On the Order of Atmospheric Scattering, Its Polarization and Computation Efficiency

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Contents

1. Background
2. Polarized Radiative Transfer
3. N-Appr method
4. N-Plus method
5. Summary
1. Properties of Atmosphere scattering:

2. More polarized instrument
   (PARASOL, 3MI, SGLI, APS, SPEX, MSPI...)

3. Errors due to neglecting polarization

4. Hyperspectral Measurements for Gases such as OCO2, TanSAT are much polarized sensitive
Errors due to neglecting polarization in O2-A Band

$\theta_0=40$, Albedo=0.3, No Aerosol

Table 2
Retrieval and smoothing errors and errors from neglecting polarization for January and July scenes in Park Falls, WI, USA

<table>
<thead>
<tr>
<th>Scenario</th>
<th>Retrieval error (ppm)</th>
<th>Polarization error (ppm)</th>
<th>Smoothing error (ppm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>January</td>
<td>1.8</td>
<td>7.4</td>
<td>1.0</td>
</tr>
<tr>
<td>July</td>
<td>0.4</td>
<td>0.4</td>
<td>0.1</td>
</tr>
</tbody>
</table>

Vijay Natraj, JQSRT, 103, 245
XCO2 retrieval errors due to neglecting polarization

Radiative Transfer Equation

\[ \mu \frac{dI(\tau, \Omega)}{d\tau} = I(\tau, \Omega) - J_1(\tau, \Omega) - J_m(\tau, \Omega) \]

\[ J_m(\tau, \Omega) = \frac{\omega}{4\pi} \int_0^{4\pi} I(\tau, \Omega') M(\tau, \Omega; \Omega') d\Omega' \]

\[ J_1(\tau, \Omega) = \frac{\omega}{4\pi} \pi F_0 \exp(-\tau / \mu_0) P(\tau, \Omega; -\Omega_0) \]

\[ I = [I, Q, U, V]^T \]
Coordinate system

\[
I = \begin{pmatrix}
I \\
Q \\
U \\
V
\end{pmatrix} \quad \tilde{P} = \begin{pmatrix}
a_1 & b_1 & 0 & 0 \\
b_1 & a_2 & 0 & 0 \\
0 & 0 & a_3 & b_2 \\
0 & 0 & -b_2 & a_4
\end{pmatrix}
\]

\[a_i, \ b_i \ \text{depend on scattering angle} \ \Theta\]

\[\tilde{M} = \tilde{L}(\pi - \i_2)\tilde{P}(\Theta)\tilde{L}(-\i_1)\]

\[O\]

\[\phi\]

\[(\mu, \phi)\]

\[Q\]

\[(\mu', \phi')\]

\[z\]

\[x\]

\[y\]
Numerical Algorithms

1. **MC**: (Kattawar & Plass, 1968; Roberti & Kummerow, 1999; Wu & Lu, 1989)
2. **Adding-doubling method** (De Haan et al., 1987; Evans & Stephens, 1991; Hansen, 1971)
3. **Inverse of matrix method** (Schulz et al., 1999; Siewert, 2000)
4. **Successive order of Scattering** (Duan, 2004; 6s; Pstar)
5. **SCIATRAN** (2009?)
N-Appr method

It is supposed that just the first a few orders are needed

How & What?

Two order scattering approximation

posium


Sergey Korkin et. al., *JQSRT* 113, 2012
N-Approximation method

\[I = I_1 + I_2 + I_3 + \ldots + I_\infty\]

\[I^* = \sum_{i=1}^{n} I_i + \sum_{i=n+1}^{\infty} [I_i, 0, 0, 0]^T\]

SOSVRT: A polarized radiative transfer model based on Successive order of Scattering
CASE A: pure molecular atmosphere with different optical depth

CASE B: molecular with aerosol, where the aerosol is supposed to be exponentially decreased with height, the scale height is set to be 1.8km for aerosol, and 7.8km for molecular
\[
\frac{(I - I_{\text{true}})}{I_{\text{true}}} \times 100\% \quad \tau=0.2
\]

\[
\frac{(I - I_{\text{true}})}{I_{\text{true}}} \times 100\% \quad \tau=0.5
\]
\[ \Delta \text{DoLP} \quad \tau = 0.2 \]

\[ \Delta \text{DoLP} \quad \tau = 0.5 \]
N-APPR  No Aerosol
\( \tau_{aerosol} = 0.2 \)

\( \tau_{aerosol} = 0.5 \)

\( \tau_{aerosol} = 1.0 \)

\( \tau_{aerosol} = 2.0 \)

\( \frac{(I - I_{true})}{I_{true}} \times 100\% \quad \tau_{mol} = 0.156 \)
$\tau_{aerosol} = 0.2$

$\tau_{aerosol} = 0.5$

$\tau_{aerosol} = 1.0$

$\tau_{aerosol} = 2.0$

$\Delta DoLP \quad \tau_{mol} = 0.156$
N-APPR with Aerosol

![Graph 1: Error maximax (\%) vs. scattering order](image1)

![Graph 2: Error maximax (\%) vs. scattering order](image2)

![Graph 3: Δ DoLP (%) vs. scattering order](image3)

![Graph 4: Δ DoLP (%) vs. scattering order](image4)
N-PLUS method

\[ I = I_1 + I_2 + I_3 \cdots + I_\infty \]

\[ I^* = \sum_{i=1}^{n} I_i + \sum_{i=n+1}^{\infty} [I_i, 0, 0, 0]^T \]

\[ \mu \frac{dI}{d\tau} = -I + \frac{\omega}{4\pi} \mathbf{M}(\mu, \phi; \mu_0, \phi_0) \mathbf{F}_0 \ e^{-\tau/\mu_0} \]

\[ + \frac{\omega}{4\pi} \int_0^{2\pi} \int_{-1}^{1} \mathbf{M}(\mu, \phi; \mu', \phi') I^*(\mu', \phi') d\mu' d\phi' \]
N-PLUS No Aerosol
N-PLUS with Aerosol
N-PLUS method, a correction of the N-appr method, is a little bit more accurate, and efficient.

The number of scattering must be carefully considered, especially for polarization sensitive instruments such as OCO2, TanSAT, where two scattering is not accurate enough.
谢谢！THANK YOU FOR YOUR TIME.