Tangential Velocity and Axial Vorticity Profiles in Vortical Flows

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Introduction

In atmospheric research, explicit assumptions are often made about the shapes of velocity or vorticity profiles of axisymmetric vortical flows. Some examples of such "vortices" are the potential vortex, the Rankine-combined vortex, the Burgers–Rott vortex, the Sullivan vortex, the Kuo vortex, the Serrin vortex, the Lamb–Oseen vortex, etc. In many such models, the **tangential velocity** away from the vortex axis (beyond the RMW) decays like r^{-1}, where r is the distance from the vortex axis, while the **axial vorticity** is zero or approaching it fast as r increases beyond the RMW.

We revisit some experimental and theoretical results at different scales and focus on the velocity/vorticity decay in the "intensification region," which, in many cases, exhibits a power law decay with $u_\theta \propto r^{-\beta}$ and/or $\omega_z \propto r^{-(\beta+1)}$ with $0 < \beta < 1$.

This phenomenon seems to be prevalent across many scales, including a bathtub vortex, the vortex chamber, tornadoes/mesocyclones, and tropical cyclones.

The Bathtub Vortex [4]

An analysis due to Klimenko [4] results in a power law for the behavior of the axial vorticity, $\omega_z \propto r^{-\alpha}$ with $\alpha = 4/3$ in the intensification region, implying an $r^{-1/3}$ component in the tangential velocity. It is argued that for "realistic vortical flows" $4/3 \le \alpha \le 3/2$.

Figure 1: A bathtub vortex. It is argued that $\omega_z \propto r^{-4/3}$ [4].

The Tornado Vortex Chamber (TVC) [7]

An experiment of Lund and Snow (1993) with the Purdue University TVC II and the laser Doppler velocimeter provides tangential velocity measurements and analysis at the 15 cm height, halfway up the convergence zone. A "best fit" of the data is provided resulting in the power law $u_\theta \propto r^{-0.63}$, and consequently $\omega_z \propto r^{-1.63}$.

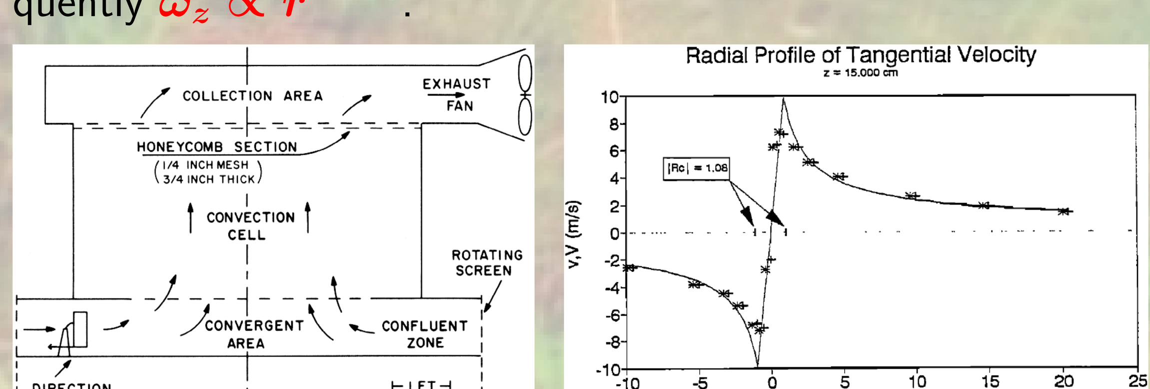

Figure 2: The PU TVC II and the observed velocity profile: $u_\theta \propto r^{-0.63}$.

Tropical Cyclones Data Analysis [8]

Mallen et al. [8] have analyzed wind speeds in 251 tropical cyclones classified as prehurricanes, minimal, and major hurricanes and came up with the values below for the decay of $u_\theta \propto r^{-\alpha}$:

Category	Mean α	Std dev α	Range α
Prehurricane	0.31	0.12	0.04–0.63
Minimal hurricane	0.35	0.14	0.05–0.64
Major hurricane	0.48	0.11	0.18–0.67
All TCs	0.37	0.14	0.04–0.67

Some results for major hurricanes are shown below. On the left, the data from the individual 72 major hurricanes analyzed together with their composite average ($\alpha = 0.48$); on the right, comparison of the average profile with some idealized vortices:

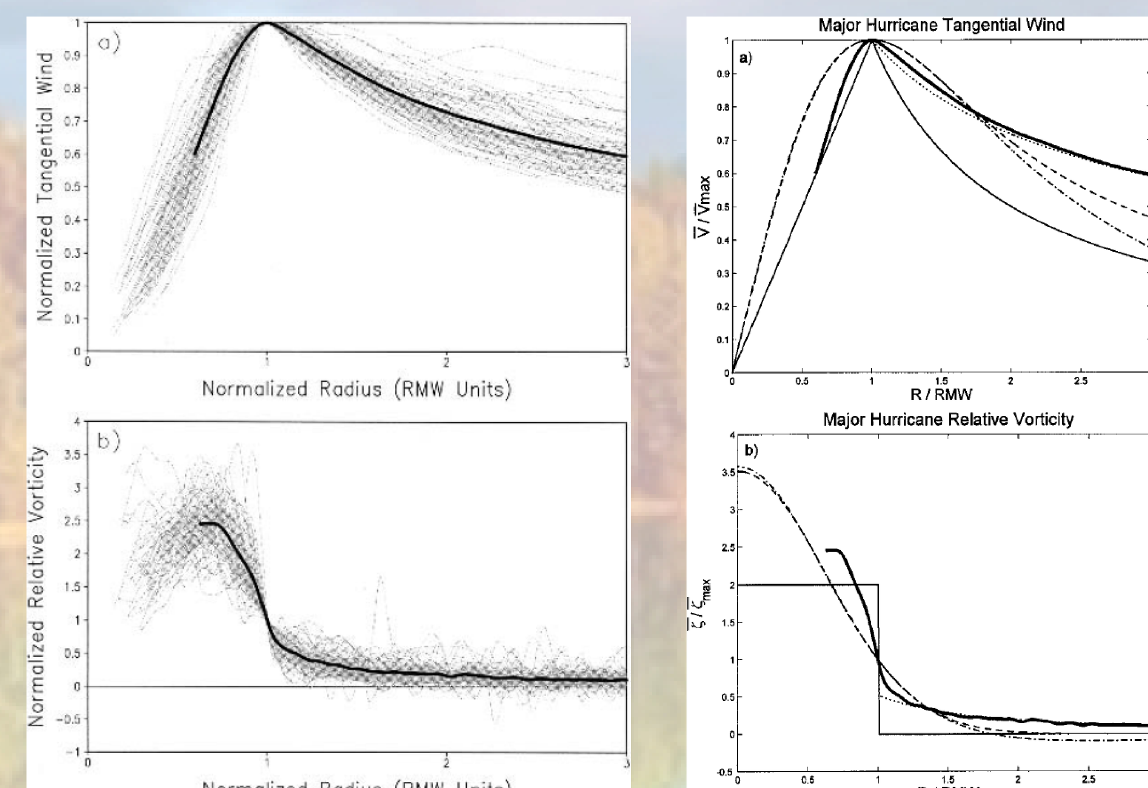

Figure 3: Composite analysis of 72 major hurricanes and comparison with some idealized vortices; here $\alpha = 0.48$ [8].

Tornado Velocity Studies [5, 6, 10, 11]

Recent radar studies show that typical velocity profiles in tornadoes exhibit similar power laws. Using Doppler radar data, Wurman et al. have demonstrated $u_\theta \propto r^{-0.67}$ for the 1998 Spencer, SD tornado (left, [10]), and $u_\theta \propto r^{-0.6}$ for the 1995 Dimmit, TX, tornado (right, [11]):

