Adjusting three-dimensional atmosphere and surface properties to fit multi-pixel polarimetric measurements

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Brian Cairns, Guillaume Bal

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3D Retrievals in broken cloud fields

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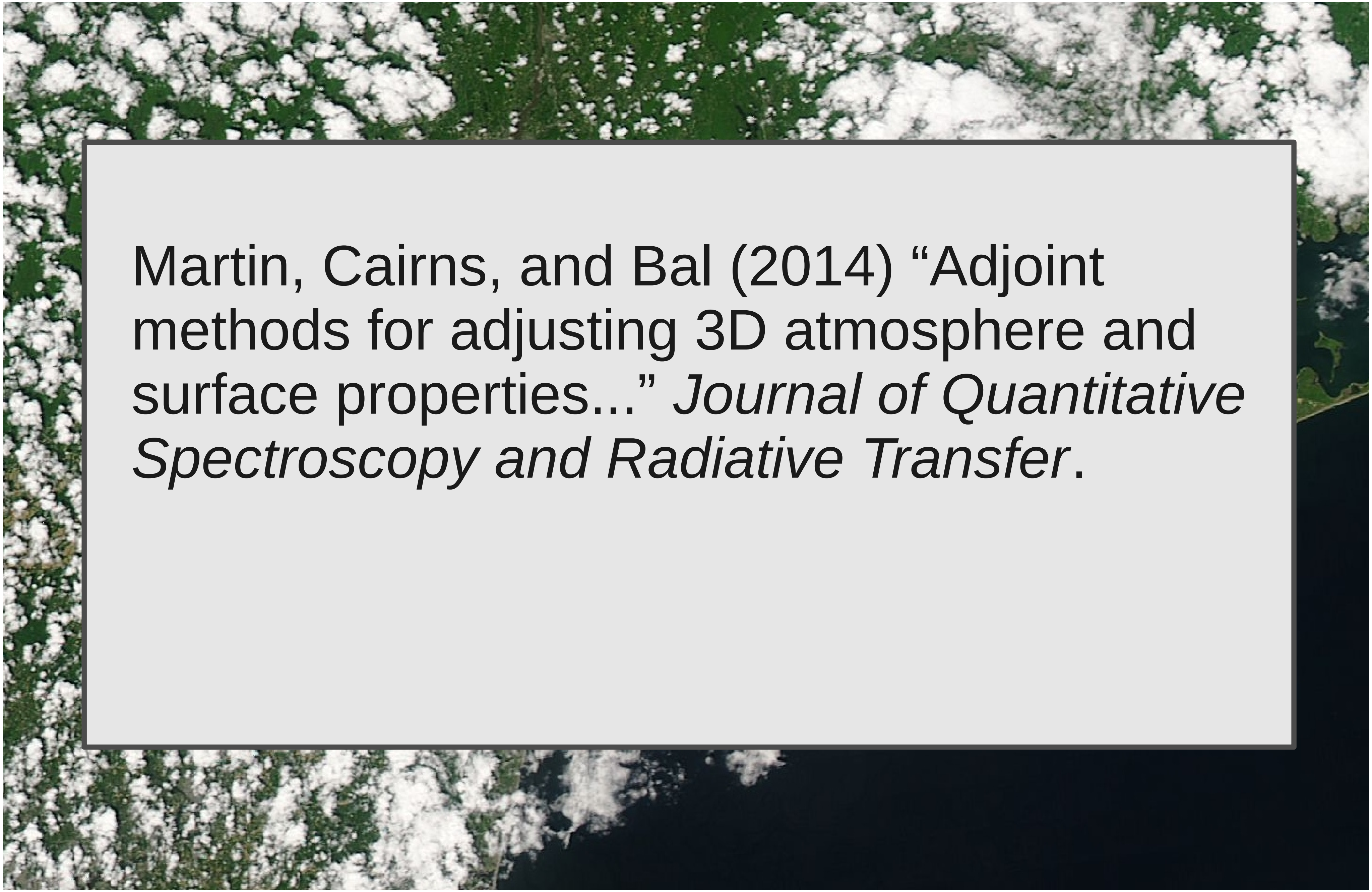
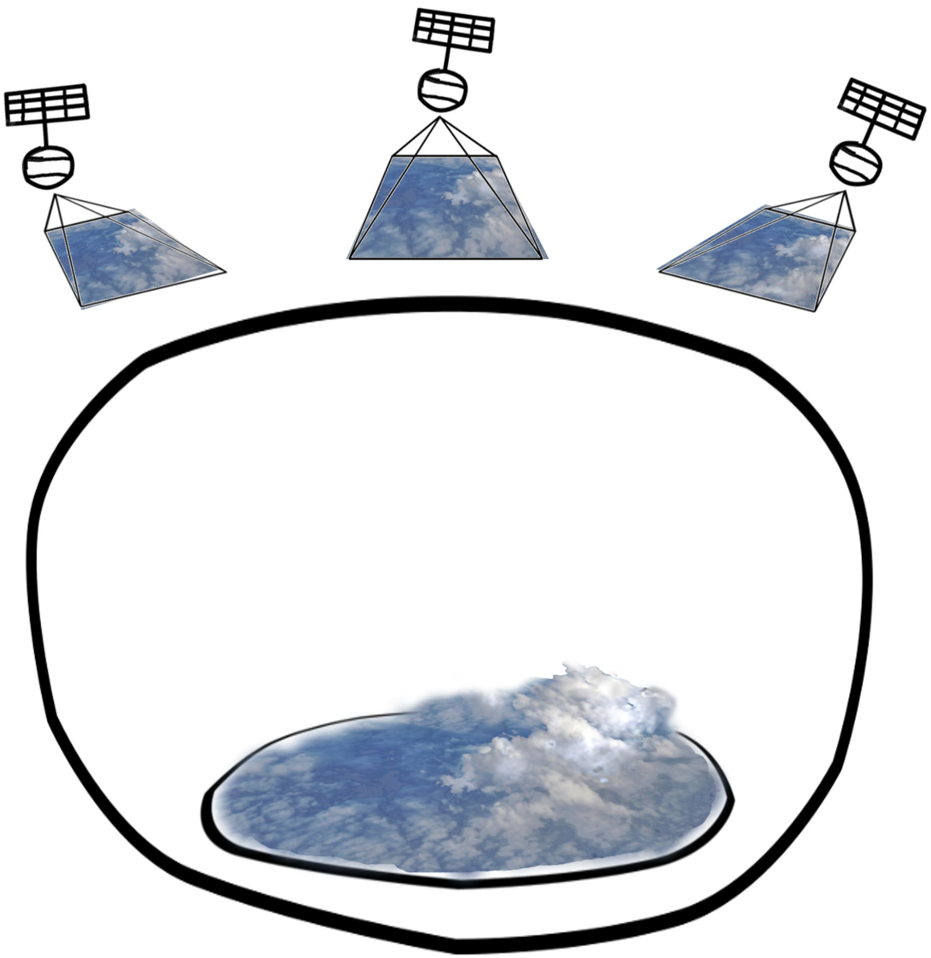


