# DEVELOPING AND BOUNDING ICE PARTICLE MASS- AND AREA-DIMENSIONAL EXPRESSIONS FOR USE IN ATMOSPHERIC MODELS AND REMOTE SENSING

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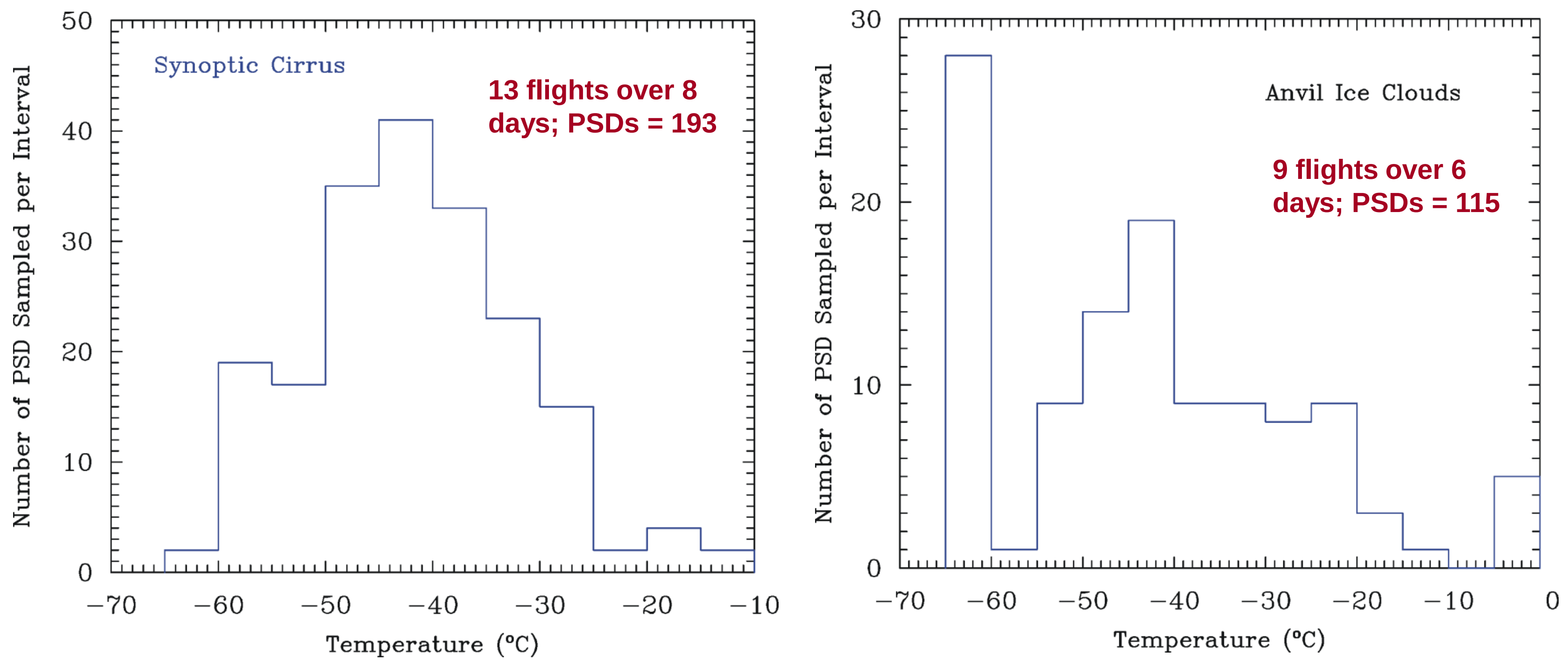
## ABSTRACT

