (innovation: 50 dBZ, observation error: 1 dBZ)

Without cross-variable covariances



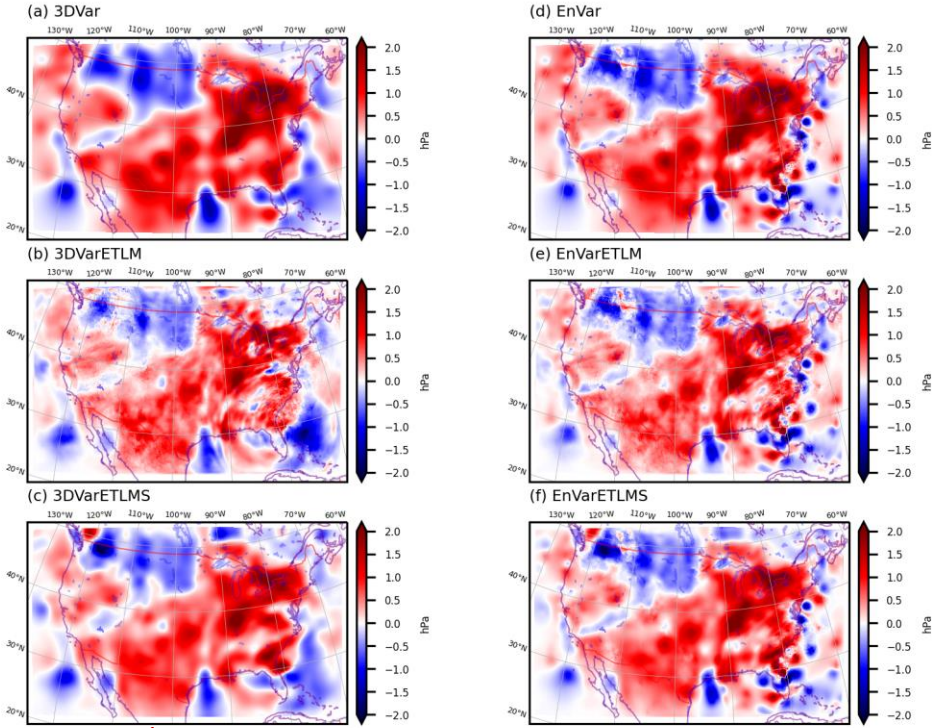
Flow-dependent analysis within localization

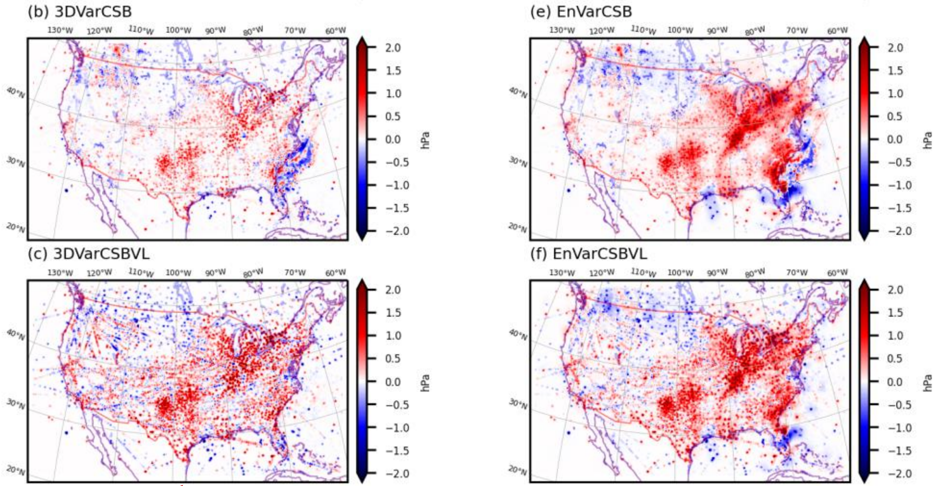
No increment

Successful hydrometeor analysis, but small correlation length for atmospheric variables