Martin, Cairns, and Bal (2014) “Adjoint methods for adjusting 3D atmosphere and surface properties...” *Journal of Quantitative Spectroscopy and Radiative Transfer*.

Cloud representation:

Three-dimensional cloud



Consider 3D retrievals to extend coverage to broken cloud fields

Solver 3D VRTE

- SHDOM [Evans,1998 and 2014]
- Adjoint derivative [Martin, 2014]

Inverse problem

- Retrieval of 1D cloud properties [Evans, 2008]
- Stability and data requirements?

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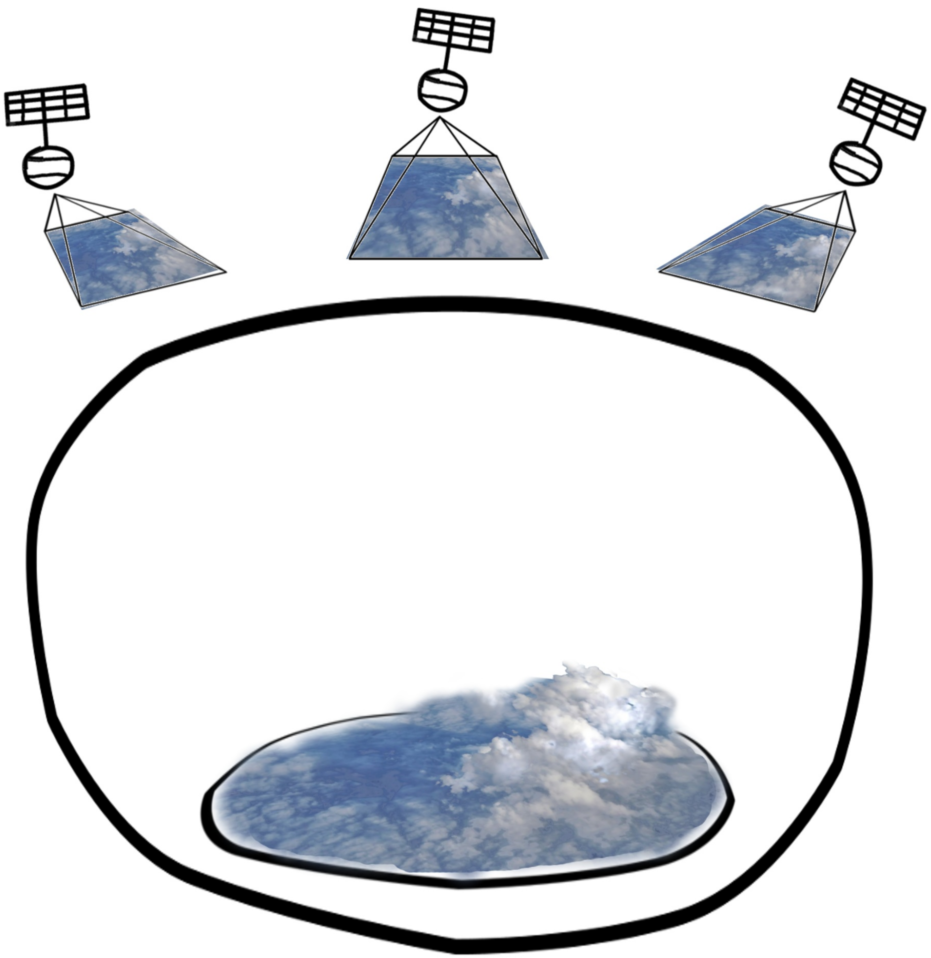

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How do we represent clouds for doing 3D retrievals of the atmosphere and surface?