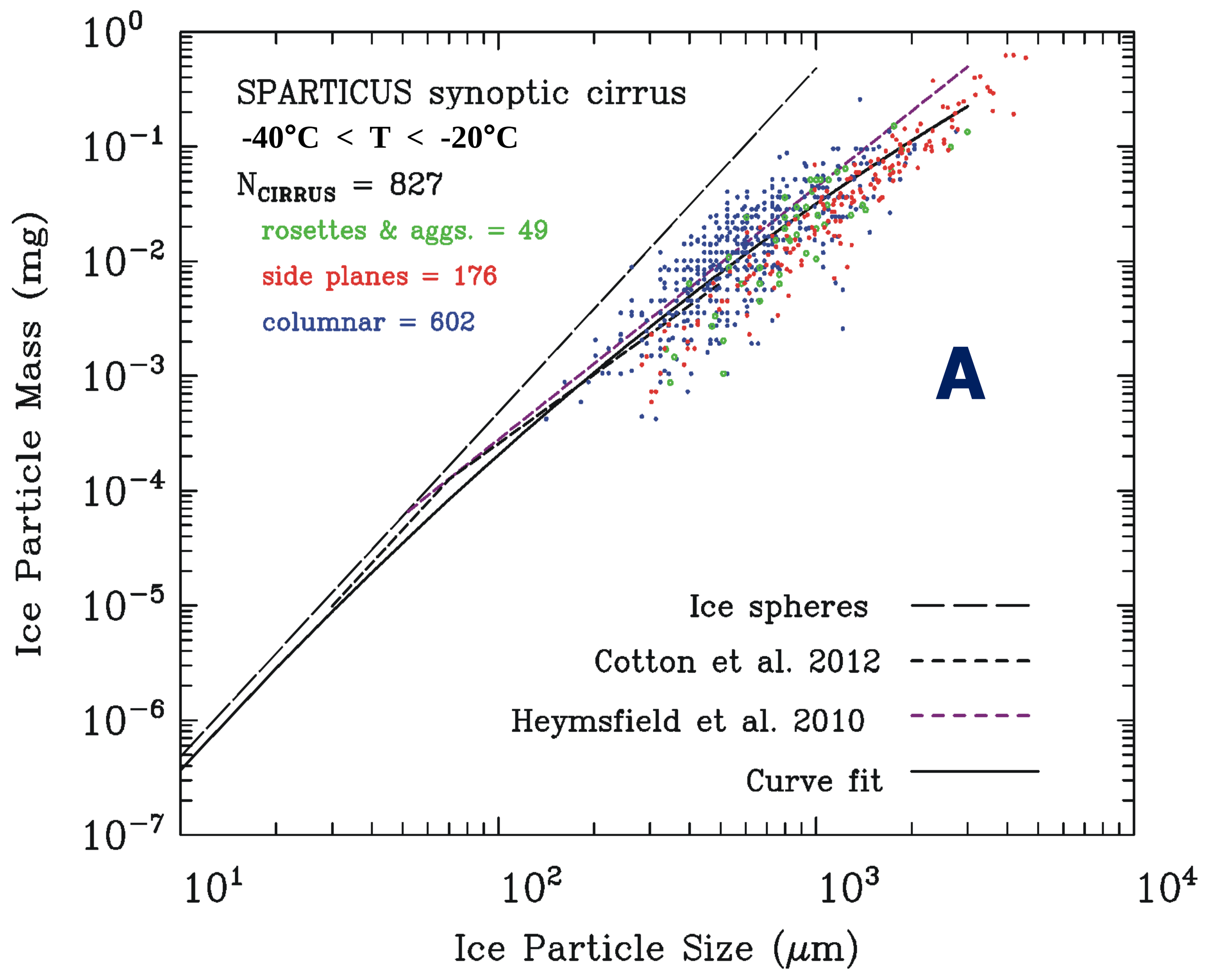
Ice particle mass- and projected area-dimension (m-D & A-D) power laws differ between small and relatively large ice particles having the same shape; a power law cannot describe the m-D or A-D relationship across all sizes. Therefore size-resolved measurements of m and A are needed to determine the general form of m-D & A-D expressions. From the general expression, m-D and A-D power laws can be extracted for the ice particle size distribution (PSD) moment of interest.

