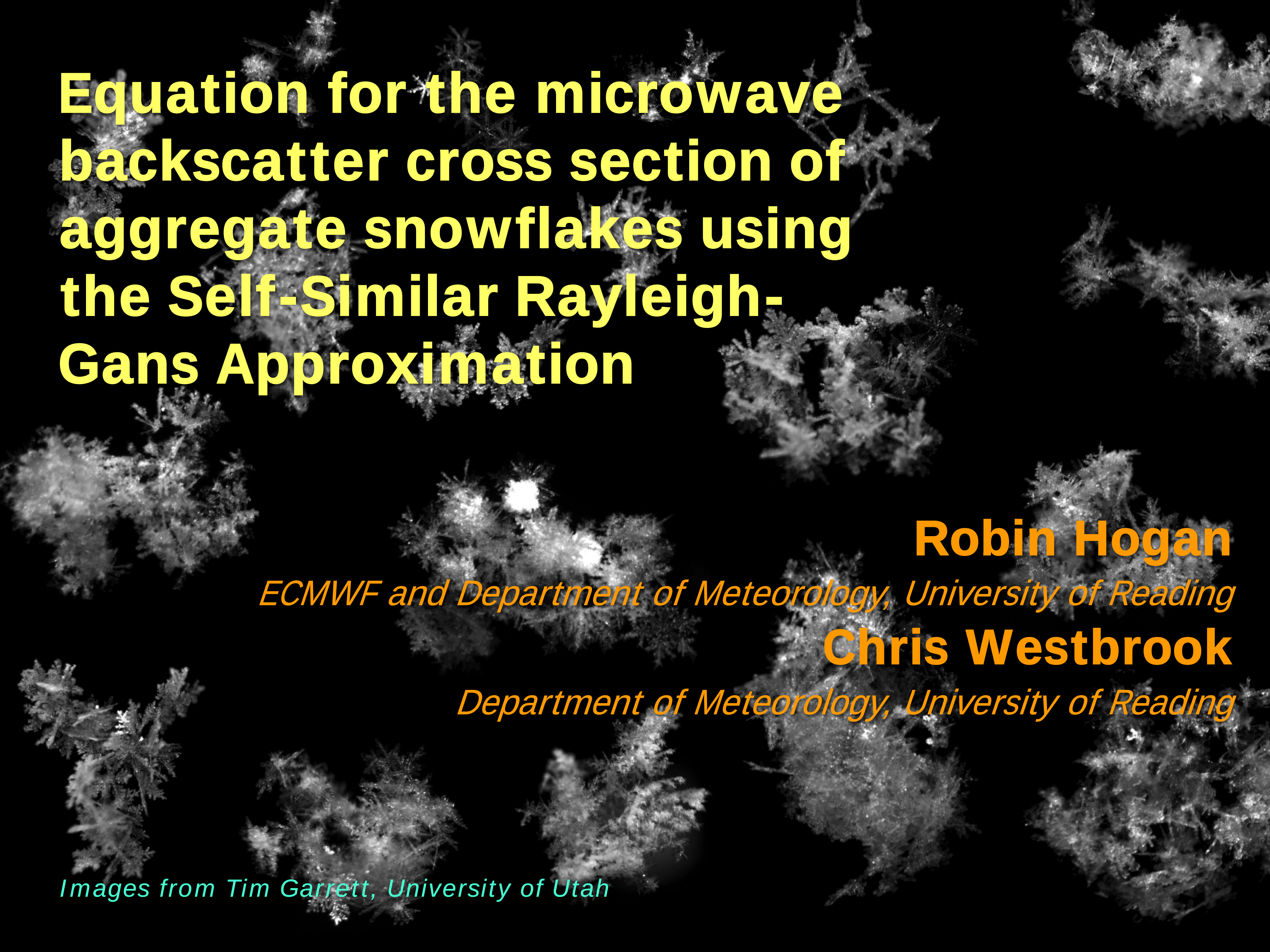

Equation for the microwave backscatter cross section of aggregate snowflakes using the Self-Similar Rayleigh-Gans Approximation