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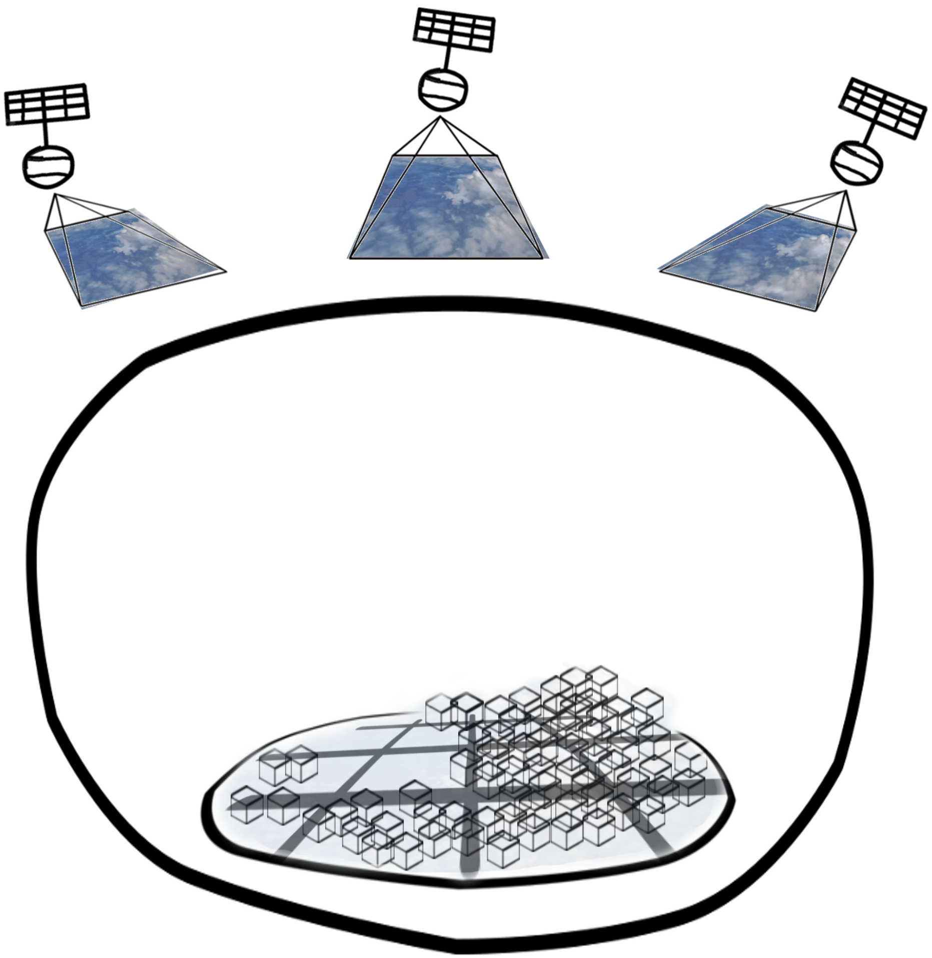

Measurements (future)

- Passive polarimetric imaging and active LIDAR and RADAR.

  ~100 images

  x

  ~10,000 pixels per image

  =

  ~1,000,000 total measurement constraints

Unknown parameters to retrieve

- *Cloud, aerosols and surface for each patch*

  ~1,000 volume and surface elements

  X

  ~10 properties (aerosol, cloud or surface)

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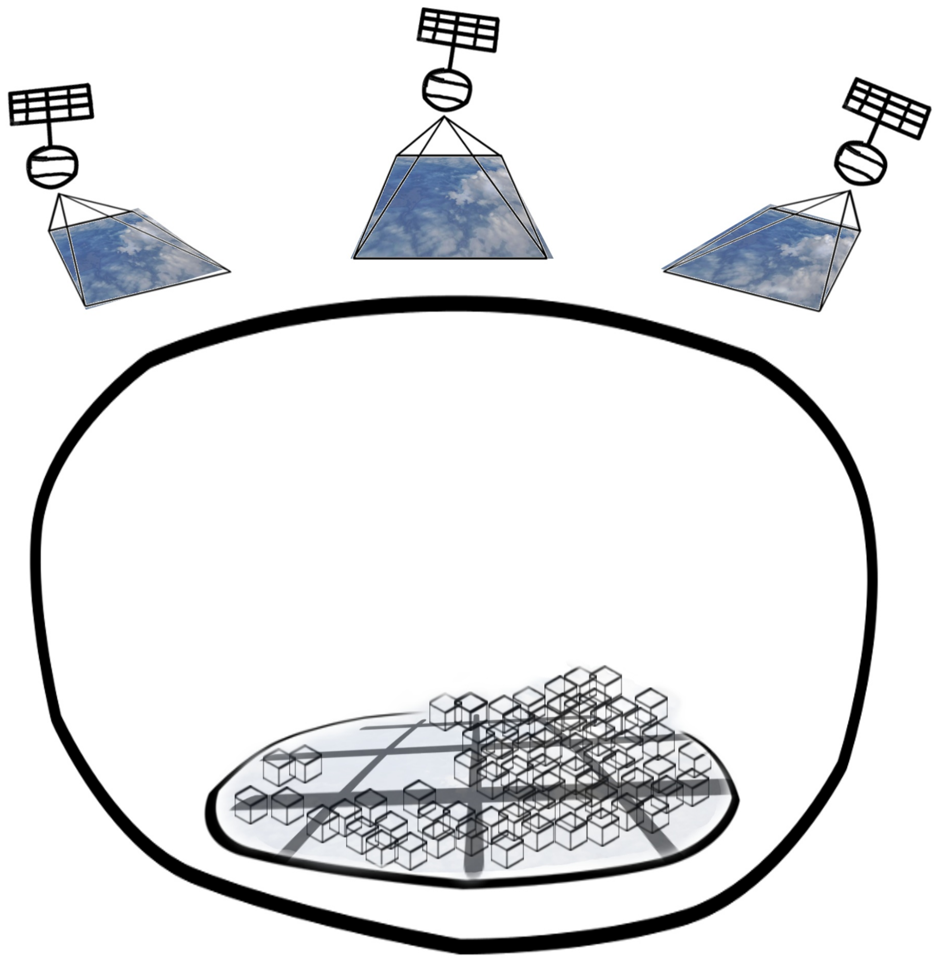