Analysis Increments

Analysis increment of surface pressure at **20220929 17z**



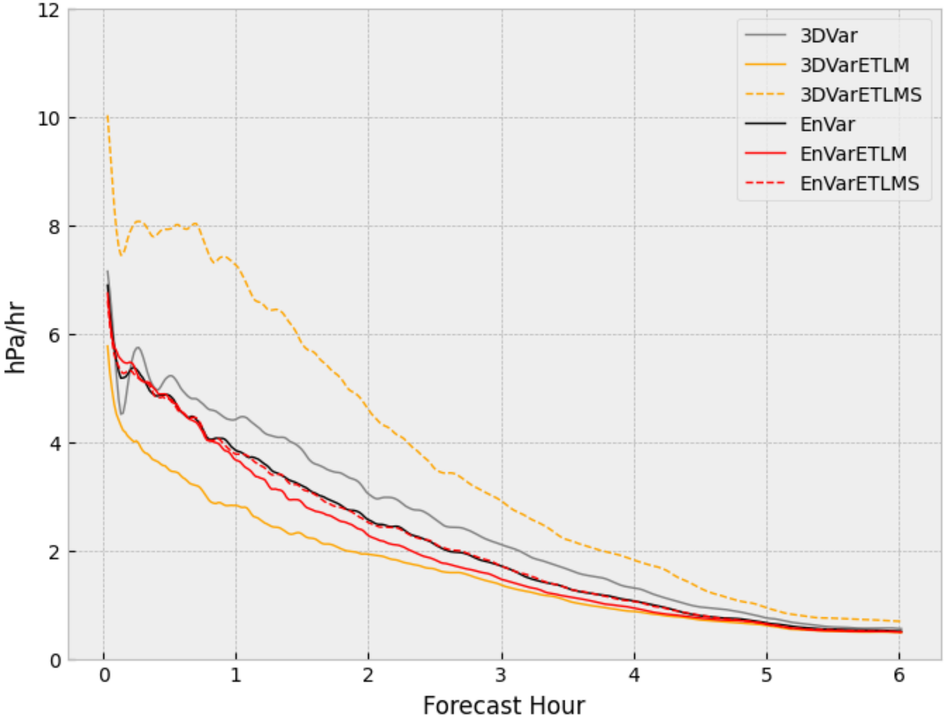


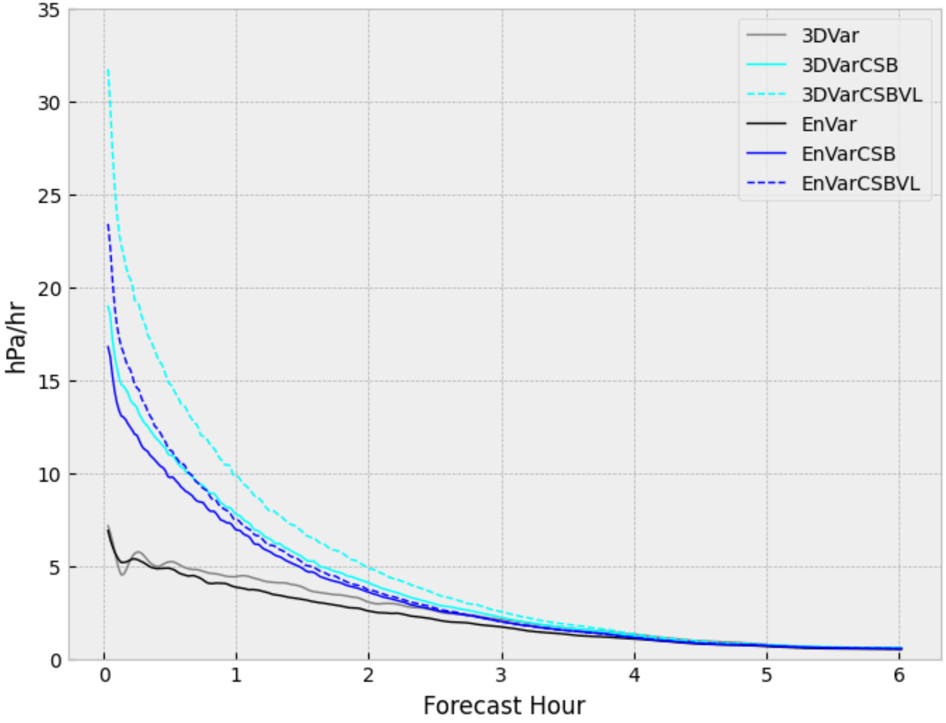
Using smoothed ensemble

Without cross-variable covariances

Pressure Tendency

Time series of pressure tendency (domain-wide mean absolute change) in the forecasts from **20220930 00z**