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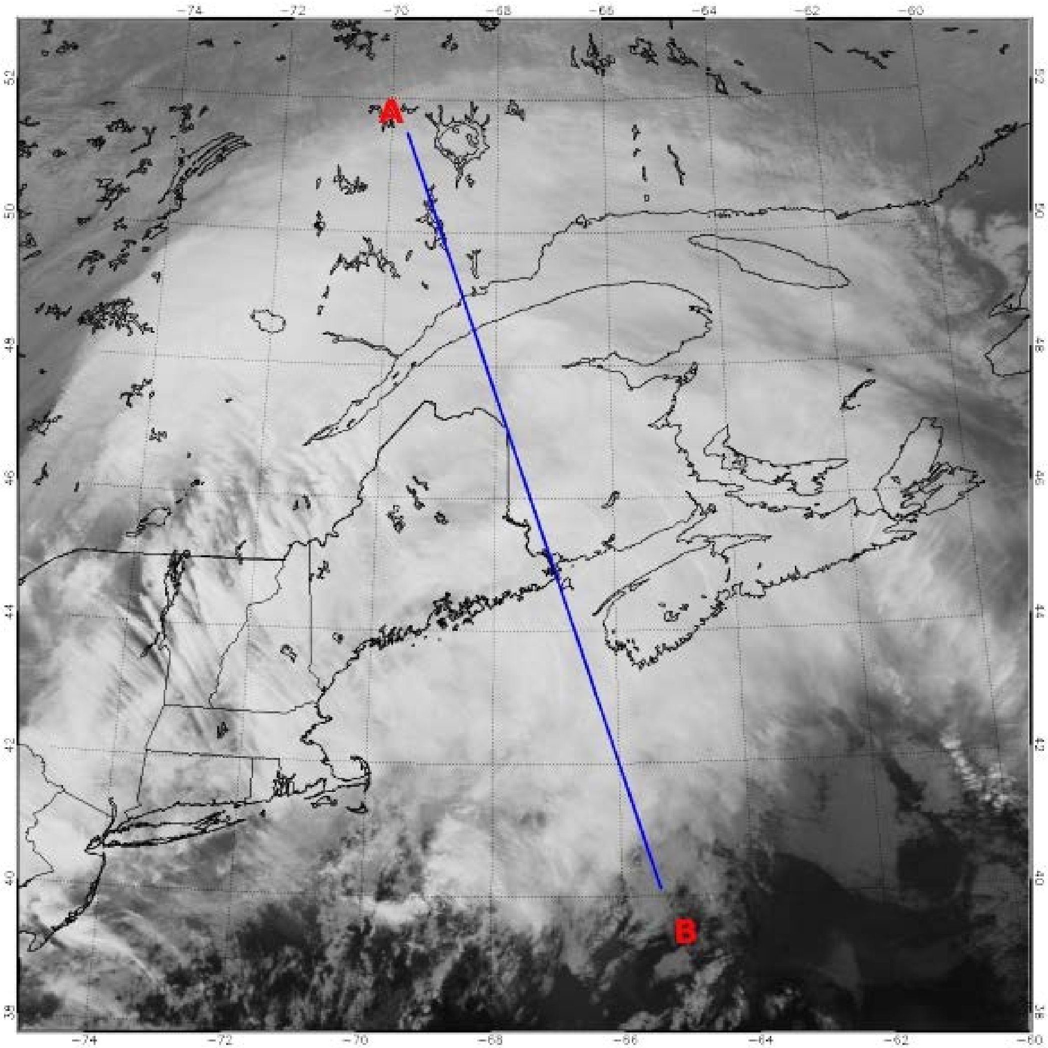
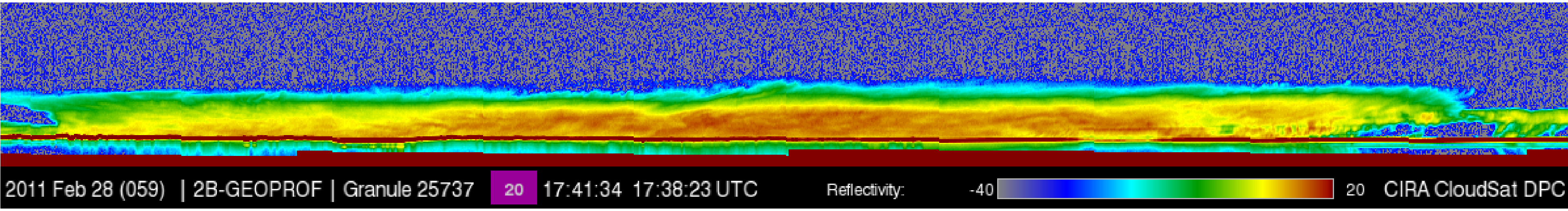
Motivation



- Global snowfall rates very uncertain, both snow at the surface and above a melting layer
- Space-borne 94-GHz radar potentially the best instrument we have
- But backscatter cross-section of snowflakes very uncertain for $D > \lambda$
- Global-mean IWP estimates dominated by deep clouds where retrievals are most uncertain