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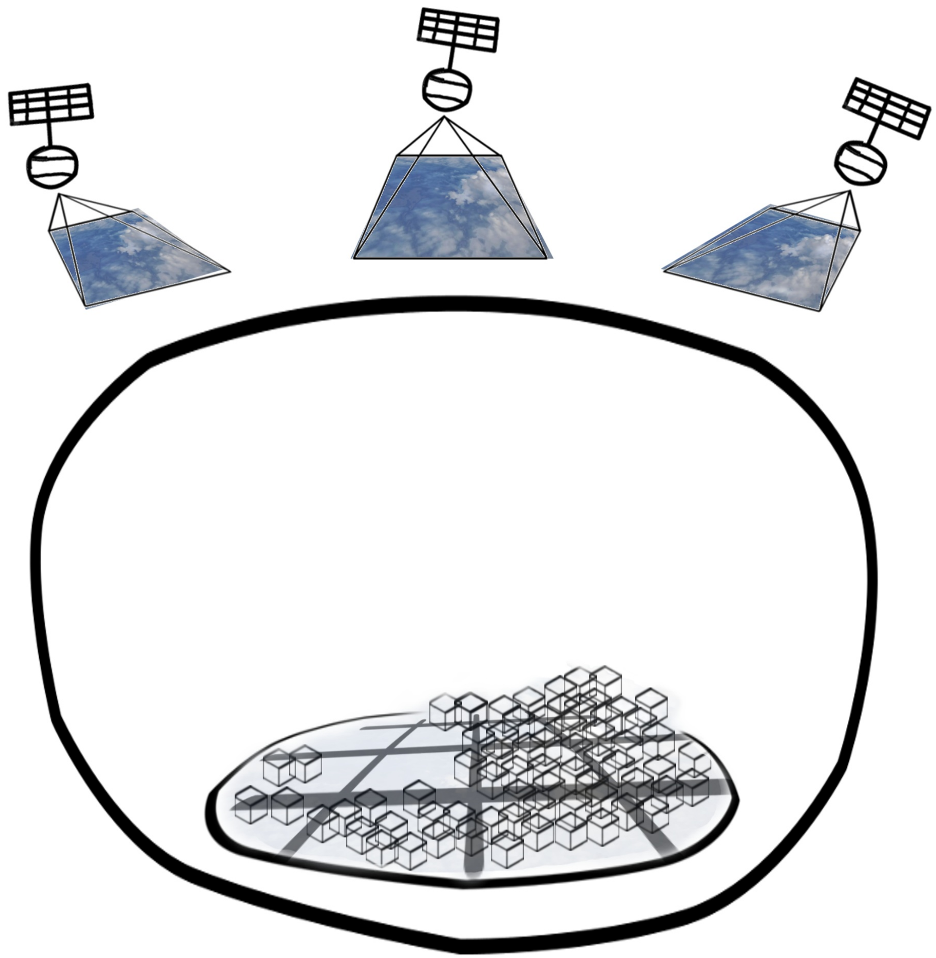
This gives a large scale inverse problem:
Adjust 10,000 unknowns to fit 1,000,000 data.

Inverse problem:
find the cloud which fits data “best”



Minimize the misfit

- Non-linear least squares problem
- Solve by iterative methods

Requires the evaluation of the

$$\Phi\left(\boldsymbol{a}\right) = \frac{1}{2}\left(\hat{\boldsymbol{y}} - \boldsymbol{y}(\boldsymbol{a})\right)^T \cdot \boldsymbol{S}_{\boldsymbol{\epsilon}}^{-1} \cdot \left(\hat{\boldsymbol{y}} - \boldsymbol{y}(\boldsymbol{a})\right)$$

And the derivative of the misfit (steepest descent)

$$-\frac{\partial \Phi(\boldsymbol{a})}{\partial a^n} = \left(\hat{\boldsymbol{y}} - \boldsymbol{y}(\boldsymbol{a})\right)^T \cdot \left(\boldsymbol{S}_{\boldsymbol{\epsilon}}^{-1}\right) \cdot \frac{\partial \boldsymbol{y}(\boldsymbol{a})}{\partial a^n}$$

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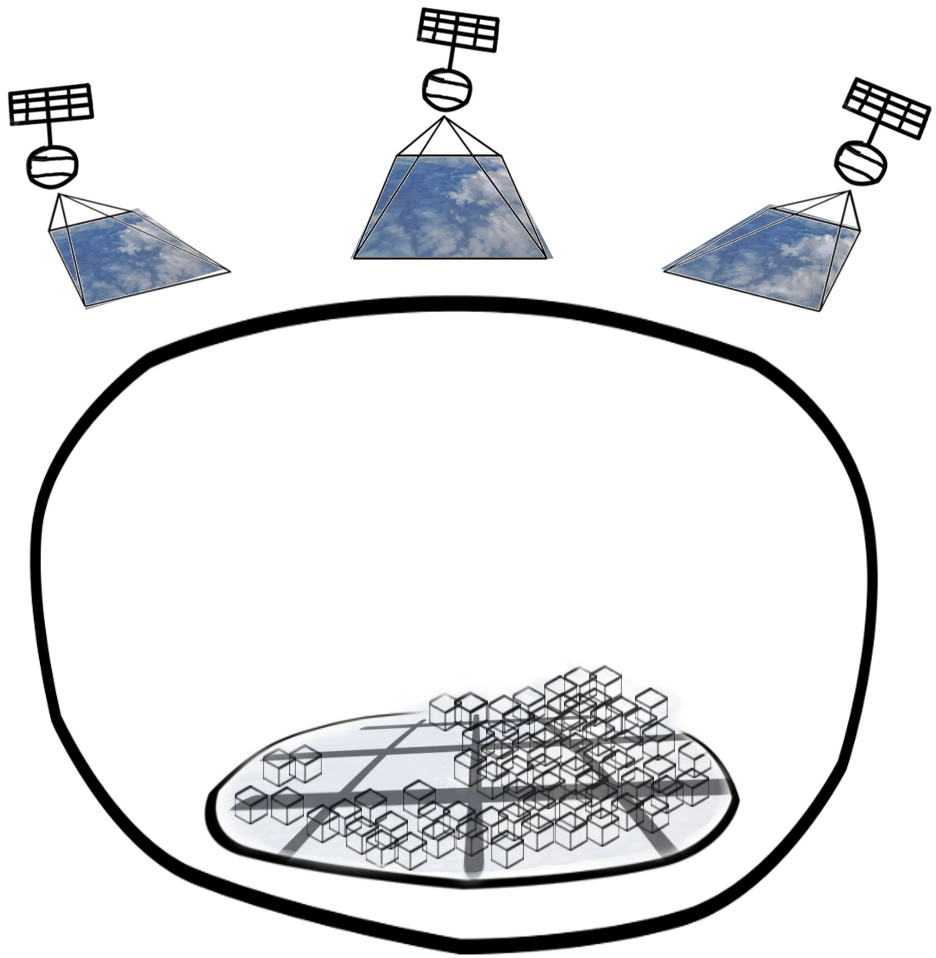