Weighted RMSE of First Guess

Weighted RMSE (WRMSE) for observations assimilated in **20220929 18-24z**

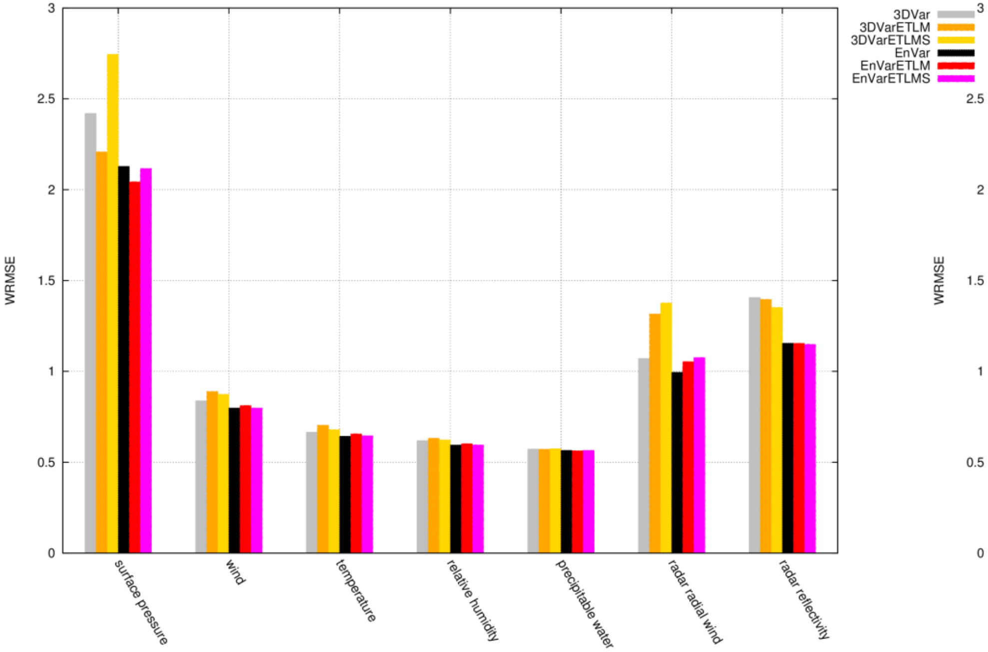
number of analyses

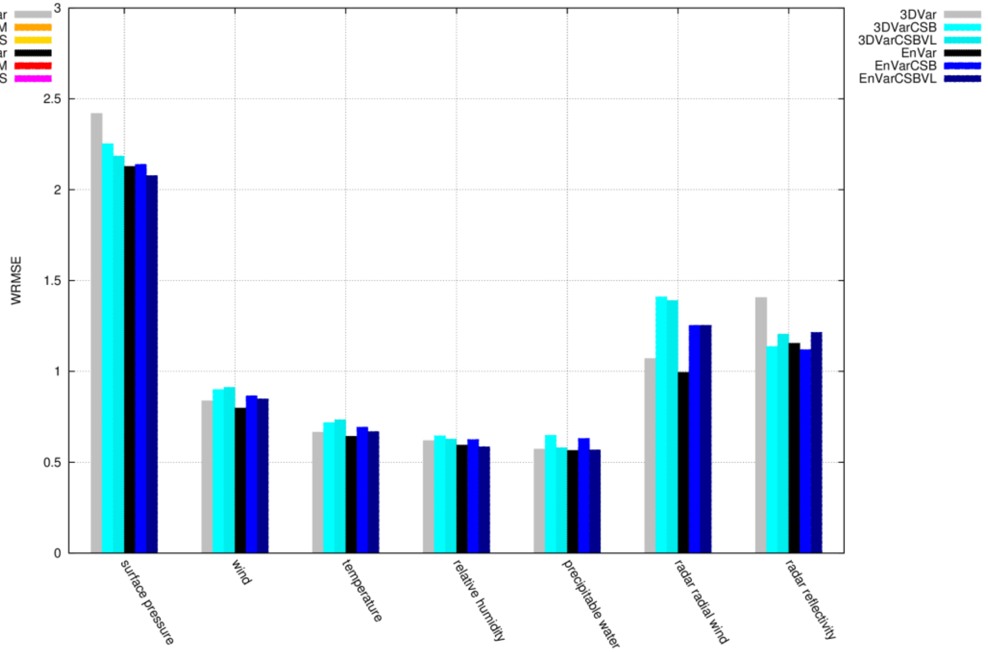
$$\mathrm{WRMSE} = \sqrt{\frac{2}{N_o}\sum_{i=1}^{N_t} J_i^{Init}}$$

(Duc and Saito 2018, *QJRMS*)

total number of observations assimilated in all analyses

initial cost at *i*-th analysis

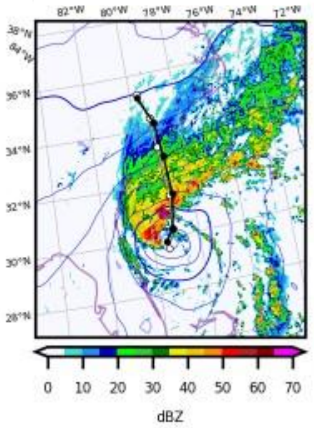




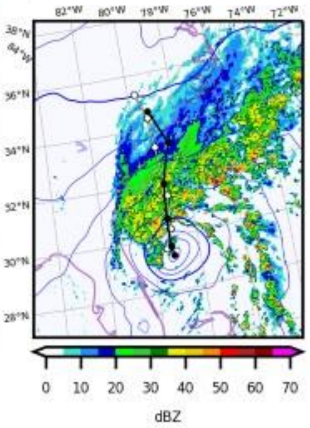
Hurricane Ian (2022) Analysis

Analysis of composite reflectivity (color, dBZ) and sea-level pressure (contours, every 4 hPa) at **20220930 00z**