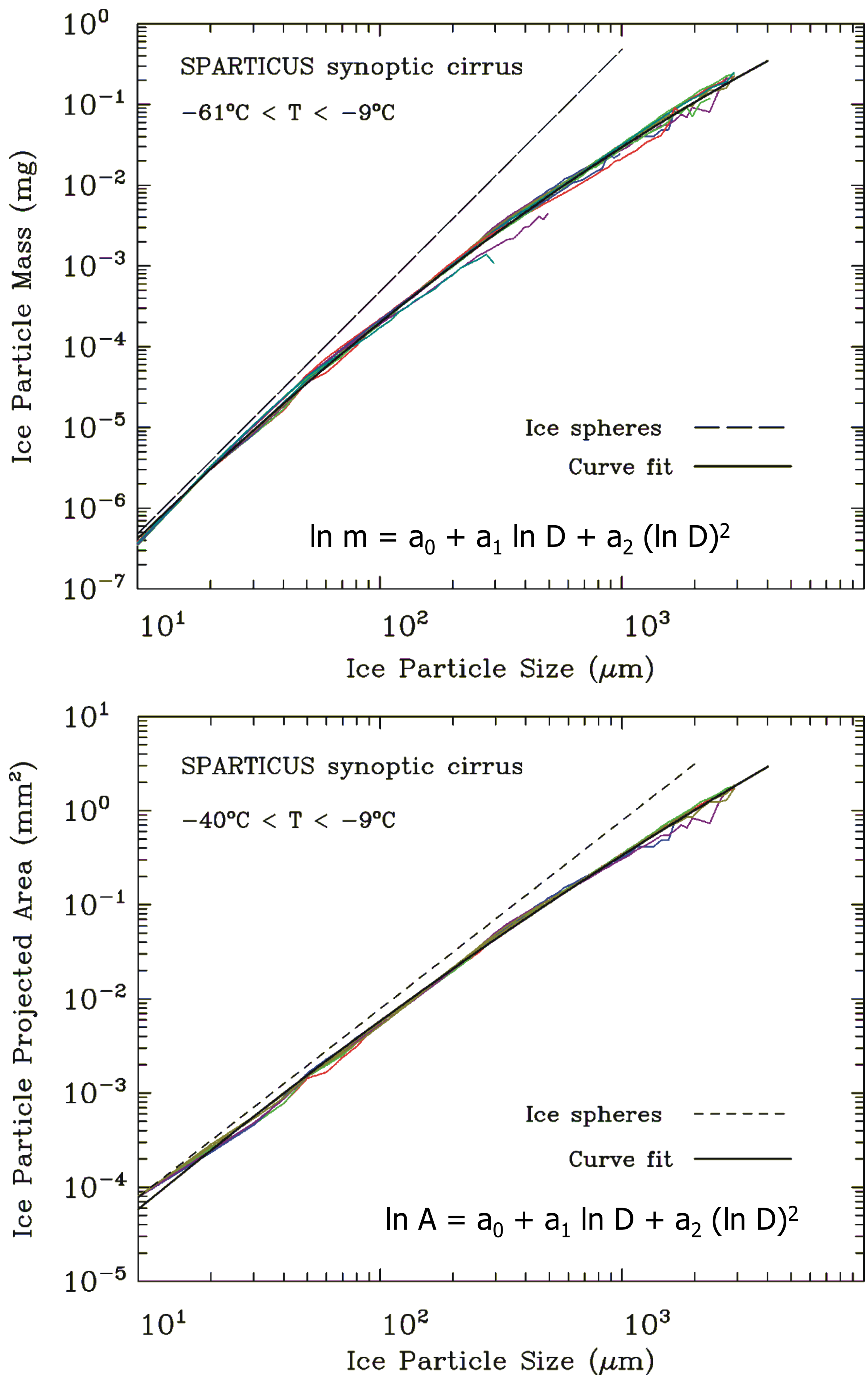
To develop m-D & A-D expressions for cirrus clouds in terms of temperature T and cloud type, the following strategy was developed: (1) use 2D-S probe size-resolved measurements of ice particle number, A and m concentration, where m is estimated from the Baker-Lawson m-A power law, to generate m-D & A-D expressions for 10 μm < D < 1280 μm; (2) test the 2D-S m-D expression for -40°C < T < -20°C against size-resolved ice particle mass measurements for this same T regime; (3) if agreement in (2) is good, then assume that the 2D-S mass estimates for T < -40°C are adequate provided PSD area ratios for T < -40°C are not much different than for -40°C < T < -20°C; (4) assume that the m-D & A-D uncertainties measured for -40°C < T < -20°C also apply at colder temperatures.