Snowstorm 28 Feb 2011: 20 cm of snow fell in New Brunswick

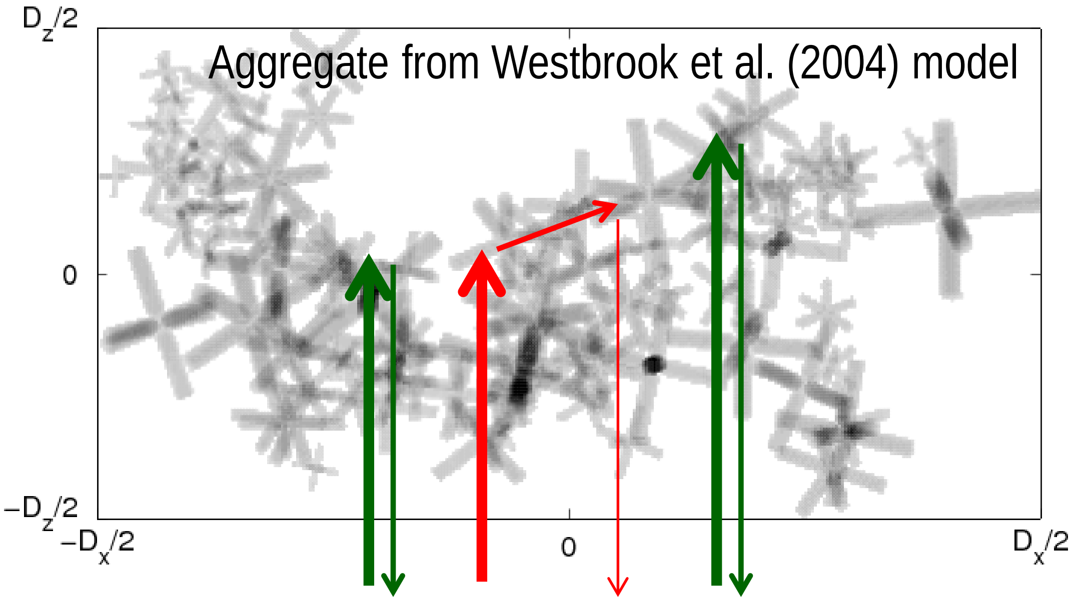


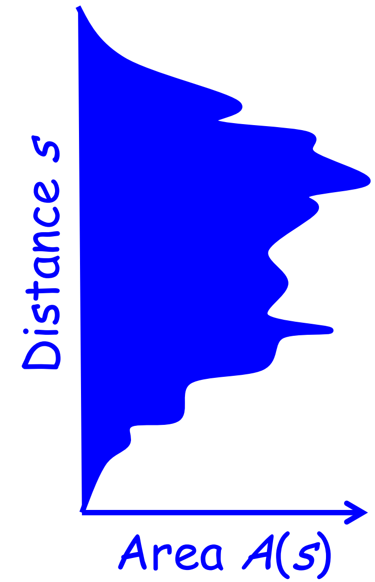
Overview

- At cm wavelengths: $D < \lambda$
  - Rayleigh theory applies: radar reflectivity proportional to mass squared
  - Snowfall rates sensitive to mass-size and fallspeed-size assumptions
- At mm wavelengths: $D > \lambda$
  - Particle *shape* important in addition to *mass*
  - Matrosov et al (2005), Hogan et al. (2012): *large ice particles can be represented by homogeneous oblate ("soft") spheroids with aspect ratio 0.6: characterize shape only by dimension in direction of propagation*
  - Petty & Huang (2010), Tyynelä et al. (2011): *soft spheroids underestimate backscatter: need to represent exact shape*
- Who's right?
  - Is brute-force computation the only way (e.g. Discrete Dipole Approximation with thousands of snowflake shapes)?
- *This talk: an equation can be derived for cross-section because*
  - *Snowflakes have fractal structure that can be described statistically*
  - *The Rayleigh-Gans approximation is applicable*

The Rayleigh-Gans approximation

- Approximate the field at any point by the incident field
- Sum backscattered returns from each volume element coherently





Backscatter depends only on *A*(*s*) function and dielectric constant ε (or refractive index $m = \varepsilon^{1/2}$)

- Rayleigh-Gans applicable if $|m - 1| \ll 1$ and $|\rho| \ll 1$
  - where $\rho = kD(m-1)$ is the phase shift across the particle and $k = 2\pi/\lambda$
- Solid ice in the microwave has $m = 1.77$, but on the scale of the wavelength the particle is mostly air so effective *m* close to 1
  - Tyynela et al. (2012) found that Rayleigh-Gans is a good approximation compared to other uncertainties, e.g. in ice structure

The Rayleigh-Gans approximation

$$\sigma_b = \frac{9k^4|K|^2}{4\pi}\left|\int_{-D/2}^{D/2} A(s)\exp(\mathrm{i}2ks)ds\right|^2$$

- Backscatter cross-section is proportional to the power in the Fourier component of $A(s)$ at the scale of half the wavelength

- Can we parameterize $A(s)$ and its variation?