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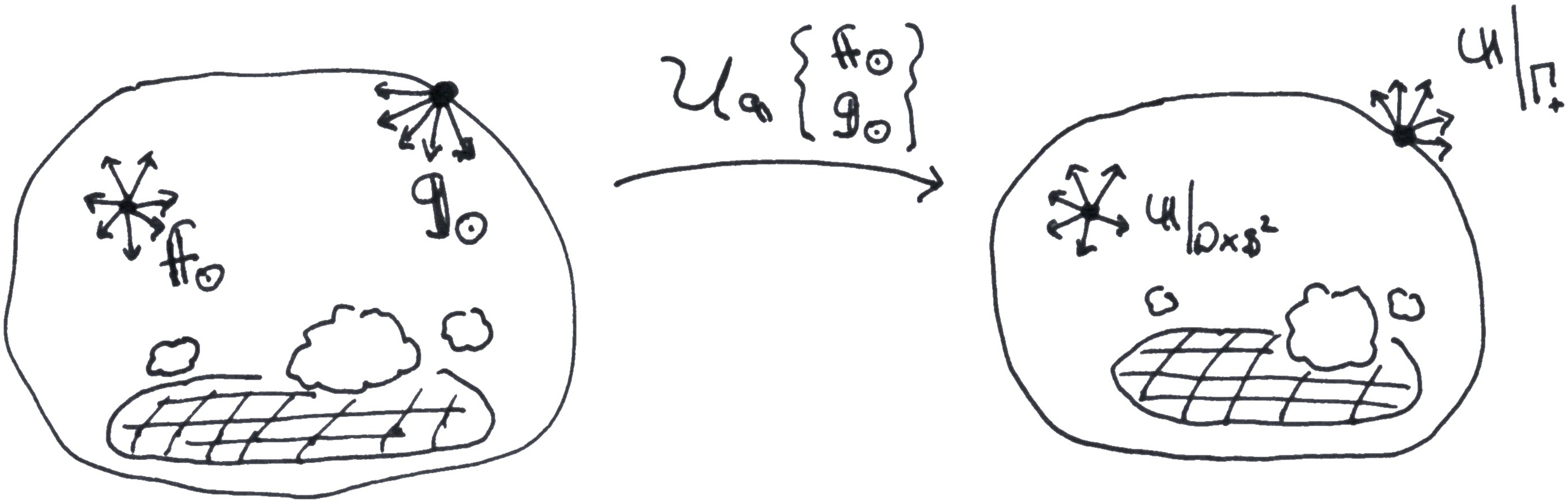
Too big to solve on your smart phone.

Adjoint method:

need a general 3D VRTE solver

For a fixed atmosphere and surface, the solver transforms volume-source and incoming-source functions into the internal and outgoing Stokes vectors.