This strategy has been implemented and is described below. The 2D-S data is from the SPARTICUS field campaign, using hundreds of PSD from synoptic and anvil cirrus clouds. The size-resolved ice particle mass measurements were obtained from a cloud seeding field study (Mitchell et al. 1990, JAM), using 827 ice particles having shapes characteristic of cirrus cloud ice particles formed between -20 and -40°C. The m-D & A-D relationships are best described by 2nd order polynomial fits. This strategy appears successful for T > -55°C, but the PSD area ratios change for T < -55°C, indicating a need for direct ice particle mass measurements at these coldest temperatures. A methodology for extracting m-D & A-D power laws from the curve fit expressions, appropriate for a given PSD moment, has been developed. This may be a convenient way for cloud and climate models to utilize these curve-fit expressions while still preserving their model architecture that is based on m-D and A-D power laws.



PSD sampling statistics for synoptic and anvil cirrus clouds. Each PSD yields one m-D and one A-D array. A is directly measured; m is estimated from the Baker-Lawson (2006) m-A power law.



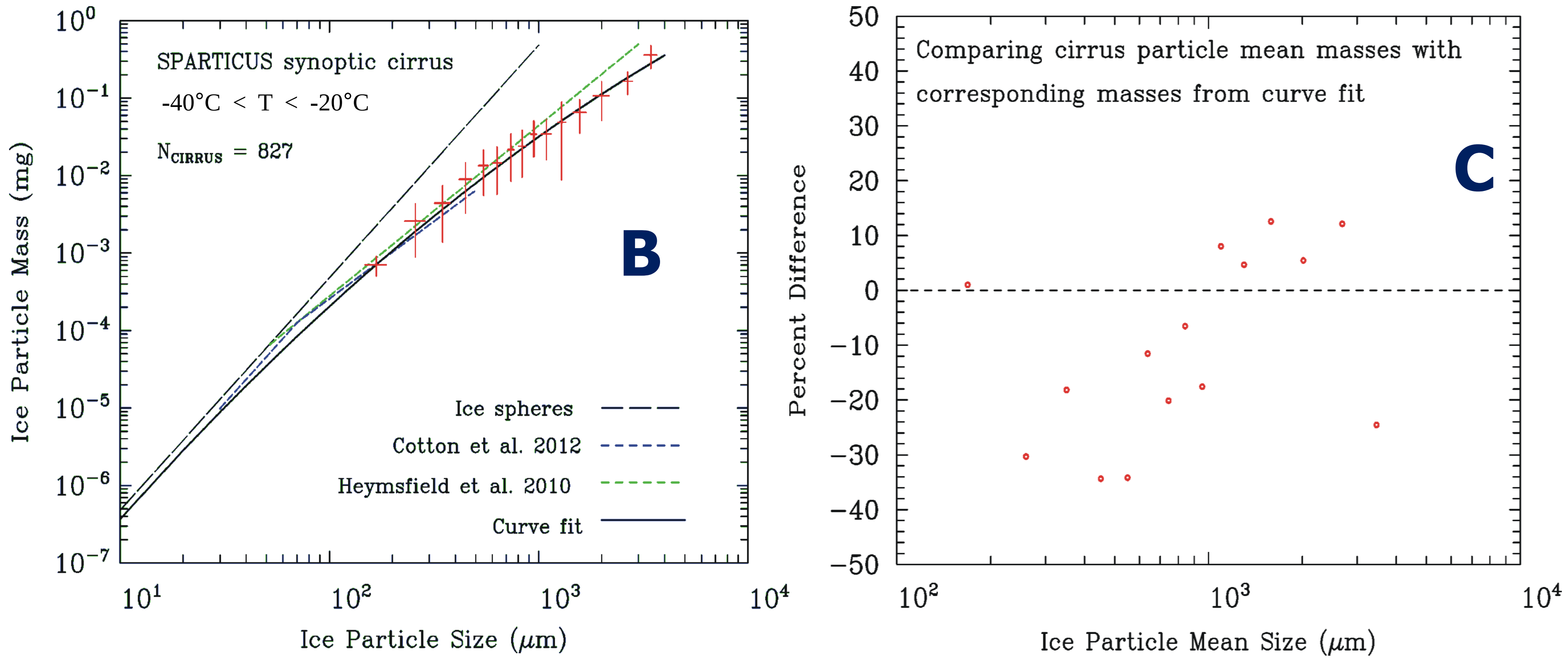


$$\ln m = a_0 + a_1 \ln D + a_2 (\ln D)^2$$

$$\ln A = a_0 + a_1 \ln D + a_2 (\ln D)^2$$

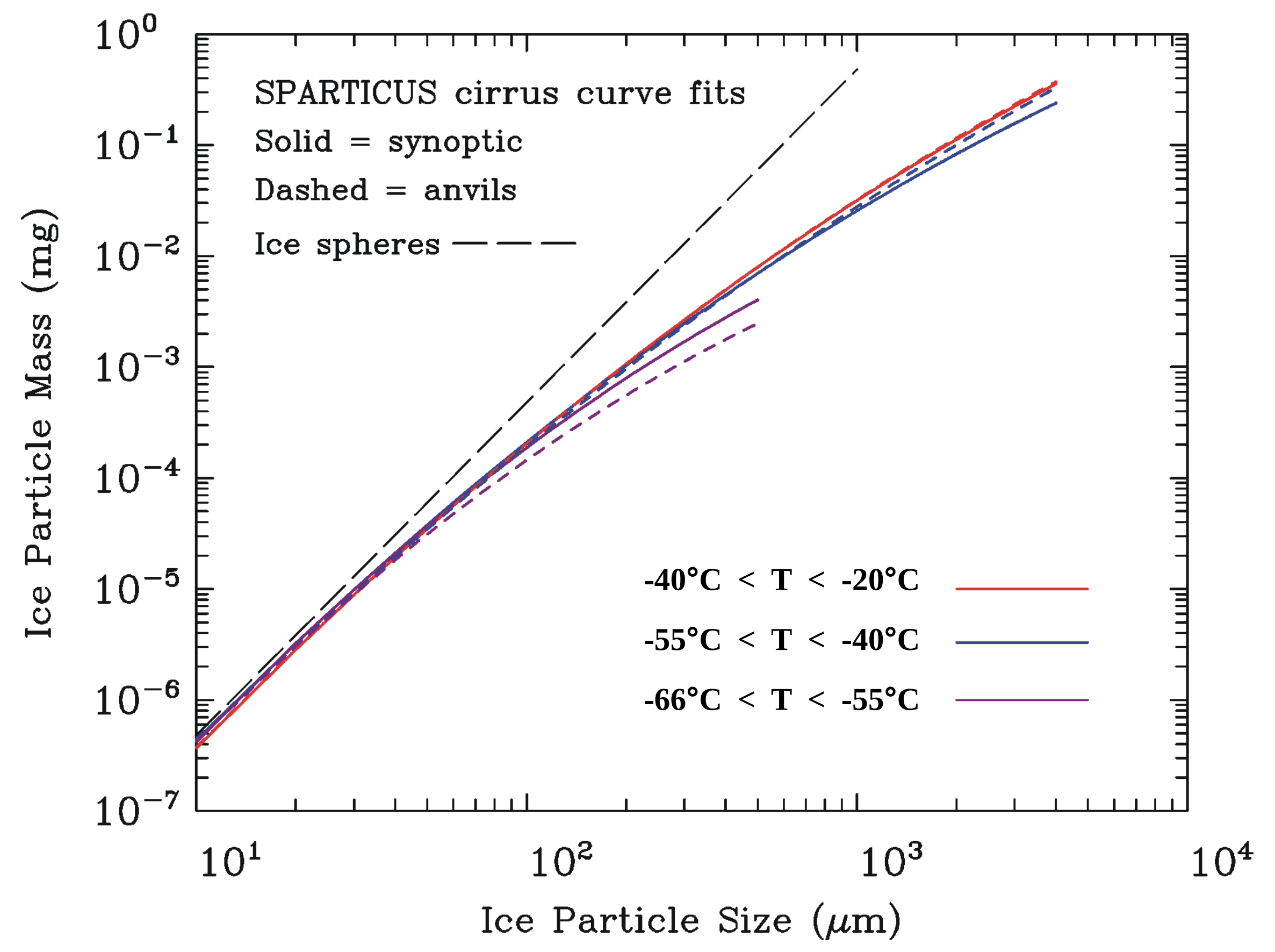
PSD were averaged over 5°C intervals to produce m-D and A-D arrays for each interval. m-D & A-D curve fits were obtained for selected temperature regimes.