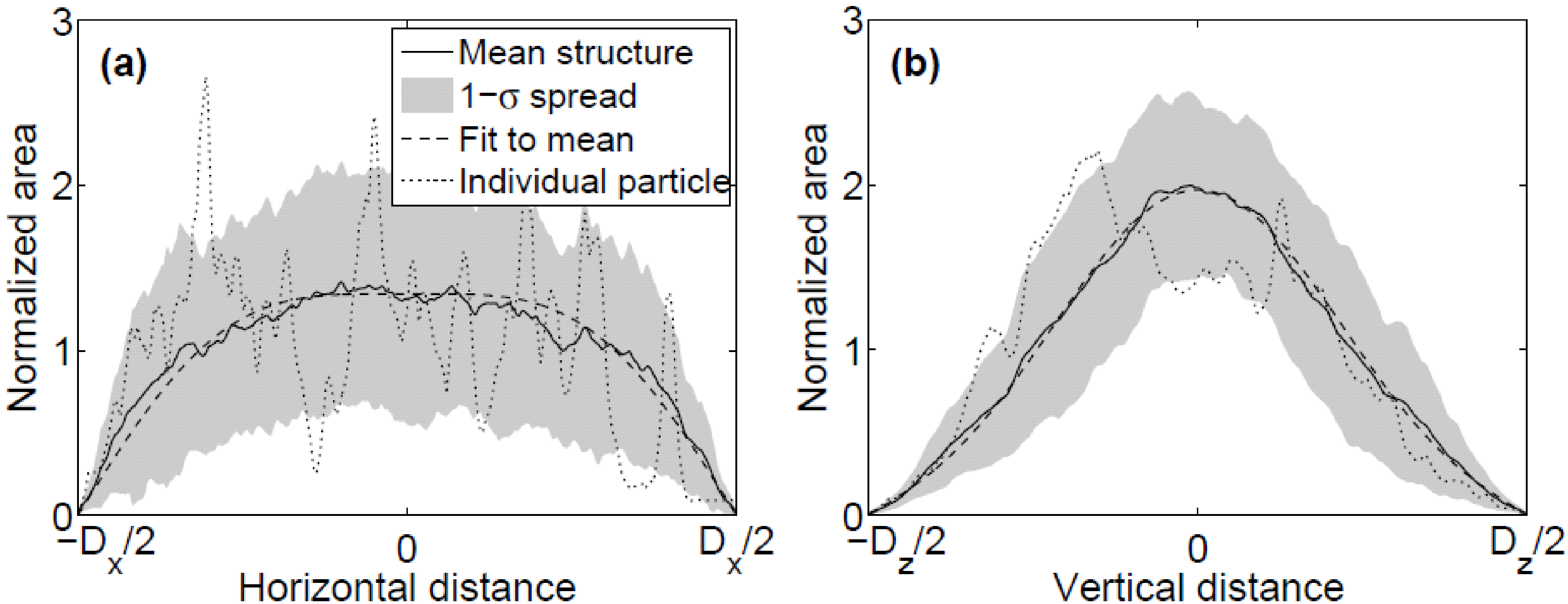
$$A(s) = a_0\left[\left(1+\frac{\kappa}{3}\right)\cos\left(\frac{\pi s}{D}\right) + \kappa\cos\left(\frac{3\pi s}{D}\right)\right] \longleftarrow \text{Mean structure, } \kappa = \textit{kurtosis parameter}$$

$$+\sum_{j=1}^{n} a'_j\cos\left(\frac{2\pi js}{D}\right) + a''_j\sin\left(\frac{2\pi js}{D}\right), \longleftarrow \text{Fluctuations from the mean}$$

