A Reynolds Averaged Navier-Stokes (RANS) Procedure for Predicting Skin Friction in Stable Shear Dominated Atmospheric Boundary Layers over Complex Terrain

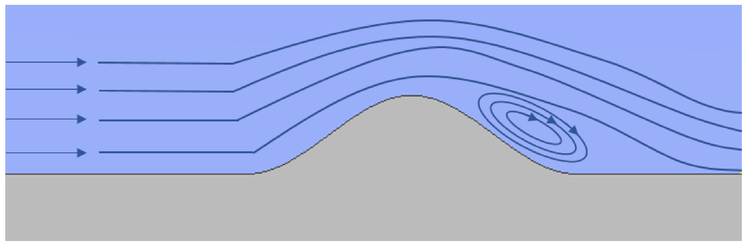
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Background

Numerical modeling of Atmospheric turbulence is a useful but expensive method of studying the atmosphere. RANS equations are time averaged equations used to numerically model turbulent flows, where the mean of a quantity in a turbulent flow is averaged over a time scale larger than the largest time scale in a turbulent motion. This removes the fluctuating component from the Reynolds' decomposition of a flow quantity, i.e. $u' in\ u = \bar{u} + u'$, which reduces the number of variables calculated by the model, reducing computational cost but reducing the results' accuracy.

RANS modelling particularly struggles to accurately capture flow separation regions, which are regions where the flow boundary layer separates from the surface and forms into a wake. In the Atmospheric Boundary Layer (ABL), separation regions are commonly formed by air flow encountering a natural or man-made perturbation, such as a hill or a building. Complex terrain is where a lot of separation happens in the ABL.

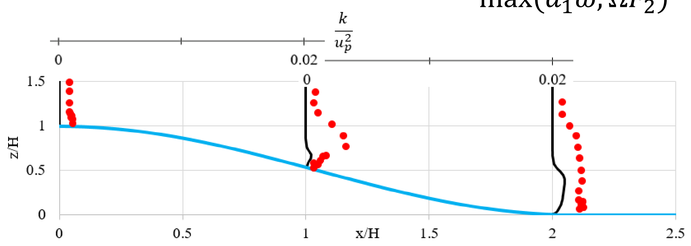


Flow Separation Following a Hill

RANS models specifically mis predict k in separation regions, which influences the calculation of the wall shear stress (τ_w) that is first and second hand-dependent on k:

$$\tau_w = \mu_t\left(2S_{xy} - \frac{2}{3}\frac{\partial u_z}{\partial x_z}\delta_{xy}\right) - \frac{2}{3}\rho k\delta_{xy}$$

$$where\ the\ turbulent\ viscosity\ \mu_t = \frac{\rho a_1 k}{\max(a_1\omega, \Omega F_2)}$$



Normalized k Flow Profiles After Passing Over a Hill [Numerical RANS Results in Black, Experimental from Takahashi (2005) in Red]

Research Objective

To provide a validated method of predicting the range of shear stress values at the surface of complex terrains in Stable ABLs using existing RANS modelling and finding areas where the modelling yields imprecise results.

Methods

The ABL modelled in this poster is a thermally stable nocturnal boundary layer that was experimentally measured over Cincinnati by Clarke (1969). The velocity profile was extracted from Clarke's paper and matched to a law of the wall velocity profile, with the ABL height being around 300 meters above ground and the free stream velocity (U_∞) being at 16.2 m/s. The topography of the University of Cincinnati and surrounding area was extracted from CADMapper. The numerical model used is the k-ω model consisting of a Turbulent Kinetic Energy (k) transport equation and a Turbulence Dissipation (ω) Transport equation.

A fully developed Turbulent upstream flow was achieved by inserting Clarke's (1969) velocity profile at the inlet of a 7000x1000x300 meter smooth flat plate geometry. Mesh sensitivity was achieved by comparing numerical skin friction (c_f), to Schultz-Grunow's (1940) empirical c_f correlation:

$$\frac{c_f}{2} = 0.185 Log_{10}(Re_x)^{-2.584}, where\ Re_x = \frac{\rho U_\infty x}{\mu}$$

The best mesh, with a starting y^+ of 31, produced an error of 12.8%. The mesh node count is 9.39 Million. The flow fully developed at Re = $6x10^9$. The fully developed velocity profile for each surface type was used as the inlet profile for that type's topography simulation.

To numerically account for surface roughness, half of the height of the first mesh cell over the surface (y_p) should be greater than the roughness height k_s, where $k_s = \frac{9.793}{c_{k_S}} z_0$.

