Dynamics of the Optimal State for Cold Pools in Shear

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1. Background:
- This study revisits the "optimal state" for gravity currents in sheared environments, which was first hypothesized by Rotunno et al. (1988) (hereafter RKW).
- Conceptual model (Fig. 1): a sheared environmental flow is turned into a vertically oriented jet at the leading edge of a cold pool.

2. Control Volume Constraints
- We perform Control Volume (CV) analyses on the flow in Fig. 1.
- Assumptions: steady, viscous flow; free slip lower boundary.
- See Bryan and Rotunno (2013) for governing equations, further assumptions, and additional details.

3. Interior Solution for Steady Inviscid Flow
- Assuming steady, inviscid, and adiabatic flow, we obtain a unique solution for the outflow profile \( v(x,0) \) (top-center of Fig. 1).
- Then we determine the shape of the free boundary between Regions I and III, and thus the entire interior flow (Fig. 2).

4. Time-Dependent Numerical Simulations
- Direct Numerical Simulations; initialized with CV constraints (1) and (2).
- Fig. 5: Comparison of the theoretical interface for steady, inviscid, isotropic flow (red curve from Section 3) with time-averaged potential temperature from the numerical simulation (gray contours).

5. Comparison of Simulation and Theory
- A vertically oriented jet emerges in the simulation and remains statistically steady (Fig. 3); transient eddies develop on the left side.
- All the important elements of the theoretical flow (Fig. 1) are seen in the numerical solution (Fig. 3), e.g.: the front-relative flow remains negligible behind the surface gust front, the "flux at right" term dominates the net generation term (blue), and the "flux at left" term (red) is negligible. Other terms oscillate about zero.

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References:

Fig. 1: Idealized schematic of the optimal state for a gravity current in a sheared environment.

Fig. 2: Numerical solution for the optimal state assuming steady, inviscid, adiabatic flow. Gray shading denotes the cold pool, black contours show streamfunction \( \psi \) (contour interval 0.05), and flow vectors are shown every fourth gridpoint. The red-dashed line indicates a 60° angle from the horizontal (Rotman et al. 1985; Xu 1992).

Fig. 3: Instantaneous snapshots from a numerical simulation at \( t = (a) 0 \), (b) 60, (c) 120, and (d) 180 min. Gray flow vectors are shown every 16th grid point (vectors with a magnitude less than 0.05 are excluded). The red contour shows non-dimensional buoyancy \( b = 0.5 \). Color shading is vorticity.