Mean uncertainty = ± 54.4%

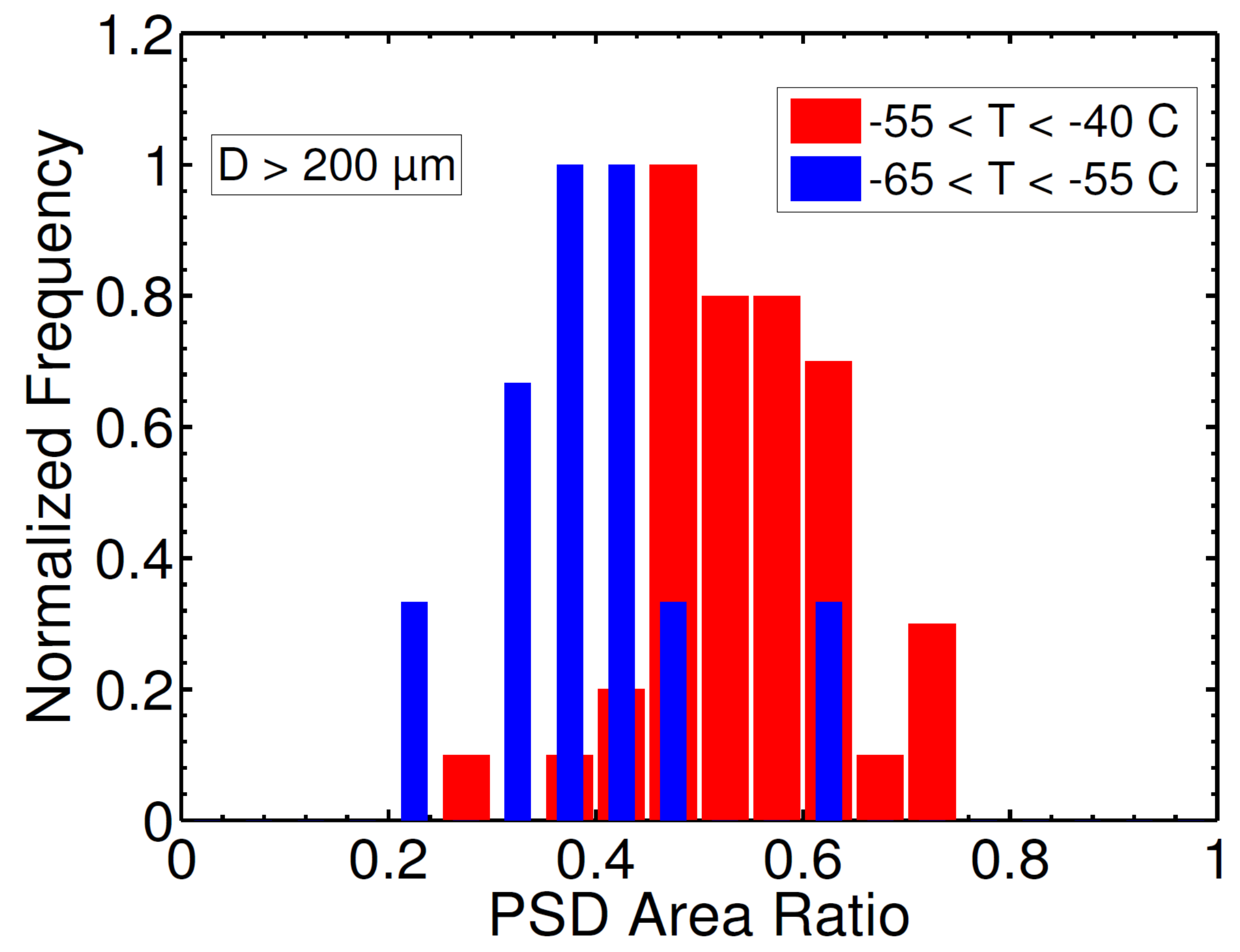


Panel A compares m-D measurements from the cloud seeding experiment for ice particle shapes characteristic of cirrus clouds with the corresponding m-D curve fit obtained from the 2D-S measurements during the SPartICus field campaign. Also compared are two m-D power laws based on cirrus in situ measurements, reported in two recent studies. The Cotton et al. and Heymsfield et al. m-D power laws yield m within 50% and 100%, respectively, of corresponding m predicted from the curve fit. These comparisons are quantified in Panel B, where mean m & D and their standard deviations are calculated for selected size intervals. Percent differences between these size-resolved mean m values and corresponding m values from