Surface Type	First Cell Height (m)	z_0 (m)	k_s (m)	y_p (m)
No Roughness	0.0012	0	0	0.0006
Snow	0.21	0.005	0.1	0.105
Grass	1.21	0.03	0.6	0.605

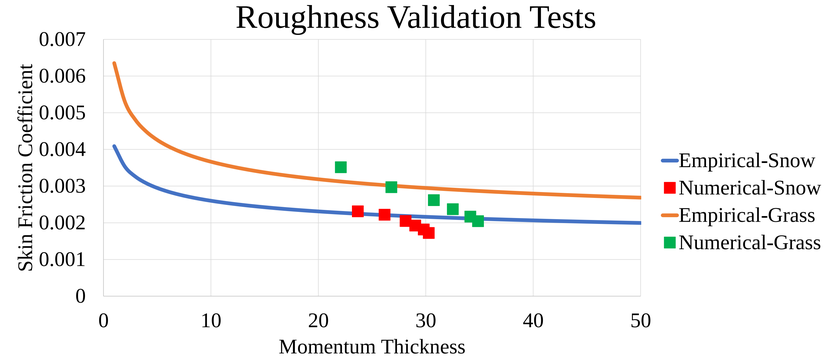
Table 1: Mesh Conditions

Findings and Conclusion

Results were recorded at the Centerline between the two side symmetry walls. For the rough surface cases, the surfaces were considering "fully rough" since their Roughness Reynolds Numbers exceeded 70 which meant that skin friction is independent of the Reynold's number, and is correlated with the Momentum Thickness (θ) of the ABL using Pimenta's (1975) Empirical Correlation:

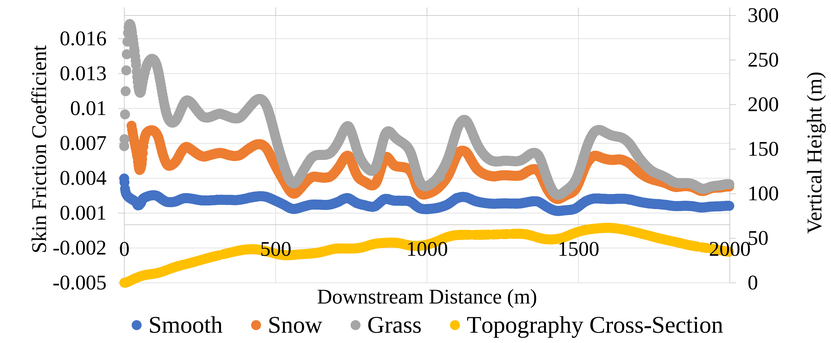
$$\frac{c_f}{2} = \frac{0.168}{\left(\ln\left(\frac{864\theta}{k_s}\right)^2\right)}$$

Roughness Validation Tests



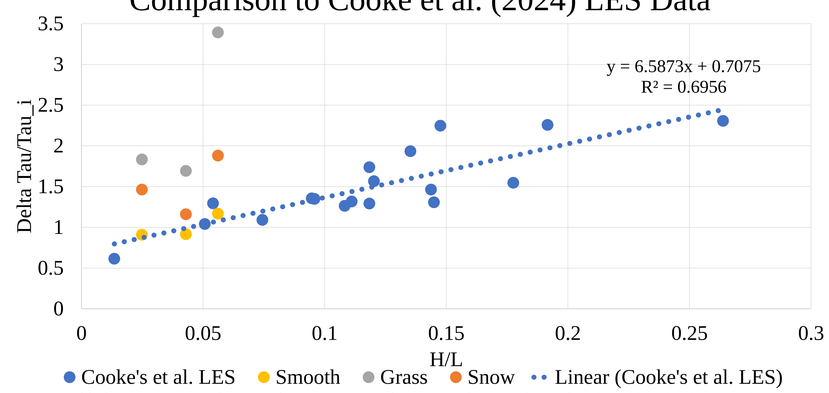
The upstream flow over the rough surfaces developed faster relative to over the smooth surface due to increased turbulence production at the surface from the dominance of pressure drag formed by the roughness.

Skin Friction Over Different Surface Roughness



Skin friction peaked at the start due to the topographical roughness transition from a flat surface to a complex surface. Increased k_s at the surface increased shear stress at hill peaks which determines that separation will happen in a larger size following hills with rough surfaces.

Comparison to Cooke et al. (2024) LES Data



To Validate, we plotted our results against Cooke's (2024) LES results. Normalized shear stress from flow over sand dunes is plotted against the Height to half width ratio of every hill the flow encountered. Our data for the Smooth surface matches best with the LES data, whereas the 2 roughness cases reported results far from the linear form of Cooke's data. We conclude that our model can produce valid shear stress values for a smooth surface, whereas improvements are needed for modelling over rough surfaces.

References:

Takahashi et. al., 2005, *Jou. Wind Eng.*, 93, 155-169; J. F. Clarke, 1969, *Mon. Wea. Rev.*, 97, 582-589; Schultz-Grunow, 1940, *NACA TM 986;* M. M. Pimenta et. al., 1975, *Stanford University;* Cooke et. al., 2024, *Proc. Natl. Acad. Sci. U.S.A.*