Multi-Fluid Modeling of Energy and Momentum Exchange in Spray-Laden Hurricane Boundary Layer

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I. MOTIVATION AND GOALS

Motivation:

Extensive observational and theoretical evidence demonstrates that sea spray strongly influences hurricane dynamics, both thermodynamically and mechanically, by altering vertical enthalpy and momentum fluxes, respectively. Studies have shown that these effects of sea spray depend sensitively on the size of spray droplets. Therefore, accounting for the droplet size distribution is crucial for accurately describing and modeling the spray effects on hurricane dynamics.

Goals:

- Develop a rigorous and comprehensive Eulerian Multi-Fluid model of a Marine Atmospheric Boundary Layer (MABL) laden with multi-sized sea spray.
- Conduct an in-depth study of the effect of sea spray on hurricane dynamics through systematic numerical and theoretical analyses of the developed MABL model.

II. EFFECTS OF OCEAN SPRAY ON TROPICAL CYCLONE

The ocean spray has two effects on hurricane dynamics:

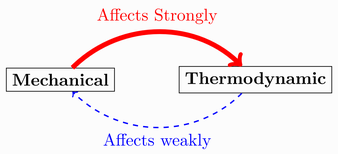
I. Thermodynamic (*Rastigejev & Suslov 2016, 2019*). Evaporating sea spray modifies latent and sensible heat fluxes in a MABL.

II. Mechanical (*Rastigejev & Suslov 2022, 2024*). Sea spray affects the vertical momentum transport.

IIa. Accelerating effect due to turbulence attenuation by spray – Droplet Slip (DS) and "Gravity Lubrication" (GL)

IIb. Decelerating effect due to spray inertia.

The thermodynamic and mechanical effects are coupled but coupling is not symmetric (*Rastigejev & Suslov, JPO 2019*)



Hence, the mechanical effect can be considered separately, that is inert spray models are sufficiently accurate for simulating the mechanical effect of sea spray.

III. MECHANICAL EFFECT OF SPRAY

The decelerating effect of spray inertia



Horizontal velocities of the injected droplets are much smaller than that of the airflow, $u_i^1 \ll u_a$

The accelerating effects of spray due to turbulence suppression, two mechanisms:

1. Droplet slip (DS) causes air-droplet "friction" that leads to the turbulence attenuation



Turbulent flow loses its energy due to air-droplet friction ⇨ turbulent transport becomes less intensive ⇨ reduction in air-ocean drag ⇨ wind acceleration

2. "Gravity Lubrication" (GL) due to upward transport of spray by the turbulent eddies

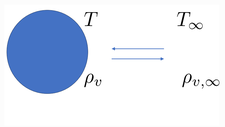


Turbulent eddies lift spray ⇨ they do work against gravity ⇨ lose their energy ⇨ turbulent transport becomes less intensive ⇨ reduction in air-ocean drag ⇨ wind acceleration

The droplet-slip effect is typically much stronger than the "gravity lubrication" effect (DS>GL)

IV. THERMODYNAMIC EFFECT OF SPRAY

Spray droplets exchange heat and mass with the surrounding turbulent airflow



The model accounts for the ventilation effect due to the air-droplet slip

V. MATHEMATICAL MODEL FOR SPRAY-LADEN MABL

Multi-fluid Eulerian model of MABL laden with polydisperse sea spray

Spray conservation

$$\frac{d}{dz}\left(d_i\frac{ds_i}{dz}+s_i a_i\right)=\frac{1}{\rho_w}\left[\frac{1}{\Delta r}(f_{i+1}-f_i)-\dot{q}_i s_i\right]-a_{0i}s_{0i}\delta_{z-z_w} \quad \leftarrow \text{volumetric density of spray}$$

$$i=1\ldots N$$

Moisture conservation

$$\frac{d}{dz}\left(\alpha_v k_a\rho_a\frac{dq}{dz}\right)=\sum_{i=1}^{N}\dot{q}_i s_i \quad \leftarrow \text{specific humidity}$$

Momentum conservation

$$\frac{d}{dz}\left(k_a\rho_a\frac{du_a}{dz}\right)=\rho_w\sum_{i=1}^{N}\frac{s_i(u_a-u_i)}{\tau_{di}} \quad \leftarrow \text{horizontal wind speed}$$

$$\frac{d}{dz}\left(k_i s_i\frac{du_i}{dz}\right)+s_{0i}a_{0i}H_{z-z_w}\frac{du_i}{dz}+\frac{s_i}{\tau_{di}}(u_a-u_i)+s_{0i}a_{0i}(U-u_i)\delta_{z-z_w}=0 \quad \text{horizontal spray motion}$$

$$\frac{dP}{dz}=-\rho g$$

Thermal energy conservation

$$\frac{d}{dz}\left(\alpha_T k_a c_{pd}\rho_d\frac{d\theta}{dz}\right)=\sum_{i=1}^{N}\gamma_i s_i\left(T_a-T_i\right) \quad \leftarrow \text{potential air temperature}$$

$$c_w\frac{d}{dz}\left(s_i d_i\frac{dT_i}{dz}+s_{0i}a_{0i}H_{z-z_w}T_i\right)=\frac{1}{\rho_w}\left[\gamma_i(T_i-T_a)-l\dot{q}_i\right]s_i-s_{0i}a_{0i}c_w T_0\delta_{z-z_w} \quad \text{spray temperature}$$

E-epsilon (k-epsilon) 1.5 order model

$$\alpha_e\frac{d}{dz}\left(\rho_a k_a\frac{de_a}{dz}\right)=\rho_a(\epsilon_a-p_a)-q_{ae}$$

$$\alpha_\epsilon\frac{d}{dz}\left(\rho_a k_a\frac{d\epsilon_a}{dz}\right)=\rho_a(d_{a\epsilon}-p_{a\epsilon})-q_{a\epsilon}$$