the indicated curve fit are less than 35% (considered small for this type of measurement). The relative agreement between the curve fit, the m-D measurements and the two power laws suggests that the curve fit is representative for warm cirrus clouds.

## Dependence of m-D expression on temperature regime and cloud type



## Evaluation of ice particle shape difference for coldest temperature regime



## Extraction of power laws from 2nd order polynomial curve fits for m-D & A-D

$$m = \alpha D^{\beta}$$

$$A = \gamma D^{\delta}$$

For a given D, we can obtain these power laws

$$\ln m = a_0 + a_1 \ln D + a_2 (\ln D)^2$$

$$d(\ln m)/d(\ln D) = \beta = a_1 + 2a_2 \ln D$$

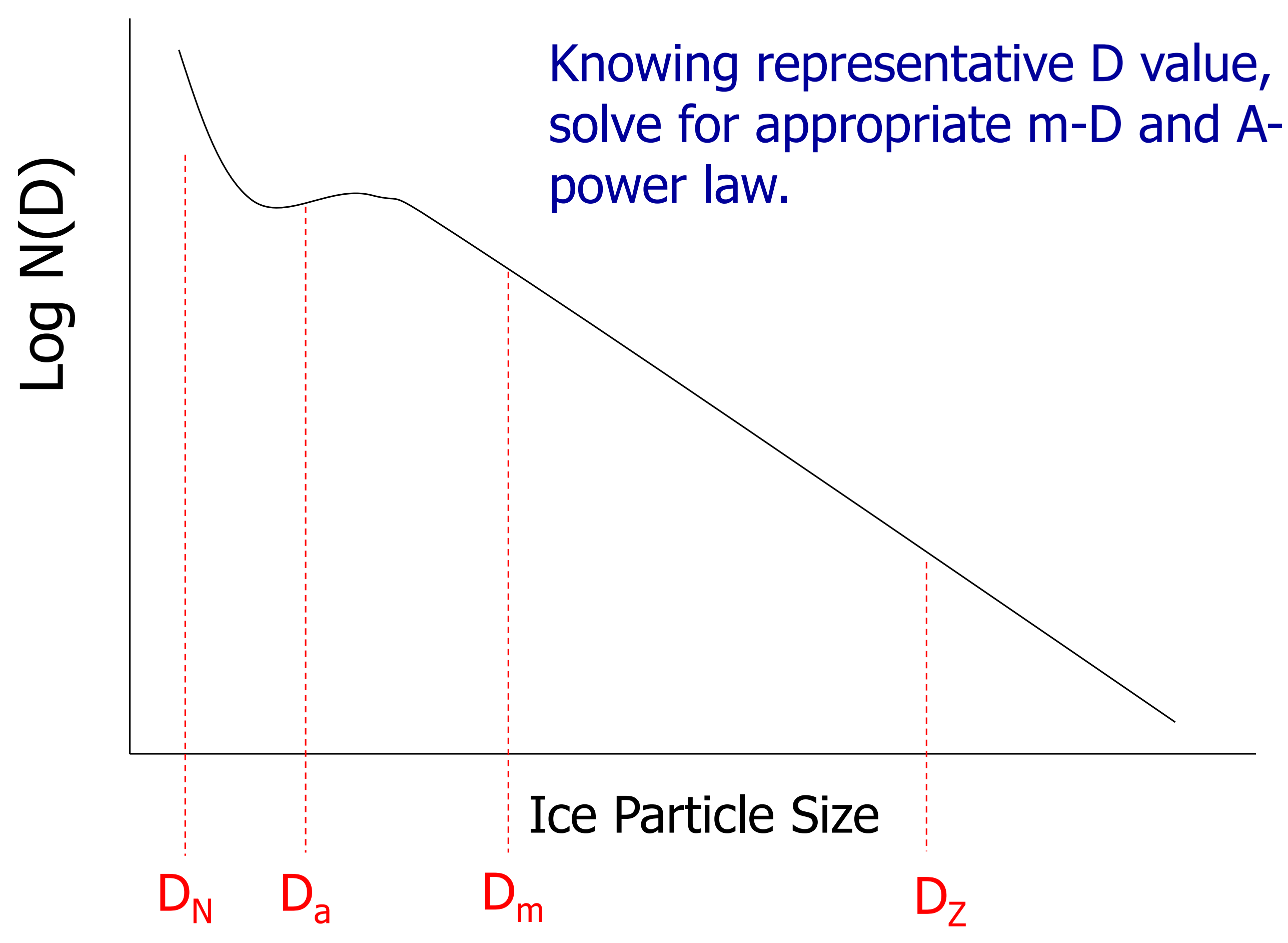
$$\alpha = \exp[a_0 + a_1 \ln D + a_2 (\ln D)^2 ] / D^{\beta}$$

