TOWARDS BETTER UNDERSTANDING OF PREFERENTIAL CONCENTRATION IN CLOUDS:

DROPLETS IN SMALL VORTICES

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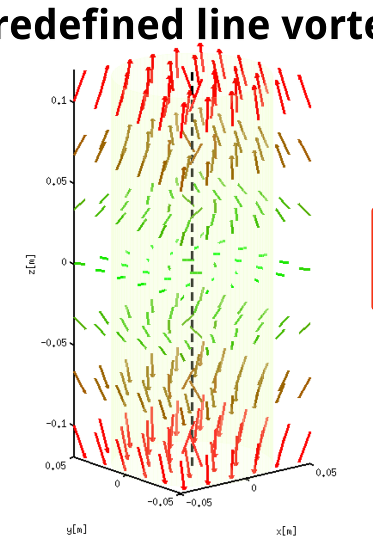
1. INTRODUCTION

Recent studies attribute the evolution of droplet size distribution in warm convective clouds to enhancement of collision-coalescence by turbulence. The aim of this study is a better understanding of enhanced collisions and preferencial concentration from numerical analysis of droplet motion in vortex tubes: small coherent structures characteristic for high Reynolds number turbulent flows. Former research of such effects by Hill[1] and by Markowicz et al. [2] was limited to horizontally oriented vortex tubes only. Herein we analyse tubes which are parallel or oblique to the direction of gravity.

2. DROPLET DYNAMICS

Motion of droplet governed by the Stokes equation was calculated in predefined velocity field. 3D numerical simulations were done for two sets of field parameters.

Predefined line vortex with stretching

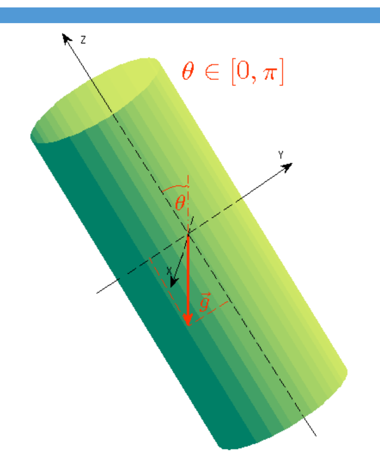


Predefined velocity field scheme

stretching with force γ

$$\vec{v_a} = -\frac{\gamma}{2} r\hat{e}_r + \frac{L_w}{r}\hat{e}_\phi + \gamma z\hat{e}_z$$

line vortex with circulation $2\pi L_w$



$\vec{g} = -g(sin\theta\hat{e_y} + cos\theta\hat{e_z})$

Scheme of velocity field in relation to gravity direction.

Simulation cases	
	"weak vortex" $\gamma = 0.5\frac{1}{s}, \quad L_w = 0.00025\frac{m^2}{s}$
	"strong vortex" $\gamma = 30\frac{1}{s}, \quad L_w = 0.025\frac{m^2}{s}$

Equation of motion

$$m\ddot{\vec{r}} = \frac{m}{\tau}(\vec{v}_a - \dot{\vec{r}}) + m\vec{g}$$

Scales of motion

$\tau = \frac{2\rho}{9\mu}R^2$

$S = \sqrt{L_w\tau}$

inertial response time

$J = \frac{g\tau}{\frac{S}{\tau}} = \frac{\text{terminal velocity}}{\text{characteristic velocity}}$

! Solutions of the equations depend on 3 nondimensional parameters : motion in a plane XY depends on K_1 and K_3, in Z-direction - on K_1 and K_2.

$K_1 = \gamma\tau \qquad K_2 = J\cos\theta \qquad K_3 = J\sin\theta$

3. DROPLET MOTION ANALYSIS

Motion in plane perpendicular to vortex axis separates from the direction parallel to it.