- where $a_0 = \frac{\pi}{2D}V,$ and V is the volume of ice in the particle

Mean structure of aggregate snowflake

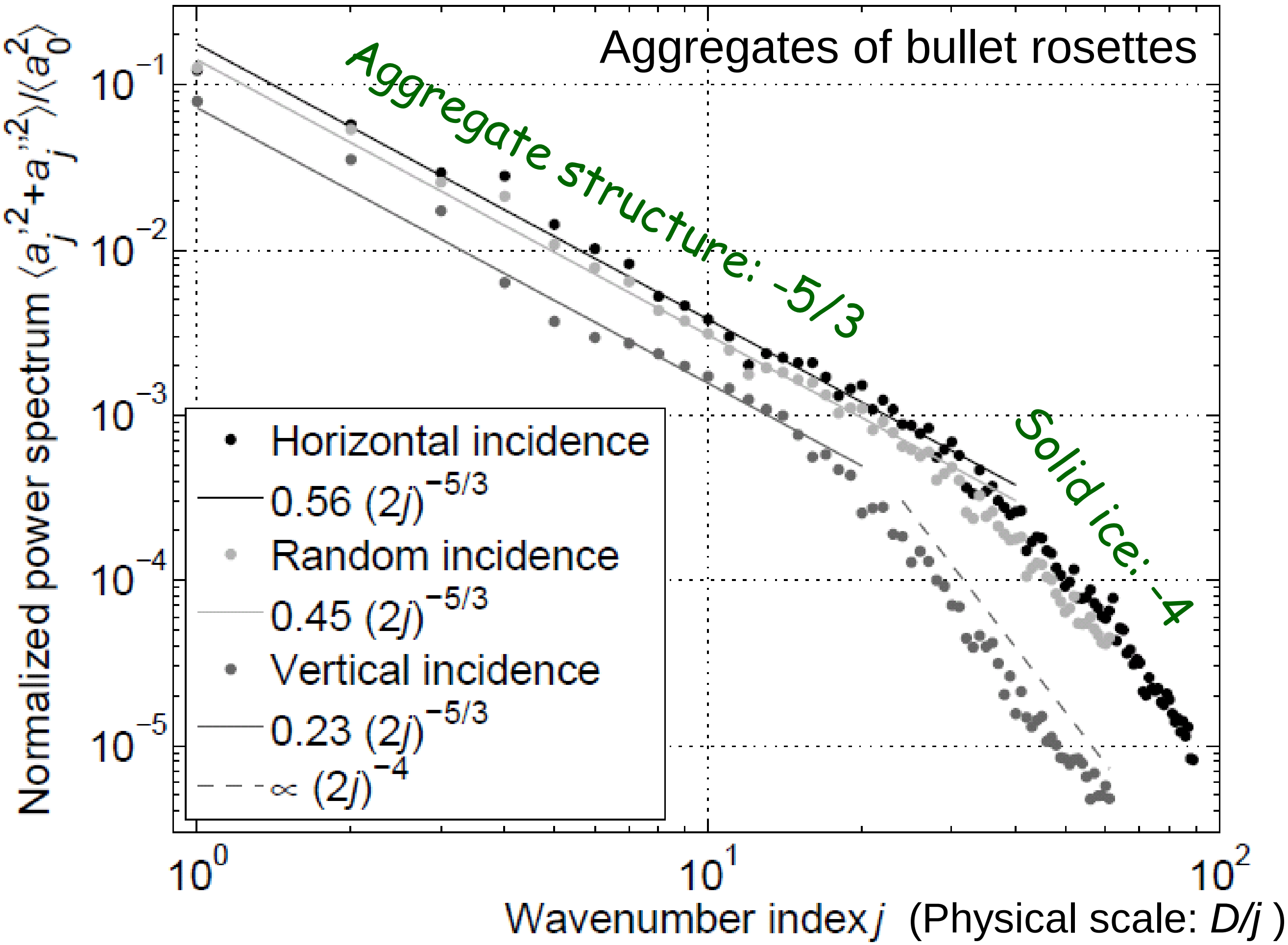
- Hydrodynamic forces cause ice particles to fall horizontally, so we need separate analysis for horizontally and vertically viewing radar



- Mean structure of 50 simulated aggregates of bullet rosettes is very well captured by the two-cosine model with kurtosis parameters of
  - κ = −0.11 for horizontal structure
  - κ = 0.19 for vertical structure
- Slightly different numbers for aggregates of other monomer crystals

Aggregate self-similar structure

- Power spectrum of fluctuations obeys a -5/3 power law
  - Why the Kolmogorov value when no turbulence involved? Coincidence?
  - Aggregates of columns and plates show the same slope