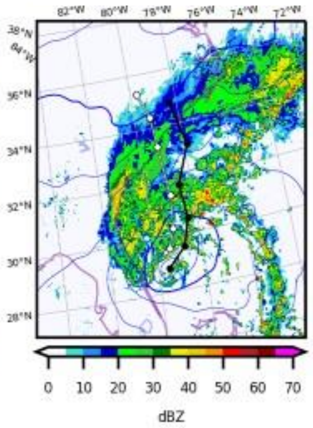
3DVar



3DVarETLM



3DVarETLMS

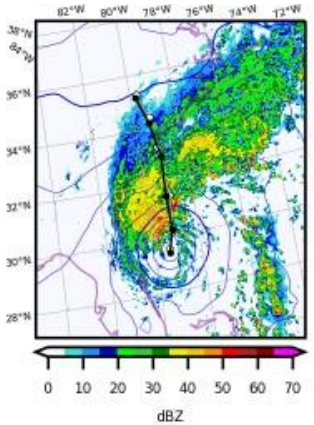


Using smoothed ensemble

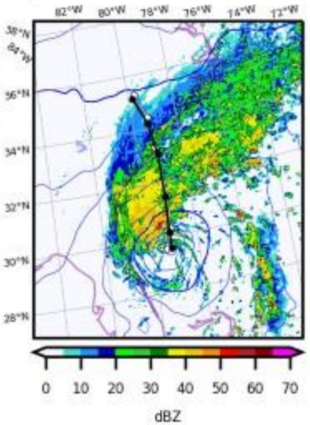
Black: Hurricane track forecast from these analyses

White: Best track

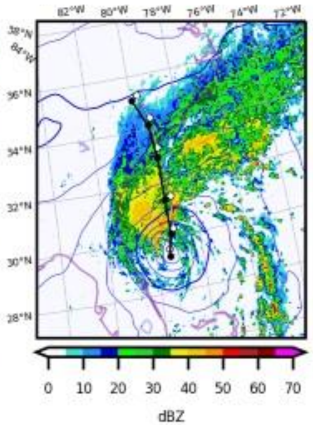
EnVar



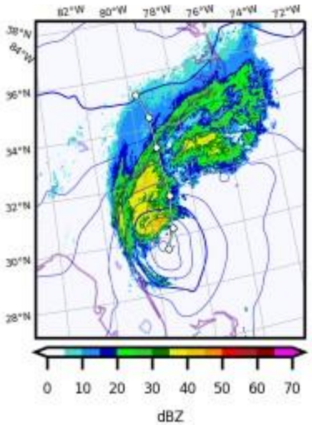
EnVarETLM



EnVarETLMS

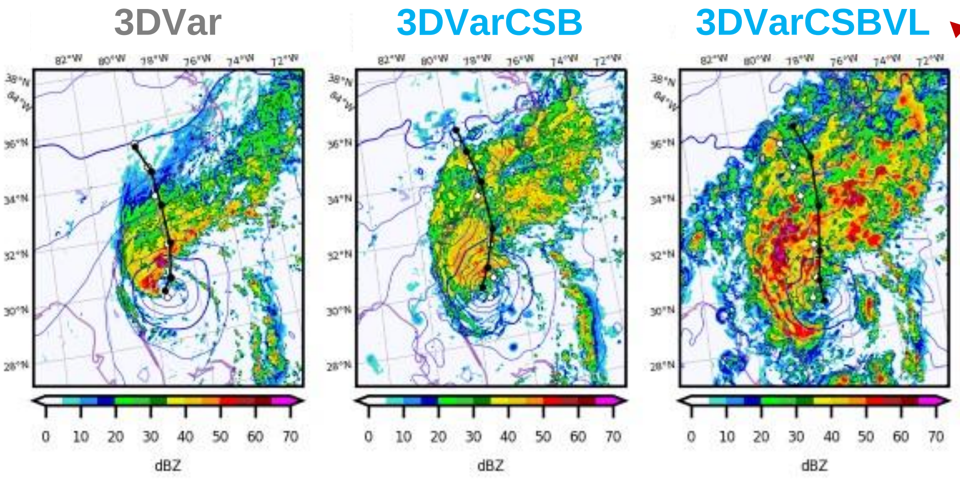


MRMS(obs)



Hurricane Ian (2022) Analysis

Analysis of composite reflectivity (color, dBZ) and sea-level pressure (contours, every 4 hPa) at **20220930 00z**



Without cross-variable covariances

Black: Hurricane track forecast from these analyses

White: Best track