Toward Faster Computation of Horizontal Localization in EnVar by Multigrid Beta Filter

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Background

In EnVar data assimilation with Gridpoint Statistical Interpolation (GSI), the static background error covariance (BEC) and localization for the ensemble BEC are computed by the recursive filter (RF, Purser et al. 2003). However, the computational efficiency of the Multigrid Beta Filter (MGBF, Purser et al. 2022) is higher than that of the RF when using large number of MPI processors for parallel computation.

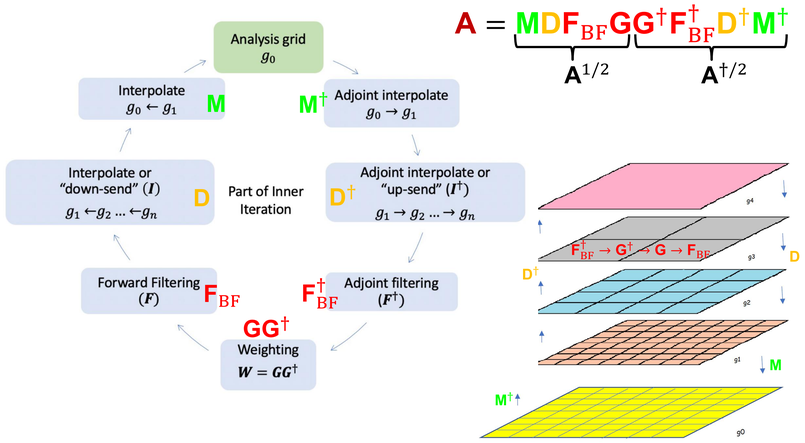
It is already clear that the MGBF makes the computation of the static BEC faster (Rancic et al. 2022). However, it has not been clarified for localization yet. In this study, we apply the MGBF also for localization and clarify how to make it faster.

Background term of cost function of GSI-based hybrid EnVar

$$J_b = \frac{1}{2}\beta_s(\delta\mathbf{x}_s)^T\mathbf{B}^{-1}(\delta\mathbf{x}_s) + \frac{1}{2}\sum_{k=1}^{K}[\beta_e(\mathbf{a}_k)^T\mathbf{A}^{-1}(\mathbf{a}_k)] \qquad \left(\frac{1}{\beta_s}+\frac{1}{\beta_e}=1\right)$$

Static BEC (B) — Localization (A)

Multigrid Beta Filter (MGBF)



$$\mathbf{A} = \underbrace{\mathbf{MDF}_{BF}\mathbf{GG}^\dagger\mathbf{F}_{BF}^\dagger\mathbf{D}^\dagger\mathbf{M}^\dagger}_{\mathbf{A}^{1/2}\qquad\qquad\mathbf{A}^{\dagger/2}}$$

- **M**: Mapping (interpolation to filter grid)
- **D**: Down-sending (interpolation to higher (up to 4th) generations)
- $\mathbf{F}_{BF}$: Horizontal beta filter (multiplying self-adjoint matrix)
- **G**: Weighting (multiplying diagonal matrix)

$$\beta(r) \propto \left[1 - \frac{r^2}{s^2(2p+3)}\right]^p$$

cf., Recursive Filter (RF)

$$\mathbf{A} = \mathbf{C}\mathbf{F}_{RF}\mathbf{F}_{RF}^\dagger\mathbf{C}^\dagger$$

- **C**: All-to-all communication (vertical parallelization → horizontal parallelization)
- $\mathbf{F}_{RF}$: Horizontal recursive filter (calculating in each direction sequentially)

$$\mathbf{F}_{RF}^{1/2} = \beta\begin{bmatrix} 1 & -\alpha_1 & -\alpha_2 & & \mathbf{0} \\ & \ddots & \ddots & \ddots & \\ & & \ddots & \ddots & -\alpha_2 \\ & & & \ddots & -\alpha_1 \\ \mathbf{0} & & & & 1 \end{bmatrix}^{-1}$$

$$\begin{cases} \mathbf{q} = \mathbf{F}_{RF}^{T/2}\mathbf{p} \\ \mathbf{s} = \mathbf{F}_{RF}^{1/2}\mathbf{q} \end{cases} \rightarrow \begin{cases} q_i = \beta p_i + \sum_{j=1}^{2}\alpha_j q_{i-j} \\ s_i = \beta q_i + \sum_{j=1}^{2}\alpha_j s_{i+j} \end{cases}$$

