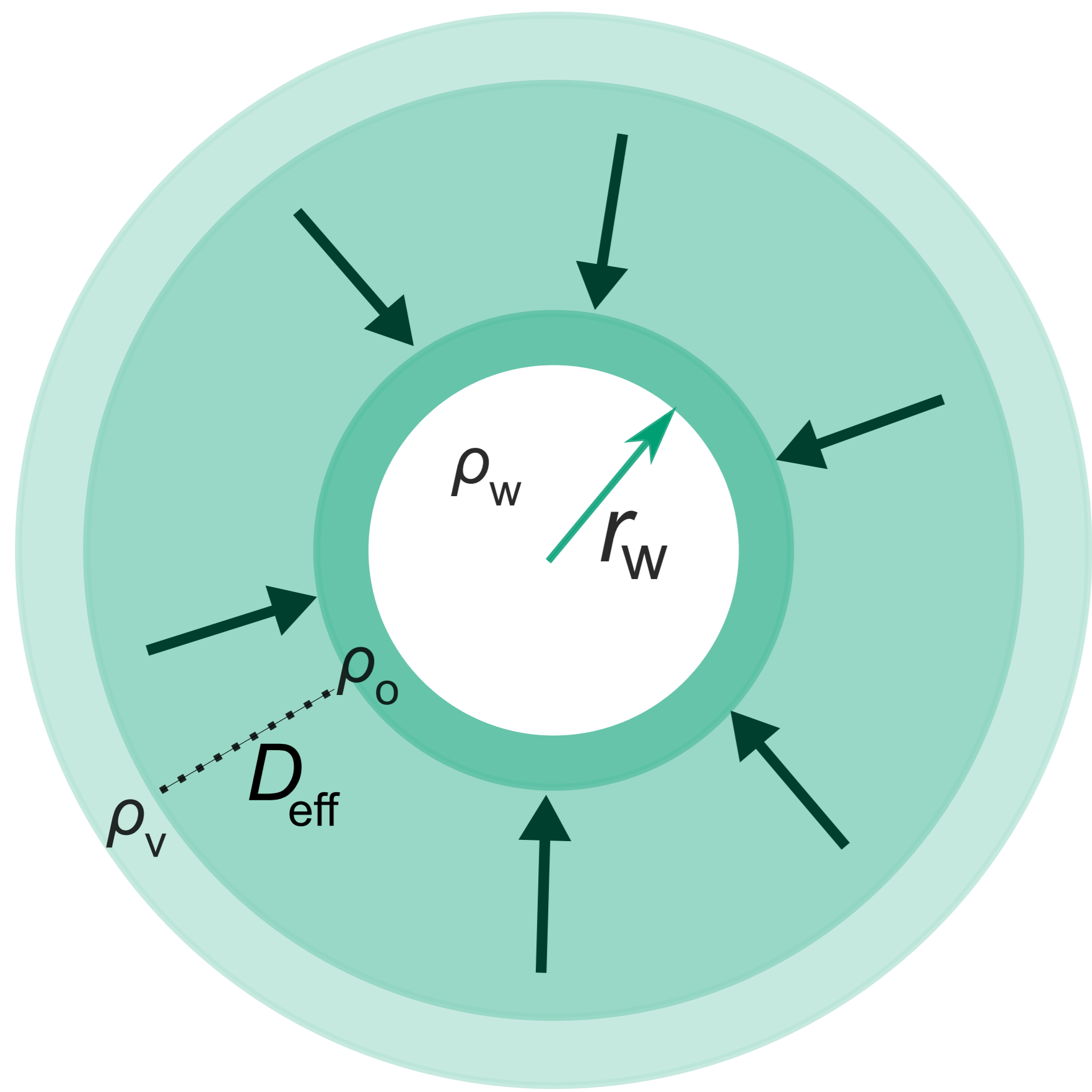


# On the CCN (de)activation nonlinearities

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droplet growth laws in a nutshell: mass and heat diffusion

phase portrait of the system: flipped Köhler curve



Fick's and Fourier's laws combined  
 spherical geometry

$$\dot{r}_w = \frac{1}{r_w} \frac{D_{\text{eff}}}{\rho_w} (\rho_v - \rho_o)$$

non-dimensional numbers:

$$RH = \rho_v / \rho_{vs}$$

$$RH_{\text{eq}} = \rho_o / \rho_{vs}$$

$$\dot{r}_w = \frac{1}{r_w} D_{\text{eff}} \frac{\rho_{vs}}{\rho_w} (RH - RH_{\text{eq}})$$

kappa-Köhler:

$$RH_{\text{eq}} \approx 1 + \frac{A}{r_w} - \frac{\kappa r_d^3}{r_w^3}$$

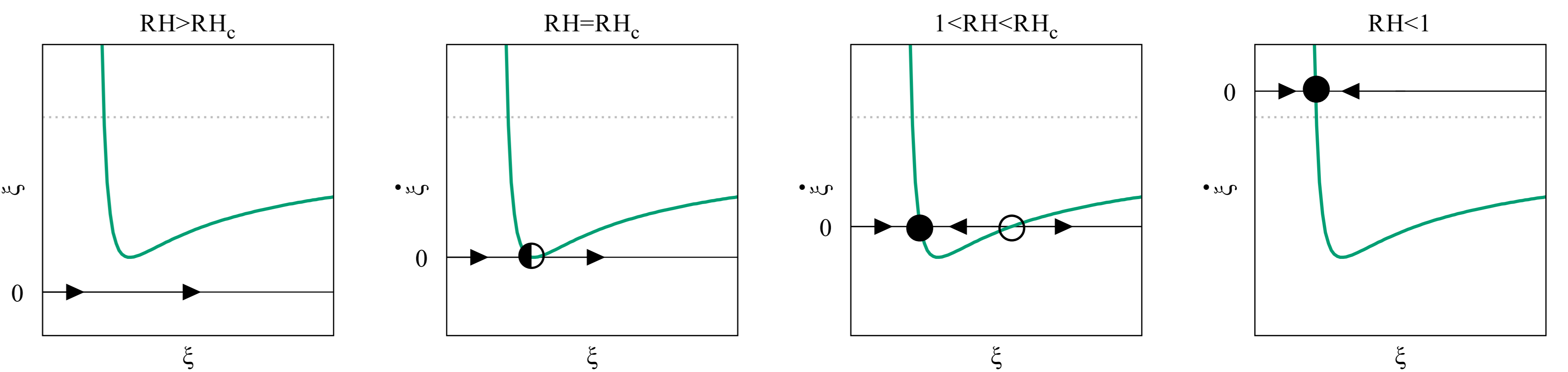
maximum at  $(r_c, RH_c)$ :

$$r_c = \sqrt{3\kappa r_d^3 / A}$$

$$RH_c = 1 + \frac{2A}{3r_c}$$

$$\xi = r_w^2 + C$$

$$\dot{\xi} = 2D_{\text{eff}} \frac{\rho_{vs}}{\rho_w} (RH - RH_{\text{eq}}(\xi))$$



saddle-node bifurcation at Köhler curve maximum

$$RH_{\text{eq}}(\xi_c) = c_0 + c_1 \xi_c + c_2 \xi_c^2 + \dots \quad \dot{\xi}_c \Big|_{\xi_c \rightarrow 0} \sim \frac{RH - RH_c}{A/(4r_c^5)} + \xi_c^2 \quad \dot{x} = r + x^2$$

bifurcations (and catastrophe) in the RH-coupled system

simple moisture budget (const T, p):

$$RH \approx \frac{\dot{\rho}_v}{\rho_{vs}} = -N \frac{4\pi \rho_w}{3\rho_{vs}} 3r_w^2 \dot{r}_w$$