+ various constitutive relationships

VI. VOLUMETRIC SPRAY GENERATION FUNCTION (VSGF)

VSGF

$$f_v(u_c,r)=s_0(u_c)p(r)$$

u_c – characteristic speed of the airflow, $u_c=u_\star$ or $u_c=u_{a10}$
r – droplet radius

VSGF — Spray production intensity — shape function

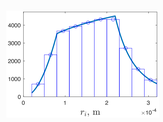
Large uncertainty over spray production rates:

- Six orders of magnitude in field experiments (the wind speeds $\lesssim$ 20 m/s)
- Two-three orders of magnitude in laboratory experiments (the wind speeds $\lesssim$ 60 m/s)

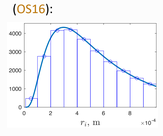
How to bridge the gap between the laboratory and the field observations when there is a 2-3 orders of magnitude difference between them ?

Shape of VSGF, p(r)

I. *Andreas 1998* (A98):



II. *Ortiz-Suslow et al. 2016* (OS16):



Spray production intensity $s_0(u_c)$

$$s_0(u_\star)=A_n u_\star^n \quad \text{(Fairall et al. 2009 and others)}$$

$$n\sim 2-16$$

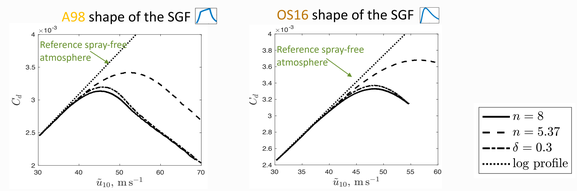
$$s_0(u_{a10})=A\exp[\delta(u_{a10}-u_r)]$$

$$\delta\sim 0.2-0.875 \quad \text{(Monahan 1986, Wu 1993)}$$

VII. NUMERICAL SIMULATION

Mechanical effect of thermally inert polydisperse spray

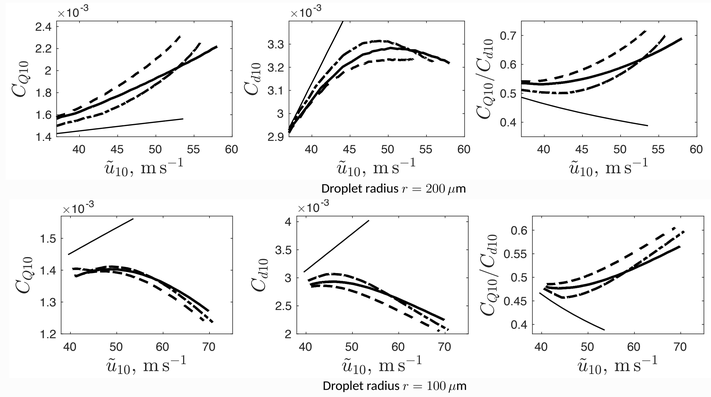
Drag coefficient C_d vs u_{10} for different correlation laws between s_0 and $u_\star$ (u_{a10})



The spray-induced drag reduction causes the wind to accelerate above the wave crest level. The reduction of the drag coefficient (C_d) occurs even at relatively low volumetric spray concentrations $\lesssim 10^{-4}$, which corresponds to the spray mass fraction of $\lesssim$10%.

Mechanical and thermodynamic effects of evaporating monodisperse spray

Drag coefficient (C_d), enthalpy exchange (C_Q) , and their ratio vs u_{10} for different correlation laws between s_0 and $u_\star$ (u_{a10})



In a spray-free atmosphere, the ratio of enthalpy exchange coefficient to air-sea drag coefficients, primarily determined by the roughness lengths for momentum and enthalpy, decreases as wind velocity increases, reaching approximately 0.3–0.4 at hurricane wind speeds exceeding 50 m/s. However, based on the balance of dissipation and production rates of kinetic energy in hurricanes, it has been suggested that this ratio should be higher, with a minimum value of 0.75 at such wind speeds. Our findings reveal that this ratio rises with increasing wind speed in the presence of spray, reaching values between 0.6 and 0.7 at high speeds for droplet radii of 100 and 200 μm. This suggests that spray can noticeably increase the ratio, bringing it closer to theoretical expectations.

VIII. CONCLUSIONS

- An Eulerian multi-fluid (MF) model has been developed to describe the dynamics of a marine atmospheric boundary layer laden with polydisperse ocean spray. This model treats air and droplets of varying sizes as distinct, interacting turbulent continua.
- The MF approach enables us to describe accurately the mechanical and thermodynamic effects of the spray, including turbulence attenuation caused by air-droplet slip (DS) and "gravity lubrication" (GL) of spray, and the impact of spray inertia and ventilation effect.
- The poly-disperse MF model of thermally inert spray has been considered for which two "shapes" of the volumetric spray generation function, A98 and OS16, have been studied. The fraction of large droplets in OS16 is much higher than that in A98. The shapes yield substantially different values of the air-sea drag coefficient.
- The wind acceleration due to DS is much stronger than due to GL and occurs even for low volumetric spray concentrations $s\lesssim 10^{-4}$ ($\lesssim$ 10% spray mass mixing ratio). This can explain a substantial air-ocean drag reduction detected in field observations.
- To date, the mono-disperse MF model of evaporating spray has been implemented. Numerical simulations reveal that sea spray significantly increases the ratio of the enthalpy exchange coefficient to the air-sea drag coefficient at high wind speeds, nearly doubling it, even at low spray concentrations. These values approach the expected minimum suggested for hurricane sustainability. Therefore, the presence of spray may explain the high wind speeds in hurricanes.

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