MGBF doesn't need all-to-all communication to change parallelization, but large MPI communication to send halo gridpoints is needed especially in case of many vertical layers and large ensemble size

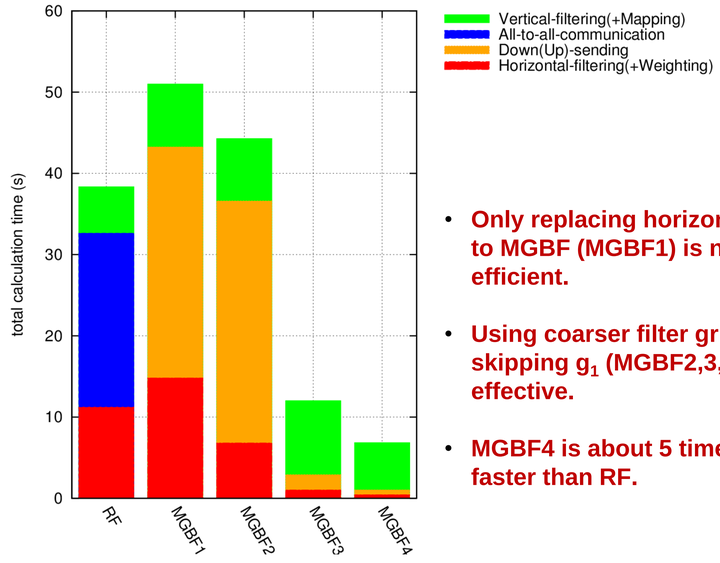
MGBF vs RF

Settings in pure EnVar tests to compare MGBF to RF

- MPI processors: $(N_{PEX},N_{PEY})=(35,21)$
- Ensemble size: $N_{ENS}=30$
- Native grid (3km): $(N_X,N_Y,N_Z)=(1820,1092,65)$
- Localization length: s=82.158(km, horizontal), 3(grids, vertical)
- Beta filter exponent: p=2
- First guess: FV3LAM CONUS 1h forecast (3km-grid) in RRFS at 16UTC on Sep 29, 2022

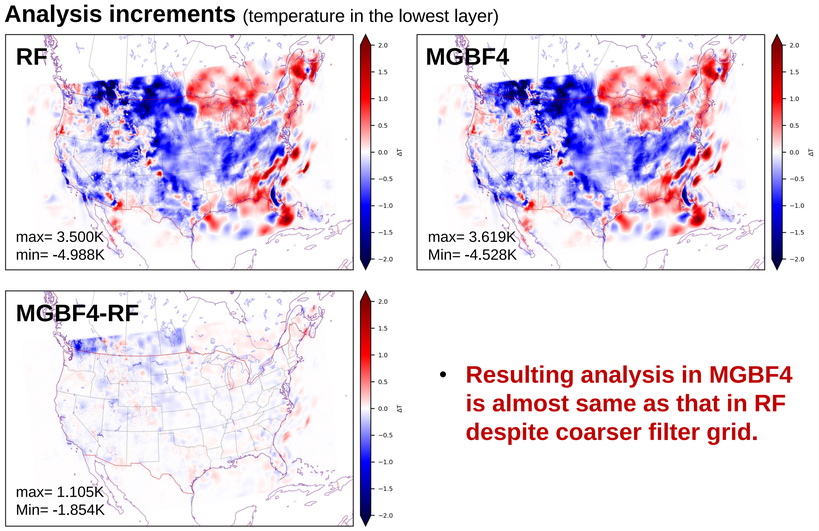
	Analysis grid (~6km)	MGBF filter grid (g_1)	Weights of (g_1,g_2, g_3,g_4) (-: not calculated)	How to calculate $\mathbf{A}^{1/2}$ (Refer to Rancic et al. 28IOAS-AOLS 15B.3 in detail)
RF	(911,547,65)	-	-	$\mathbf{F}_{RFv}\mathbf{C}\mathbf{F}_{RFh}$ (require all-to-all communication)
MGBF1	(910,546,65)	(910,546,65)	(0,0,0,1)	$\mathbf{F}_{RFv}\mathbf{D}\mathbf{F}_{BFh}^{(g_{1-4})}\mathbf{G}$ (horizontal-RF → horizontal-MGBF)
MGBF2	(910,546,65)	(910,546,65)	(-,0,0,1)	$\mathbf{F}_{RFv}\mathbf{D}\mathbf{F}_{BFh}^{(g_{2-4})}\mathbf{G}$ (skip 1st generation)
MGBF3	(910,546,65)	(**280**,**168**,65)	(-,1,-,-)	$\mathbf{F}_{RFv}\mathbf{M}\mathbf{D}\mathbf{F}_{BFh}^{(g_2)}\mathbf{G}$ (insert mapping to coarser filter grid)
MGBF4	(910,546,65)	(**280**,**168**,**33**)	(-,1,-,-)	$\mathbf{M}\mathbf{F}_{BFv}\mathbf{D}\mathbf{F}_{BFh}^{(g_2)}\mathbf{G}$ (vertical-RF → coarser vertical-BF)