New equation for backscatter

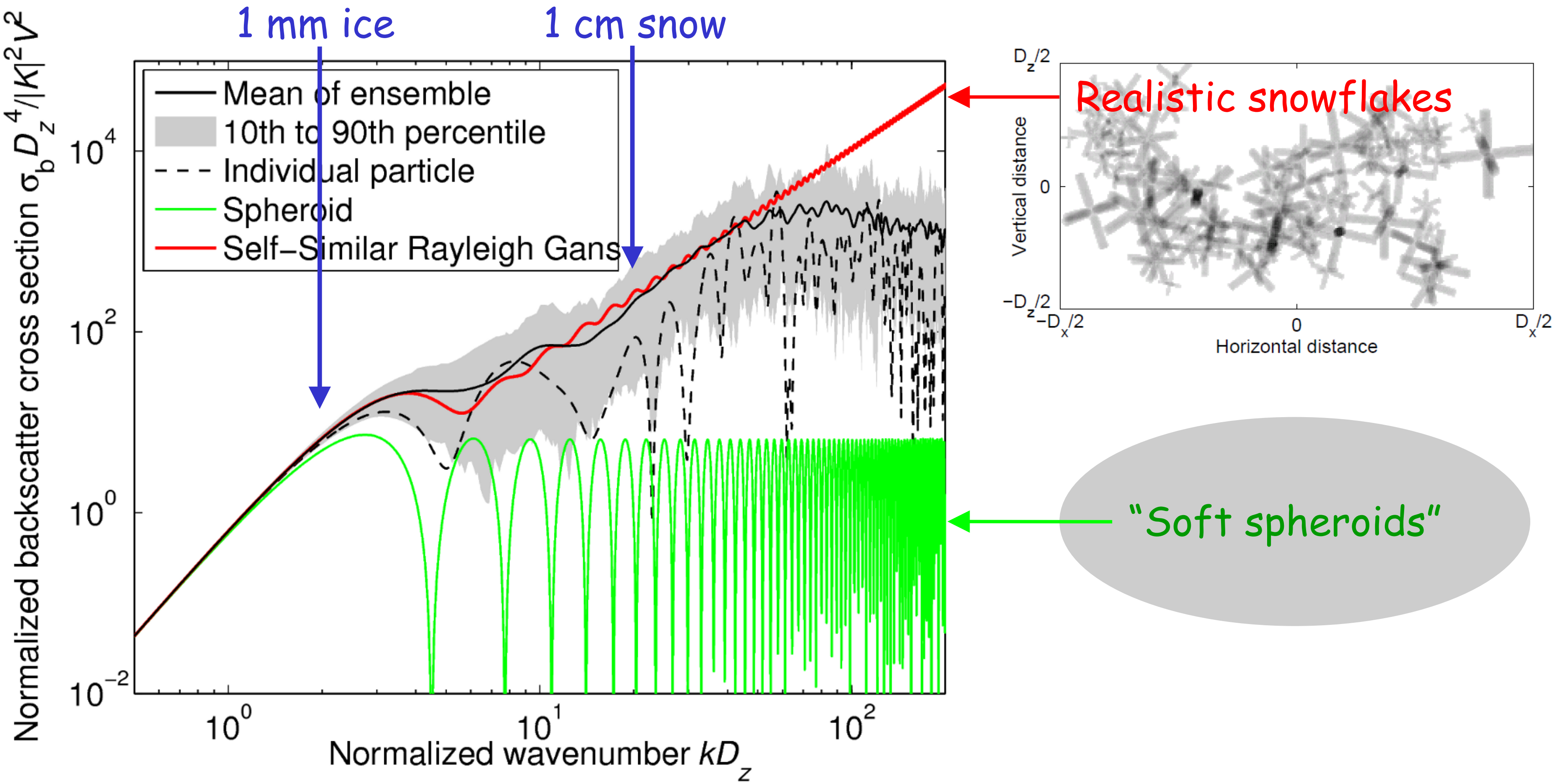
- Assumptions:
  - Power-law: $\langle a_j'^2 + a_j''^2 \rangle / \langle a_0^2 \rangle = \beta(2j)^{-\gamma}$
  - Fluctuations at different scales are uncorrelated: $\langle a_j' a_k' \rangle = \langle a_j'' a_k'' \rangle = 0$
  - Sins and cosine terms at the same scale are uncorrelated: $\langle a_j' a_j'' \rangle = 0$
- Leads to the *Self-Similar Rayleigh-Gans approximation* for ensemble-mean backscatter cross-section:

$$\langle \sigma_b \rangle = \frac{9k^4\pi|K|^2V^2}{16} \left\{ \cos^2(x) \left[\left(1 + \frac{\kappa}{3}\right) \left(\frac{1}{2x+\pi} - \frac{1}{2x-\pi} \right) - \kappa \left(\frac{1}{2x+3\pi} - \frac{1}{2x-3\pi} \right) \right]^2 + \beta \sum_{j=1}^{n} (2j)^{-\gamma} \sin^2(x) \left[\frac{1}{(2x+2\pi j)^2} + \frac{1}{(2x-2\pi j)^2} \right] \right\},$$

  - where $x = kD$
- The {} term describes only the effect of the shape of the particle; the total volume of ice (i.e. the mass) is described only by *V*
  - For small particles the {} term reduces to $4/\pi^2$, yielding the Rayleigh approximation