3.1. Z direction

Analytical solution for $z(0) = z_0,\ \dot{z}(0) = 0$:

$$z(t) = \frac{z_0 - z_{0b}}{\lambda_+ - \lambda_-}\left[\lambda_+ exp(\lambda_- t) - \lambda_- exp(\lambda_+ t)\right] + z_{0b}$$

$$\lambda = \lambda(K_1),\ \lambda_+ > 0,\ \lambda_- < 0$$

! Motion in Z direction depends only on initial position of a droplet

$z_0 < z_{0b}$ — exponential decrease

$z_0 = z_{0b} \propto \gamma^{-1}R^2\cos\theta$ — unstable, steady position

$z_0 > z_{0b}$ — exponential increase

3.2. XY plane

Simple case: vortex parallel to gravity

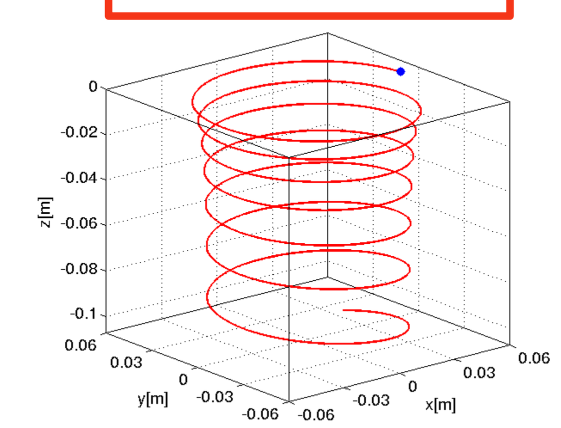
! Droplet gets on its stable, steady orbit, where radial viscous and centrifugal forces equalize.

Orbit radius

$r_{orb} = \sqrt[4]{2L_w^2\tau\gamma^{-1}} \propto \sqrt{R}$

Orbit angular velocity

$\omega_{orb} = \sqrt[2]{\frac{\gamma}{2\tau}} \propto R$



Droplet trajectory in "weak vortex", $R = 10\mu m$, t=3.5 s

Vortex non-parallel to gravity, $\theta \neq 0$

! Gravity dependence destroys axial symmetry.

- stationary points
- no stationary orbits
- limit cycles

existence conditions

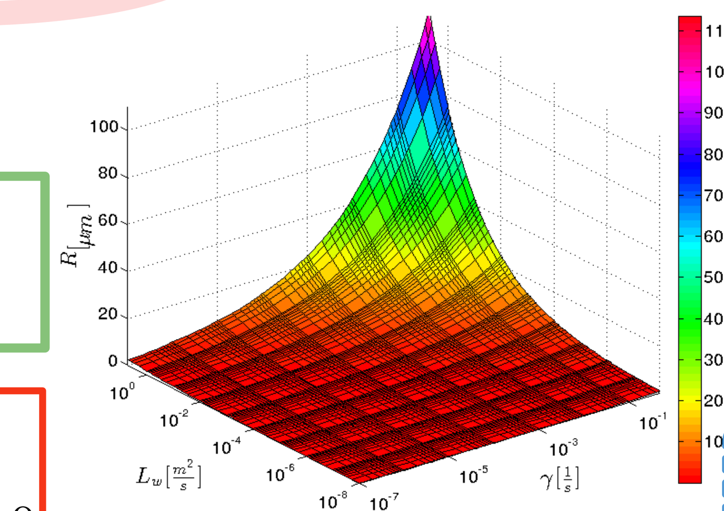
$$\frac{L_w}{\gamma\tau^2} \leq 1 \qquad \sin\theta \geq \sqrt{\frac{L_w}{\gamma\tau^2}}$$

position

$$x_{st} = K_3^{-1} \geq 0$$

$$y_{st} = -\frac{K_3}{K_1}\left(1 \mp \sqrt{1 - \left(\frac{K_1}{K_3^2}\right)^2}\right) \leq 0$$

Plot of theoretical prediction of stationary point positions for range of droplets.



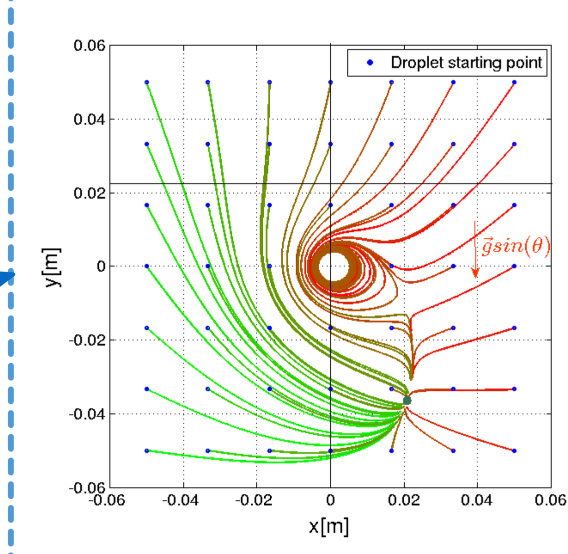
Existence condition drawn as boundary droplet radius R_{gr} surface.