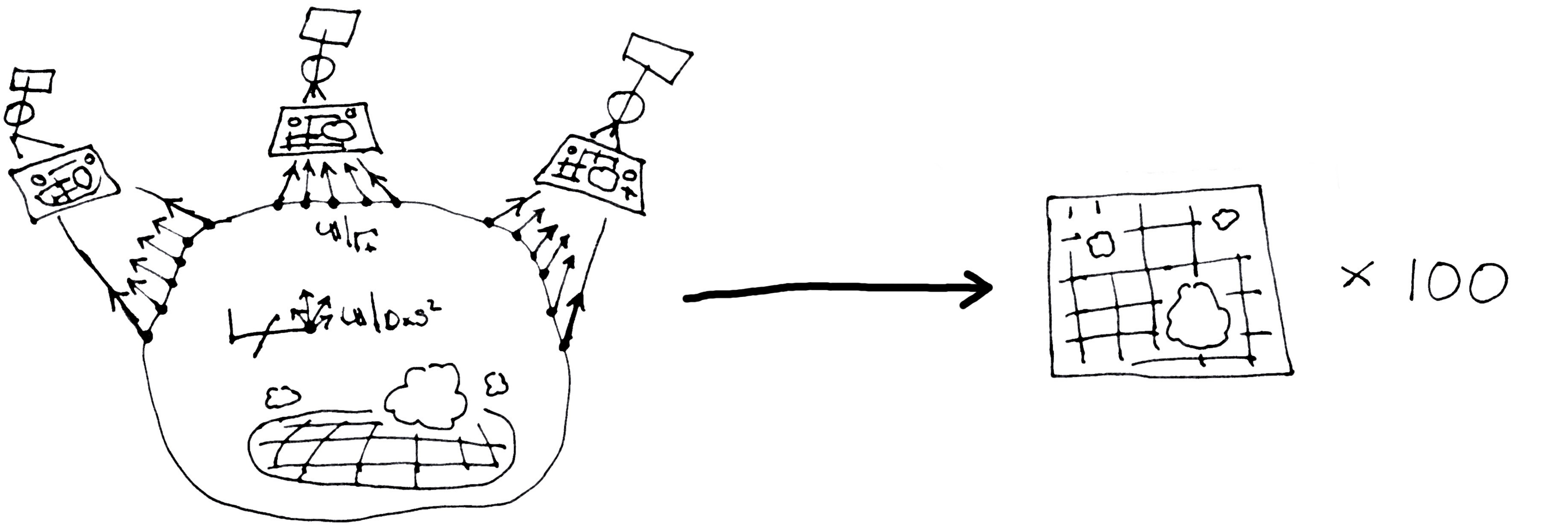


$$\mathcal{U}_{\boldsymbol{a}} \left\{ \begin{array}{c} \boldsymbol{f}_\odot \\ \boldsymbol{g}_\odot \end{array} \right\} = \left\{ \begin{array}{cc} \mathcal{T}_{00} & \mathcal{T}_{-0} \\ \mathcal{T}_{0+} & \mathcal{T}_{-+} \end{array} \right\} \sum_{k=0}^{\infty} \left\{ \begin{array}{cc} \mathcal{Z}\mathcal{T}_{00} & \mathcal{Z}\mathcal{T}_{-0} \\ \mathcal{R}\mathcal{T}_{0+} & \mathcal{R}\mathcal{T}_{-+} \end{array} \right\}^k \left\{ \begin{array}{c} \boldsymbol{f}_\odot \\ \boldsymbol{g}_\odot \end{array} \right\}$$

Adjoint method:

compute measurements

Integrate the Stokes vector solution over the polarimetric response of each pixel.

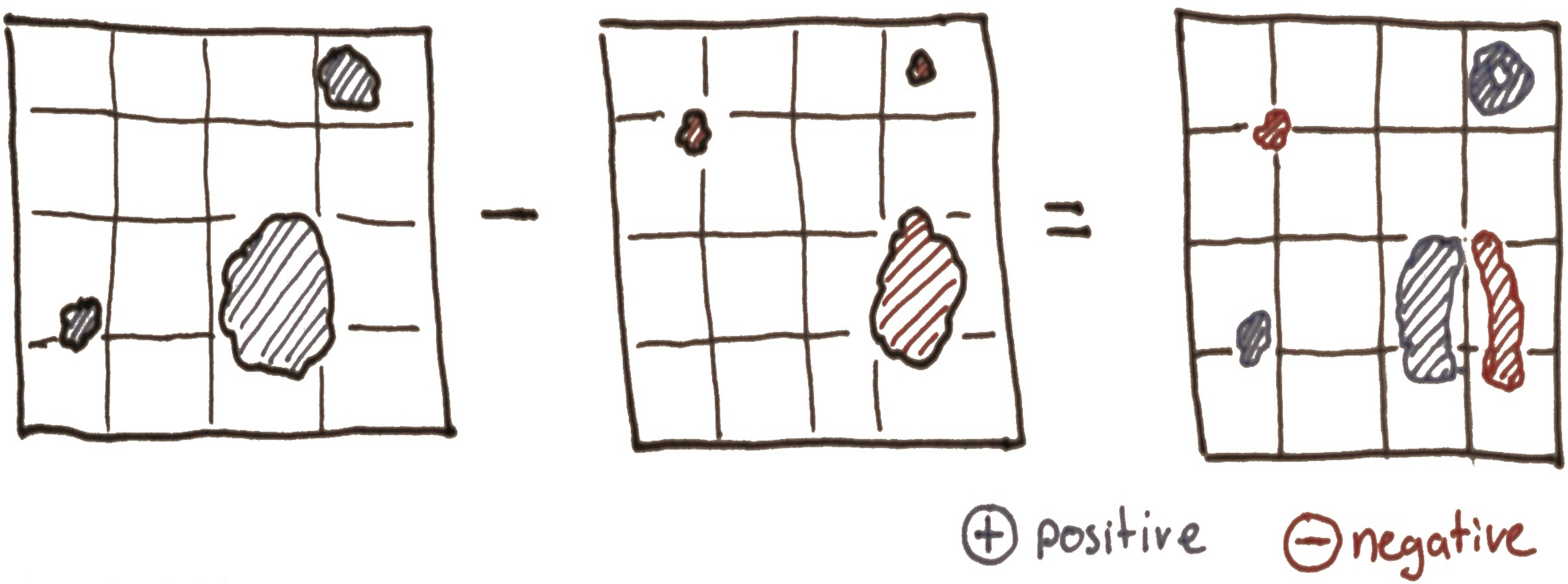


$$y^m(\boldsymbol{a}) = \left\langle \begin{Bmatrix} \boldsymbol{p}^m_\odot \\ \boldsymbol{q}^m_\odot \end{Bmatrix}, \mathcal{U}_{\boldsymbol{a}} \begin{Bmatrix} \boldsymbol{f}_\odot \\ \boldsymbol{g}_\odot \end{Bmatrix} \right\rangle_{D\times\mathbb{S}^2\oplus\Gamma_+}$$

