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A Constraint on Convection

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https://www.earthenergyflows.com/Zagoni-EGU2024-Trenberths-Greenhouse-Geometry_Full-v03-480.mp4



The Players

Manabe and Möller (1961), Monthly Weather Review
– Refer to Emden (1913) and Gold (1908)
Robert Emden [brother-in-law of Karl Schwarzschild] (1913)
– Refers to Schwarzschild (1906) and Gold (1908)
– Schwarzschild (1906): Radiative Equilibrium
– Gold (1908): Convective Equilibrium
Emden (1913): Radiative-Convective Equilibrium
Reproduces the set of equations of Schwarzschild for *B*, *A* and *E*:

und die Lösung des Problems

$$(31)\qquad \begin{cases} B = -k\gamma m + B_0 \\ A = -k\gamma m + A_0 \\ E = -k\gamma m + E_0. \end{cases}$$

B_0 und A_0 sind dem jeweiligen Spezialfalle anzupassende Integrationskonstanten. Stets ist

$$(32)\qquad \gamma = \frac{B_0 - A_0}{2}; \qquad E_0 = \frac{B_0 + A_0}{2}.$$

Ansatz und Lösung dieses Strahlungsproblems sind von K. Schwarzschild (loc. cit.) gegeben.

Schwarzschild's (1906) formulation (same content, different form):

$$(11)\qquad E = \frac{A_0}{2}(1+m), \quad A = \frac{A_0}{2}(2+m), \quad B = \frac{A_0}{2}m.$$

From Schwarzschild's (1906, Eq. 11) formulation it is evident that
$A - E = A_0/2$.
In Emden's formalism, it is also true but not striking at the first sight.
In the English translation of Emden (1913) in the Monthly Weather Review (1916) it is given in an even less evident form.
Bottom line: $A - E = A_0/2$ is missing from Manabe and Möller (1961).

$A - E = A_0/2$ textbook appearences

R. Goody (1964) Atmospheric Radiation, Oxford, Eq. 2.115
J. Houghton (1977, 1986, 2002) Cambridge, Eq. 2.13, Fig. 2.4
R. Chamberlain (1979, 1987) Academic Press, Eq. 1.2.29, Fig. 1.4
Goody and Yung (1989) Atmospheric Radiation Eq. 2.146
M. Salby (1996, 2012) Physics of the Atmospheres, Eq. 8.67, Fig. 8.20
Visconti (2001) Findamentals of Atmospheric Physics, Springer, Eq. 3.49
Vardavas and Taylor (2007) Radiation and Climate, Oxford, Fig. 2.11
Pierrehumbert (2008), Principles of Planetary Climates, Cambridge, Eq. 4.45
Zdunkowski et al. (2008) Radiation in the Atmosphere, Cambridge Fig. 6.7
D. Andrews (2010) An Introduction to Atmospheric Physics, pp. 85-86.
Ambaum (2021) Thermal Physics of the Atm., Royal Met Soc Eq. 20.56

University lecture note:
G. Stephens 2003, Colorado AT622 Section 6, Eqs. 6.10a-6.10b, Fig. 6.3a.

Missing from:
Manabe and Möller (1961)
Manabe and Strickler (1964)
Manabe and Wetherald (1967)
Manabe and Wetherald (1975)
Ramanathan and Coakley (1978)
Ramanathan, Lian, Cess (1979)
The Charney report (1979)
Theory of Climate (1983)
IPCC Reports (1990 - 2022)
All the climate studies in journal articles

Observational validation of $A - E = A_0/2$

Early data:
CERES EBAF-Surface Edition 2.8
Data Quality Summary (March 27, 2015)
March 2000 through February 2010:

The net radiation at the surface, clear-sky:
Surface (SW down – SW up + LW down – LW up) = OLR/2
243.9 – 29.7 + 316.0 – 398.0 = 265.7/2 – 0.6 Wm^{-2}
Loeb et al. (2012): Earth heat uptake for July 2005–June 2010 = 0.58 ± 0.38 Wm^{-2}
The equation is valid within Earth Energy Imbalance.

Recent data (CERES EBAF Ed4.1, 22 years):
SFC SW down – SW up + LW down – LW up
= TOA LW /2
240.87 – 29.07 + 317.40 – 398.52 = 266.01 /2
– 2.33 Wm^{-2}

Net radiation at the surface R_N = LH + SH = OLR/2.
Net radiation at the surface = Convection = Half of Outgoing Longwave Radiation at TOA.