Droplets of R above boundary have their stationary points for a range of angles θ, droplets of R_{gr} on the boundary have stationary points only for horizontal vortex ($\theta = \frac{\pi}{2}$).

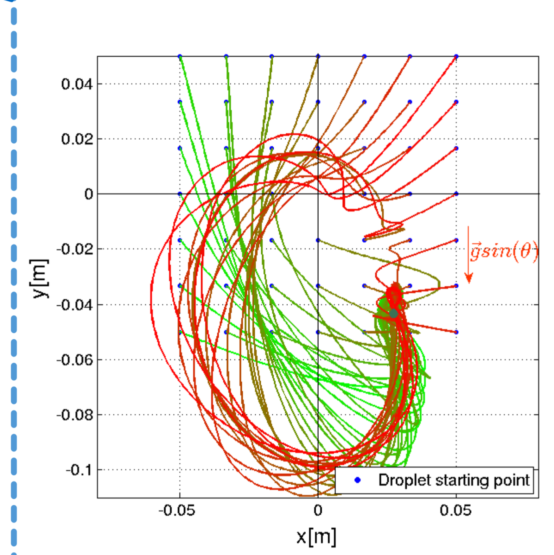
Boundary droplet radius values	
	"weak vortex" $R_{gr} = 9,56\mu m$
	"strong vortex" $R_{gr} = 84,07\mu m$