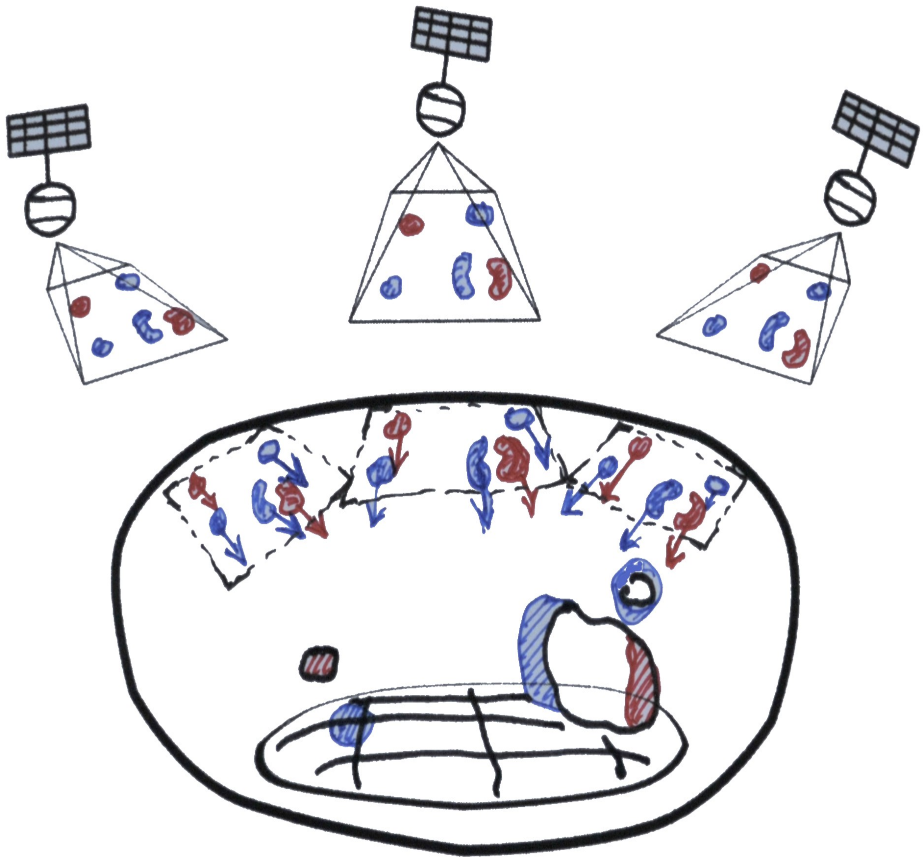
Adjoint method:

compute the measurement residual

y_{data} $y(a)$



Adjoint method:
solve the adjoint 3D VRTE



The measurement residual is on the wrong domain

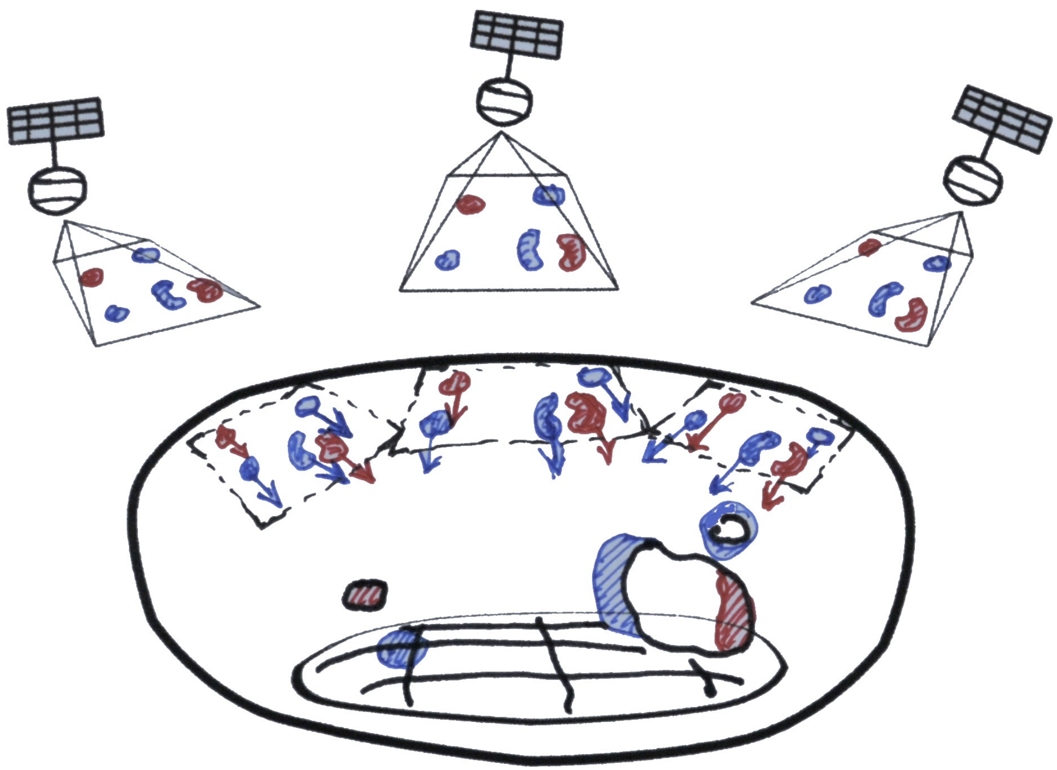
- 2D images in space

The adjoint solution is on the right domain

- 3D atmosphere and surface

$$\mathcal{U}_{\boldsymbol{a}}^{*}\left\{\begin{array}{c}\boldsymbol{p}_{\odot}\\ \boldsymbol{q}_{\odot}\end{array}\right\}=\left\{\begin{array}{cc}\mathcal{T}_{00}^{*} & \mathcal{T}_{0+}^{*}\\ \mathcal{T}_{-0}^{*} & \mathcal{T}_{-+}^{*}\end{array}\right\}\sum_{k=0}^{\infty}\left\{\begin{array}{cc}\mathcal{Z}^{*}\mathcal{T}_{00}^{*} & \mathcal{Z}^{*}\mathcal{T}_{0+}^{*}\\ \mathcal{R}^{*}\mathcal{T}_{-0}^{*} & \mathcal{R}^{*}\mathcal{T}_{-+}^{*}\end{array}\right\}^{k}\left\{\begin{array}{c}\boldsymbol{p}_{\odot}\\ \boldsymbol{q}_{\odot}\end{array}\right\}$$

Adjoint method:
compute the steepest descent of misfit



The adjoint Stokes-vector solution is on the correct 3D domain.