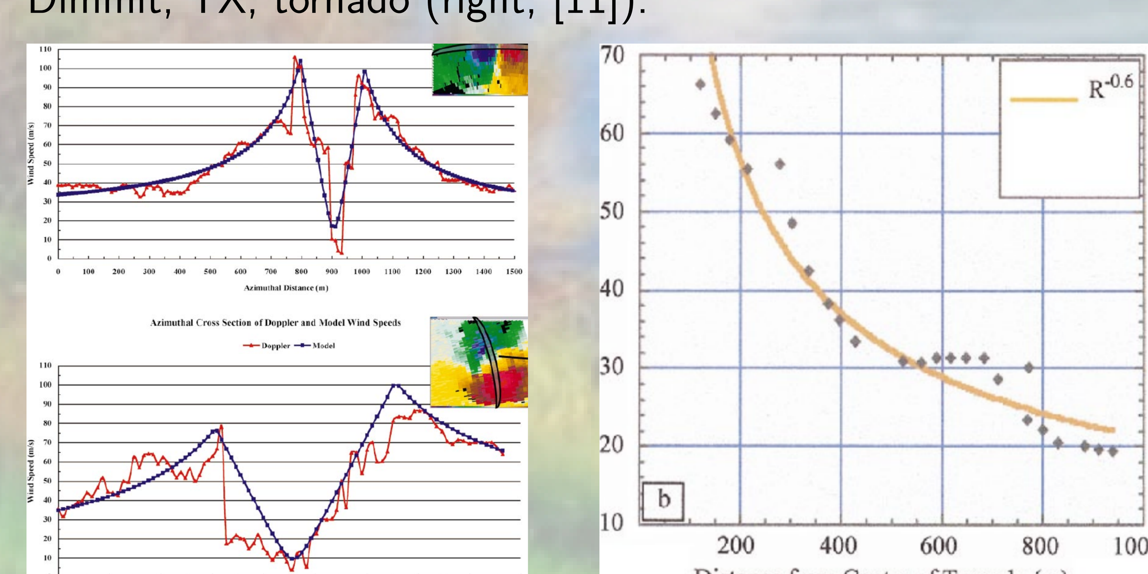

Figure 4: Doppler velocity decay for the 1998 Spencer, SD, and the 1995 Dimmit, TX tornadoes [10, 11].

More recent studies [5, 6] found variability in the decay coefficients over time. In one case increasing magnitude of the coefficient could be correlated to the intensity of the tornado; in the other it was speculated this variability could be due to the roughness of the surface along the path of the tornado.

Mesocyclone/Tornado Vorticity Study [3]

Vorticity and pseudovorticity have been analyzed by Cai [3] for five nontornadic and tornadic storms (Superior, NE; San Angelo, TX; Hays & Garden City, KS; Kellerville, TX). Pseudovorticity has been found to obey $\omega_z \propto r^{-\alpha}$, with $\alpha = 0.42$, 0.81, 1.02, 1.50, 1.60, respectively, indicating a correlation between the magnitude of the exponent and the intensity of the mesocyclone. Also, a "tendency for the vorticity lines to become steeper as the mesocyclone becomes stronger" is demonstrated for Garden City (tornadic) and Hays (nontornadic).

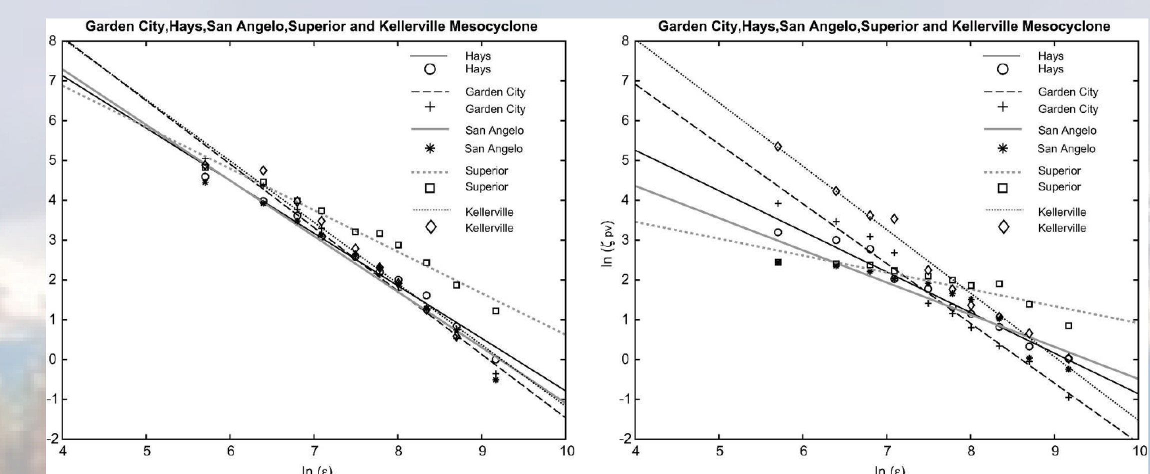

Figure 5: Vorticity (left) and pseudovorticity (right) rates of decay. Pseudovorticity seems to differentiate better between various storm strengths [3].