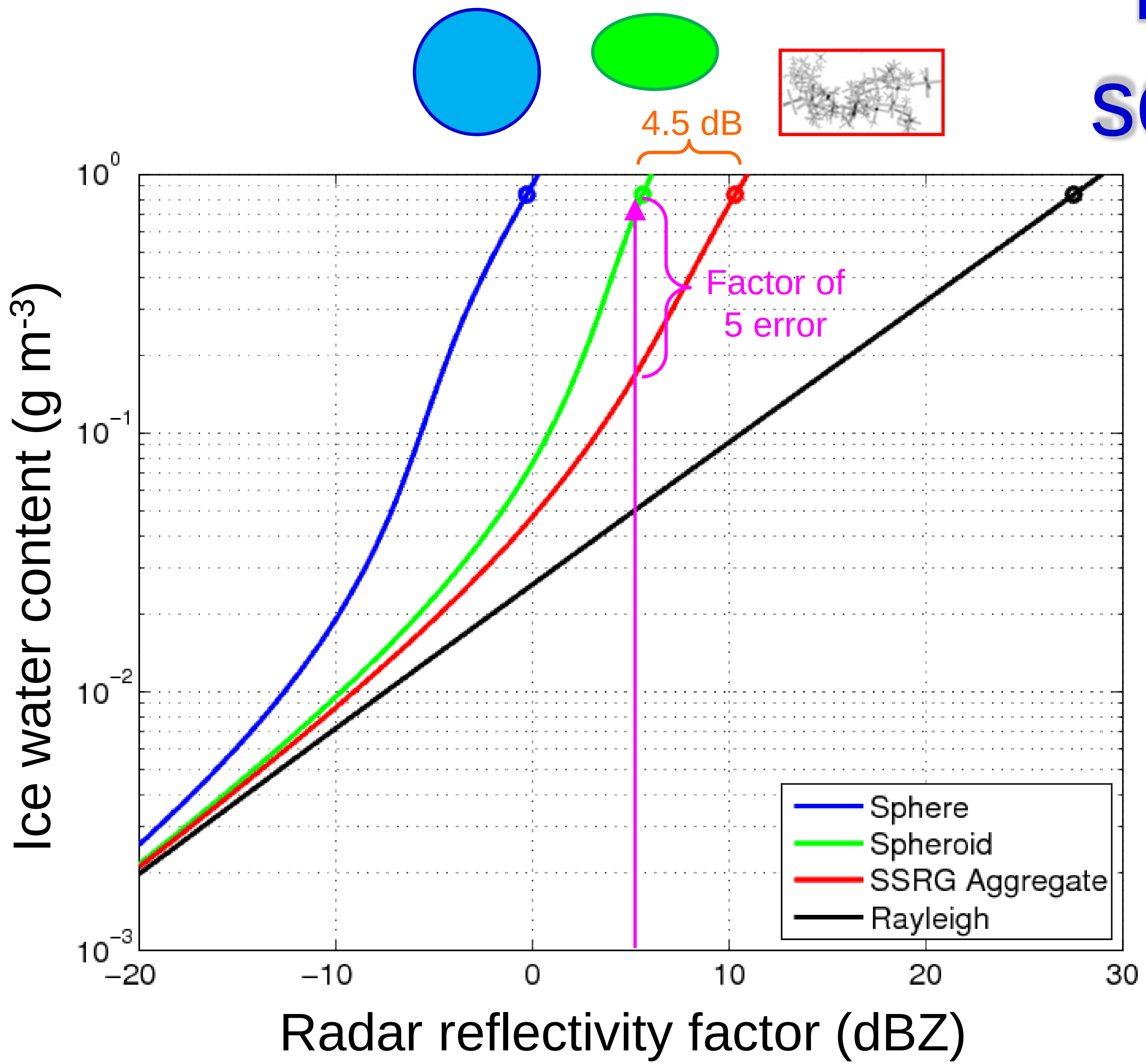
Hogan and Westbrook (JAS 2014, in press)

Radar scattering by ice



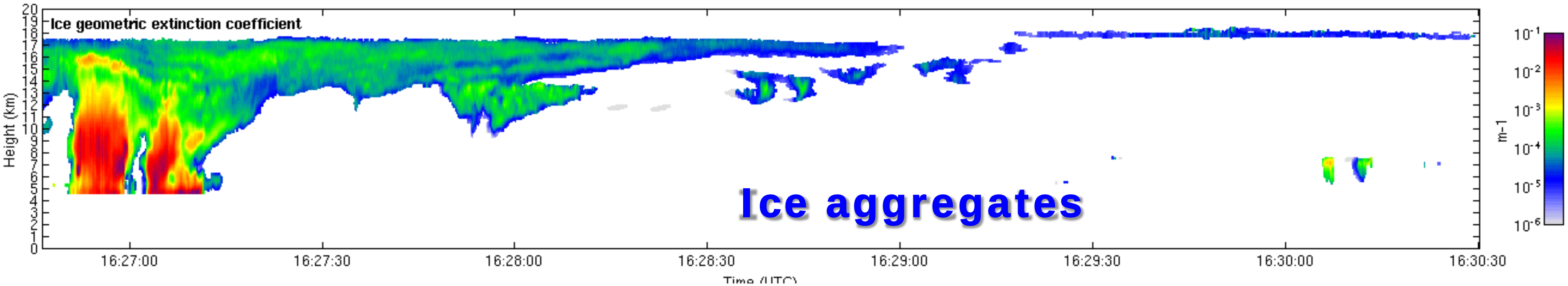
- Internal structures on scale of wavelength lead to significantly higher backscatter than "soft spheroids" (proposed by Hogan et al. 2012)
- The new SSRG equation matches the ensemble-mean well