Compute the steepest descent of the misfit function.

- This is the right-hand-side of Newton's equations for the parameter adjustment.

$$-\frac{\partial \Phi}{\partial a^n} = \left\langle \mathcal{U}_{\boldsymbol{a}}^{*} \begin{Bmatrix} \boldsymbol{\Delta p}_{\odot} \\ \boldsymbol{\Delta q}_{\odot} \end{Bmatrix}, \begin{Bmatrix} \boldsymbol{\Delta f}_{\odot}^{n} \\ \boldsymbol{\Delta g}_{\odot}^{n} \end{Bmatrix} \right\rangle_{D \times \mathbb{S}^2 \oplus \Gamma_-}$$

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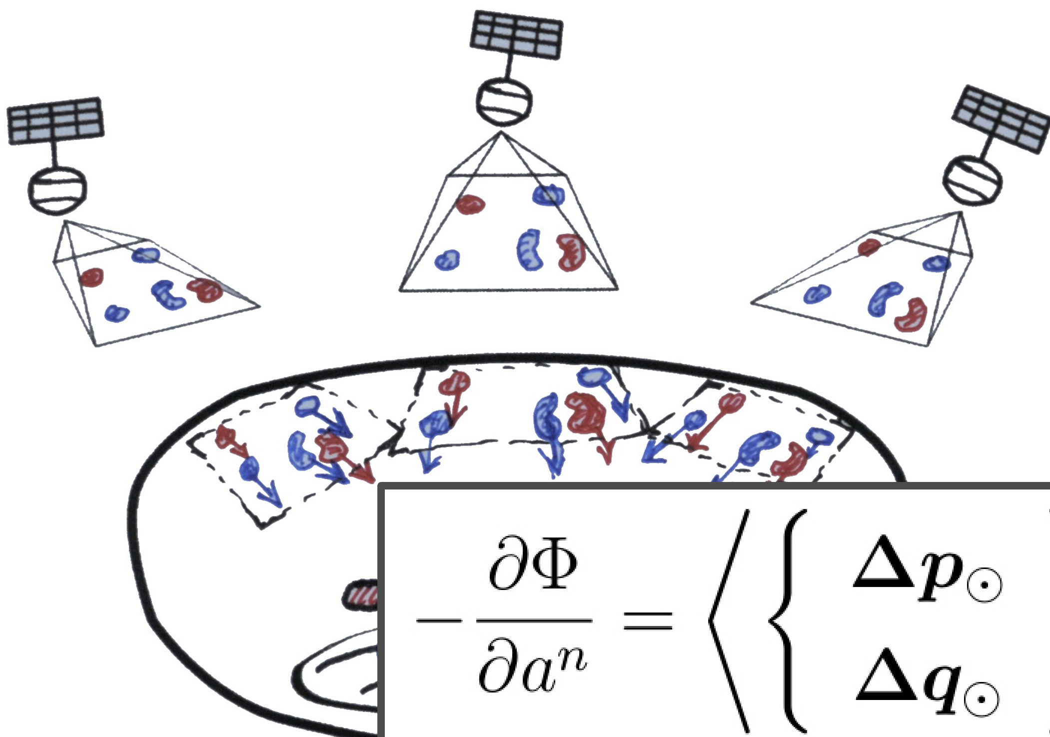

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How do we tell the computer?

computers know linear algebra.

Adjust parameters with a step, b, which solves approximate Newton's equations:

$$\left(\nabla\nabla\Phi(\boldsymbol{a}) + \nabla\nabla\Phi_{\text{prior}}(\boldsymbol{a})\right) \cdot \boldsymbol{b} = -\left(\nabla\Phi(\boldsymbol{a}) + \nabla\Phi_{\text{prior}}(\boldsymbol{a})\right)$$

Approximate the second derivative with the gradient (Broyden-Fletcher-Goldfarb-Shanno):

$$\nabla\nabla\Phi(\boldsymbol{a}_k) \approx \boldsymbol{H}_k\left(\boldsymbol{a}_0, \cdots, \boldsymbol{a}_k, \nabla\Phi(\boldsymbol{a}_0), \cdots, \nabla\Phi(\boldsymbol{a}_k)\right)$$

How do we tell the computer?

computers know linear algebra.

Use the derivative:

grad(Φ),

To setup the linear system:

Ax=b.

Adjoint method:

scalable adjustments to 3D properties

Iterative minimization of the misfit function with only two calls to the 3D VRTE (per wavelength):

- Solve the 3D VRTE once to compute the residual
- Solve the adjoint 3D VRTE once calculate the derivative
- Solve a system of linear equations for the parameter adjustment

Procedure scales to very large problems with . . .

- Many measurement constraints
- Many unknown cloud, aerosol and surface properties

Adjoint method makes 3D retrievals with the 3D VRTE worth discussing

- Future project 1: Test derivative calculations and performance
- Future project 2: Synthetic retrievals and inverse problem analysis

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Probably still can't do 3D cloud retrievals on your smart phone.

Adjusting three-dimensional
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Thank you!!!

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