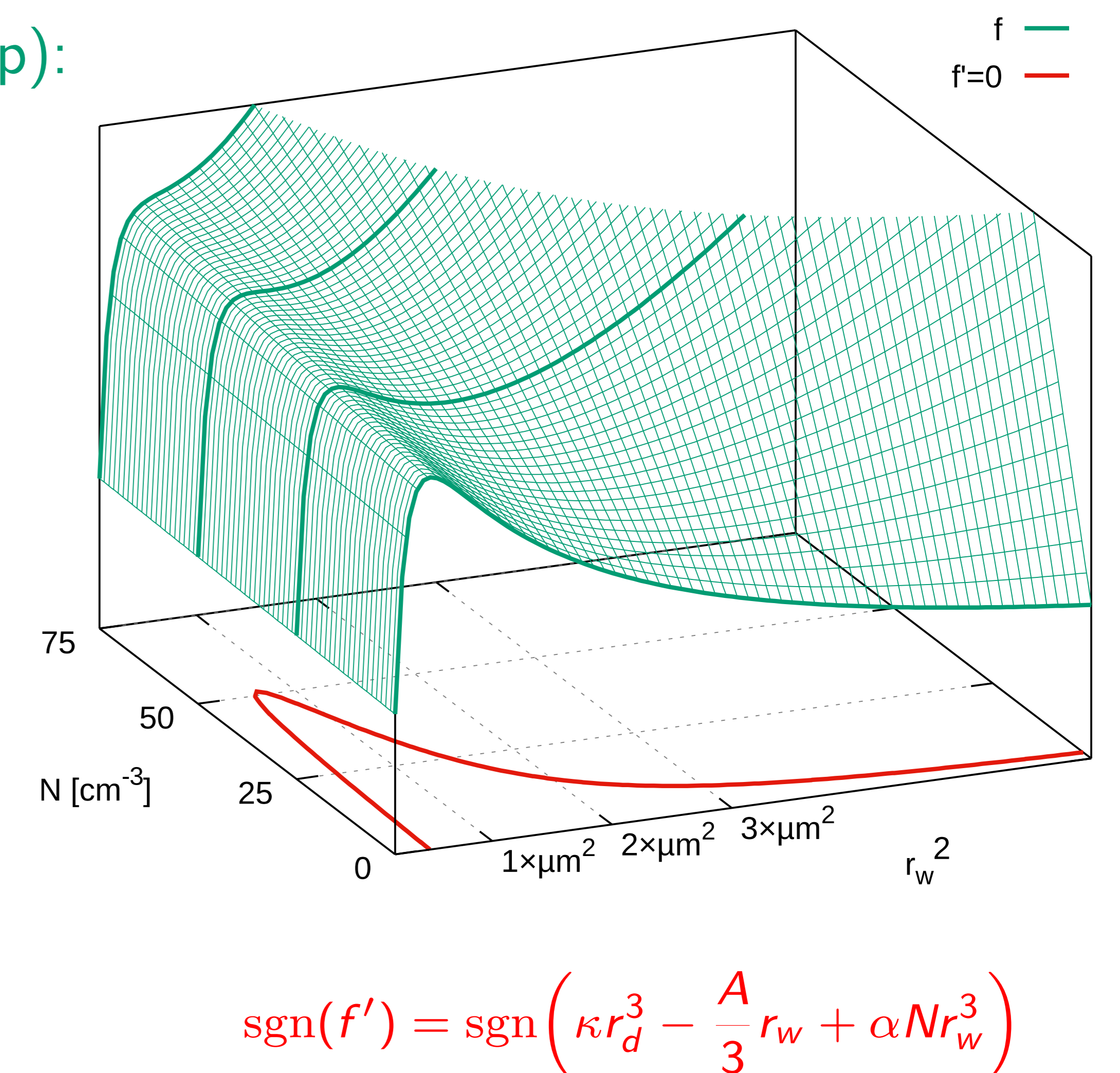
integrating in time:

$$RH = RH_0 - \alpha N r_w^3$$

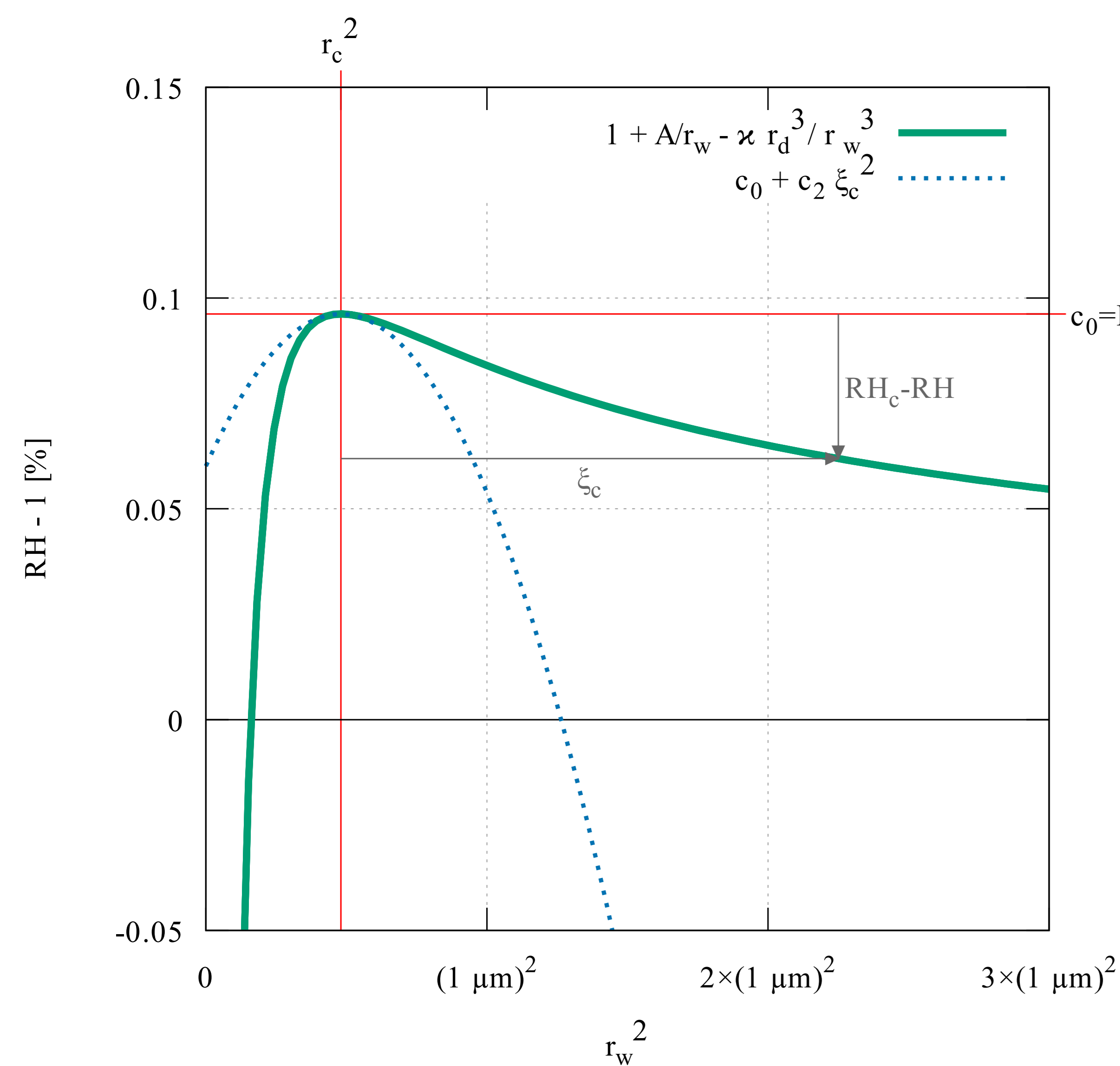
new phase portrait:

$$\dot{\xi} \sim (RH_0 - 1) - \underbrace{\left( \frac{A}{\xi^2} - \frac{\kappa r_d^3}{\xi^3} + \alpha N \xi^3 \right)}_f$$

regime-controlling params: RH, N



$$\text{sgn}(f') = \text{sgn}\left(\kappa r_d^3 - \frac{A}{3} r_w + \alpha N r_w^3\right)$$



- analytical estimation of the CCN activation timescale
- relevance to particle-based microphysics (super droplets)
- corroboration with numerical integration (cusp catastrophe related hysteresis)

seminar next week at Laramie  
 UNIVERSITY OF WYOMING  
 Dept. of Atmospheric Science  
 Tue., July 17, 3:10 pm, EN6085



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