4. NUMERICAL SIMULATIONS

4.1. Trajectories of droplets of uniform size in XY plane

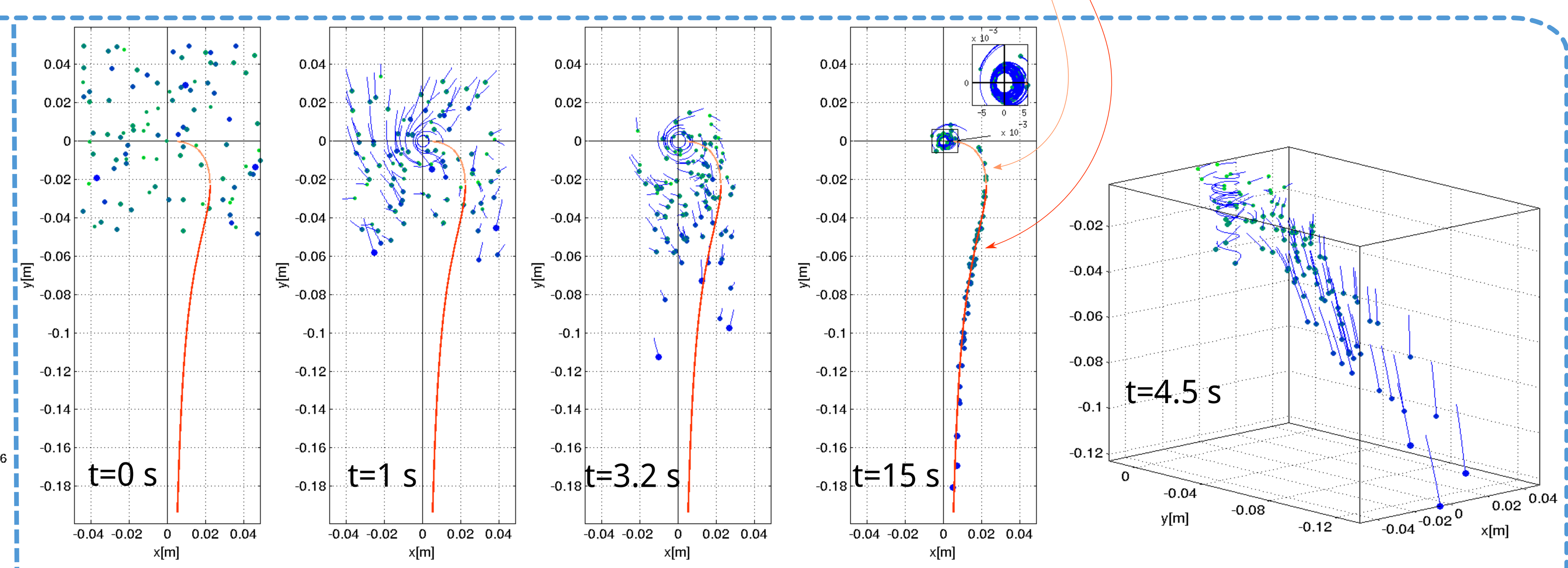


$R = 10\mu m$

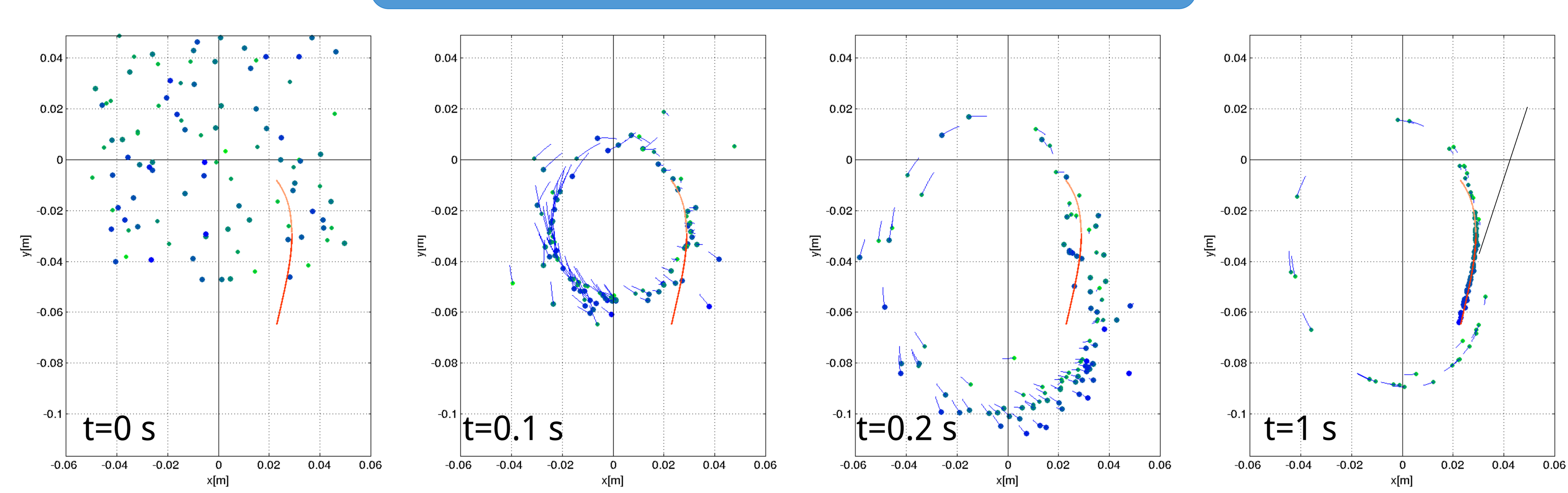


$R = 87\mu m$

4.2. Frames of motion of randomly distributed droplets



$R \in [1\mu m, 20\mu m]$



$R \in [75\mu m, 95\mu m]$

5. CONCLUSIONS

Features such as stationary orbits, stationary points and limit cycles were identified as three-dimensional structures leading to enhancement of preferential concentration. Their conditions of existence were described. Theoretical results were in a good agreement with numerical simulations.

6. REFERENCES

1) R.J.Hill., Geometric collision rates and trajectories of cloud droplets falling into a Burgers vortex. Phys. Fluids **17** : 037103 1-23 2005.

2) K. Bajer, K.M. Markowicz, S.P. Malinowski. Influence of the small-scale turbulence structure on the concentration of cloud droplets. Proceedings of 13th Conference on Clouds and Precipitation, 159-162, IAMAP, Reno, Nevada, USA, 14-18 August 2000.

3) Shaw, R. A., Particle-turbulence interactions in atmospheric clouds. Annu. Rev. Fluid Mech. **35** (2003), 183-227.