Total calculation time in localization



- **Only replacing horizontal-RF to MGBF (MGBF1) is not efficient.**
- **Using coarser filter grid and skipping g_1 (MGBF2,3,4) are effective.**
- **MGBF4 is about 5 times faster than RF.**

Analysis increments (temperature in the lowest layer)



- **Resulting analysis in MGBF4 is almost same as that in RF despite coarser filter grid.**

Summary

- MGBF can be used for computation of localization as well as static BEC.
- MGBF achieves high parallelization efficiency without all-to-all communication.
- However, MGBF is not necessarily faster than RF in case of many vertical layers and large ensemble size.
- **Using coarser filter grid** and **skipping g1**, MGBF becomes about 3-5 times faster than RF without significant change of analysis increment.

How to Apply MGBF-based localization to Scale/Variable-Dependent Localization (SDL/VDL)

Yokota et al. (2024)

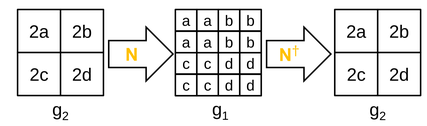
2-SDL (or 2-VDL) is formulated as:

$$\mathbf{A} = \begin{bmatrix} \mathbf{A}_1^{1/2} & \mathbf{0} \\ \mathbf{0} & \mathbf{A}_2^{1/2} \end{bmatrix}\begin{bmatrix} \mathbf{I} & c\mathbf{I} \\ c\mathbf{I} & \mathbf{I} \end{bmatrix}\begin{bmatrix} \mathbf{A}_1^{\dagger/2} & \mathbf{0} \\ \mathbf{0} & \mathbf{A}_2^{\dagger/2} \end{bmatrix} \quad (0 \le c \le 1)$$

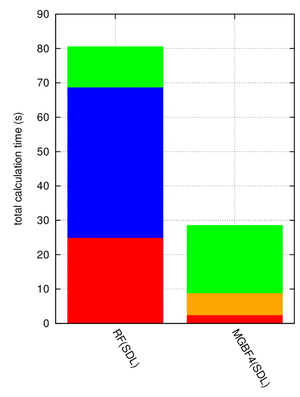
To apply MGBF for each localization ($\mathbf{A}_1$ and $\mathbf{A}_2$),

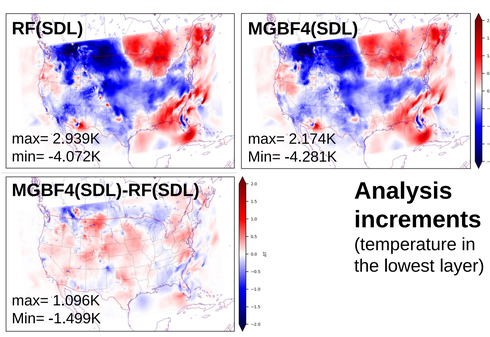
$$\mathbf{A}_1^{1/2} = \mathbf{MDF}_{BF1}\mathbf{G}_1\mathbf{N}^\dagger$$
$$\mathbf{A}_2^{1/2} = \mathbf{MDF}_{BF2}\mathbf{G}_2\mathbf{N}^\dagger$$

Here, different localization lengths and weights are used in $\mathbf{F}_{BF1(2)}\mathbf{G}_{1(2)}$, and **N** is inserted ($\mathbf{N}^\dagger\mathbf{N} = \mathbf{I}$).



Total calculation time in localization (2-SDL)





Analysis increments (temperature in the lowest layer)

References

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