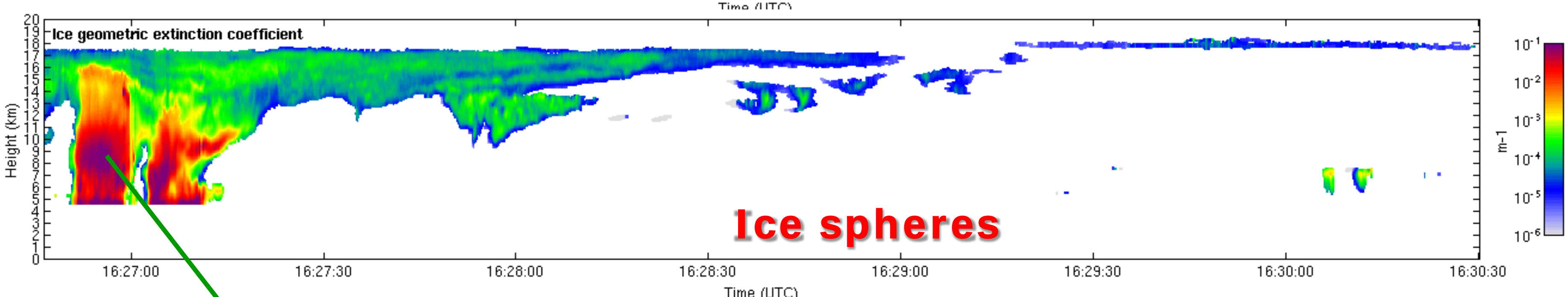
Impact of scattering model

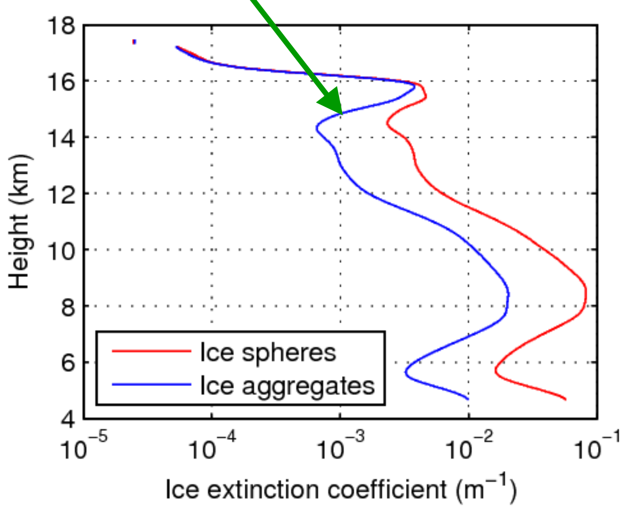


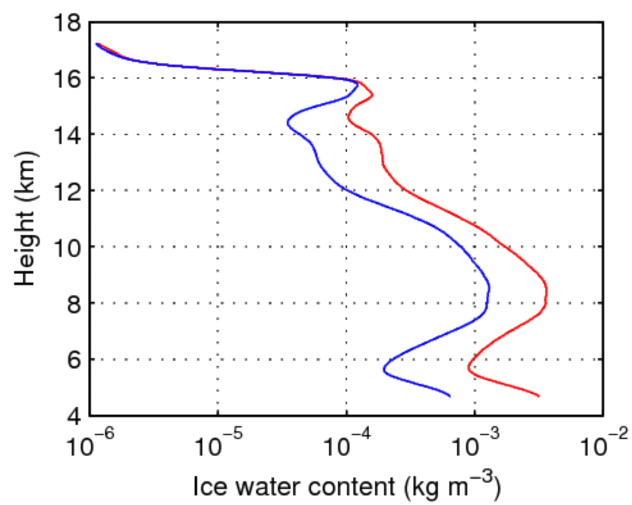
- Field et al. (2005) size distributions at 0°C
- Circles indicate D_0 of 7 mm reported from aircraft (Heymsfield et al. 2008)
- Lawson et al. (1998) reported D_0=37 mm: 17 dB difference

Example of impact on IWC retrievals









- Spheres can lead to overestimate of water content and extinction of factor of 3
- All 94-GHz radar retrievals affected in same way

Summary

- A new "Self-Similar Rayleigh Gans" equation has been proposed for backscatter cross-section of ice aggregates observed by radar
  - Far less computationally costly than DDA
  - The effects of *shape* and *mass* on backscattering are cleanly separated
  - Could also be applied to light scattering by some aerosol aggregates
- New equation predicts much higher 94-GHz backscatter for $D > \lambda$ than the "soft spheroid" model, which only works for $D < \lambda$
  - Use of soft spheroids in 94-GHz radar retrievals can lead to factor 3-5 error in IWC and snowfall rate in thick cloud and snow
- Remaining uncertainties in ice/snow retrievals
  - Mass-size relationship: *riming increases density but EarthCARE's Doppler will help*
  - Aspect ratio: *difference between 0.6 and 0.45 around the same as difference between soft-spheroids and realistic particles*
  - Fallspeed relationship: *improved with Heymsfield & Westbrook (2010)*
- Aggregate structure exhibits a power law with a slope of -5/3: why?

Hogan and Westbrook (JAS 2014, in press)