## Uncertainty analysis for β

| Temperature Range | Ice Particle Size (μm) | | | |
|---|---|---|---|---|
| | 50 | 150 | 500 | 1500 |
| | Power β | | | |
| -15 < T ≤ -10°C | 2.610 | 2.382 | 2.131 | 1.903 |
| -20 < T ≤ -15°C | 2.619 | 2.392 | 2.142 | 1.915 |
| -25 < T ≤ -20°C | 2.668 | 2.341 | 1.982 | 1.655 |
| -30 < T ≤ -25°C | 2.674 | 2.350 | 1.993 | 1.668 |
| -35 < T ≤ -30°C | 2.660 | 2.369 | 2.050 | 1.759 |
| -40 < T ≤ -35°C | 2.604 | 2.367 | 2.106 | 1.869 |
| -45 < T ≤ -40°C | 2.572 | 2.251 | 1.900 | 1.579 |
| -50 < T ≤ -45°C | 2.560 | 2.304 | 2.024 | 1.768 |
| -55 < T ≤ -50°C | 2.591 | 2.268 | 1.914 | 1.591 |
| -60 < T ≤ -55°C | 2.491 | 2.069 | 1.607 | ---- |
| -65 < T ≤ -60°C | 2.462 | 1.932 | 1.351 | ---- |
| Mean β | 2.592 | 2.275 | 1.927 | 1.745 |
| Standard Deviation of β | 0.0684 | 0.146 | 0.242 | 0.130 |
| Mean Uncertainty (%) | 7.27 | | | |

Uncertainties were lower for anvil cirrus and A-D power laws for synoptic and anvil cirrus. Thus most uncertainty is associated with prefactor α.

## Application to Cloud Modeling: Median Dimensions (dividing cloud property of interest into equal parts)



Knowing representative D value, solve for appropriate m-D and A-D power law.

$$N(D) = N_o D^{\nu} \exp(-\lambda D)$$

$$\lambda = \left( \frac{\alpha\, \Gamma(\beta+\nu+1)\, N}{\Gamma(\nu+1)\, IWC} \right)^{1/\beta}$$

Used in CAM5

To a good approximation, λ is obtained by evaluating α & β at D = 500 μm. Then estimate the D for the cloud property or process of interest by evaluating β and δ at D = 500 μm:

| | |
|---|---|
| $D_N = (\nu + 0.67)/\lambda$ | Median number conc. dimension |
| $D_a = (\delta + \nu + 0.67)/\lambda$ | Median area dimension |
| $D_m = (\beta + \nu + 0.67)/\lambda$ | Median mass dimension |
| $D_Z = (2\beta + \nu + 0.67)/\lambda$ | Median radar reflectivity dimension |

Then calculate α, β, γ and δ for the selected D value. Formally, this is an iterative solution, but practically, only one iteration is needed for most applications due to the slow change in β and δ with respect to D.

## Contrasting new scheme with CAM5