Some Theoretical Work

Tropical cyclones, Mallen et al. [8]

In **Mallen et al.** [8], references are provided in which it is argued that the velocity decay for tropical cyclones should satisfy $u_\theta \propto r^{-1/2}$. This is based on the argument that in a steady-state hurricane in **gradient wind balance**, in which the boundary layer friction drag satisfies $F \propto u_\theta^2$, the frictional torque's radial gradient, $\dfrac{\partial(rF)}{\partial r}$, vanishes. This immediately yields the power law above.

Buldakov, Egorov, Sychev [2]

Buldakov et al. [2] seek steady-state similarity **solutions in the viscous vortex core** which match external inviscid similarity solutions with a power-law decay $u_\theta \propto r^{1/n-2}$. The form of the sought solutions is

$$u_r = \frac{K^n U(\eta)}{\nu^{n-1} z^n} \qquad u_\theta = \frac{K^{2n} V(\eta)}{(\nu z)^{2n-1}} \qquad u_z = \frac{K^{2n} W(\eta)}{(\nu z)^{2n-1}}$$

where $\eta = (K^n r)/(\nu z)^n$ is a nondimensional parameter. They numerically discover no solutions for $n > 0.618$, two solutions for $0.5 < n < 0.618$, and one for $n = 0.618$. (The threshold value corresponds to $u_\theta \propto r^{-0.382}$.)

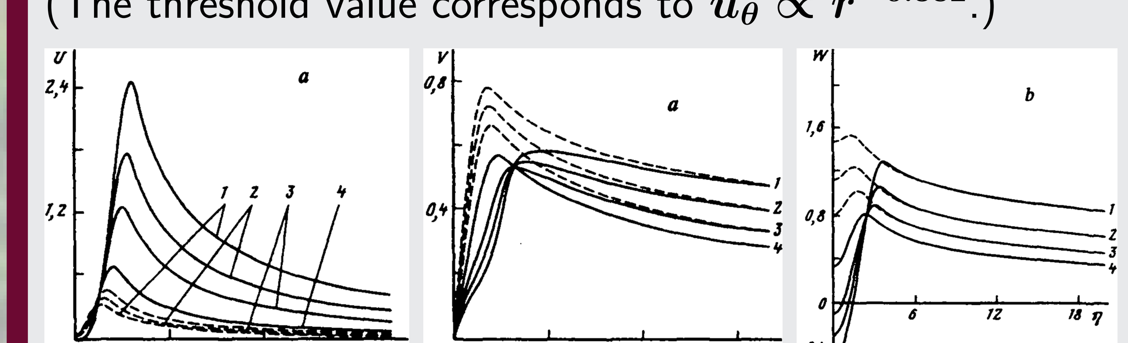

Figure 6: The numerically found solutions for $U(\eta)$, $V(\eta)$, $W(\eta)$ [2].

Klimenko [4]

Klimenko [4] investigates **"intensive" vortices**, in which converging radial flow intensifies the flow rotation, which in turn leads to further amplification of vorticity as follows from an asymptotic, multiscale analysis of the interaction between vorticity and velocity of the flow. Various flow scales are analyzed, including **bathtub vortices, firewhirls, supercell tornadoes, and tropical cyclones**. It is found that the axial vorticity in the intensification region should satisfy $\omega_z \propto r^{-\alpha}$ with $4/3 \le \alpha \le 3/2$.

Authors' work [1]

In [1], the authors look for **solutions to the Navier–Stokes and Euler equations** (with appropriate boundary conditions) in **spherical** coordinates (R, α, θ)

$$u_R = \frac{G(x)}{r^b} \qquad u_\alpha = \frac{F(x)}{r^b} \qquad u_\theta = \frac{\Omega(x)}{r^b}$$

where $x = \cos\alpha$, $r = R\sin\alpha$, and $b > 0$. (The Navier–Stokes case with $b = 1$ was analyzed in [9].)

Main results:

- The NS equations with constant viscosity do not admit a solution of the above form if $b \neq 1$.
- A trivial solution is found to hold for all $b > 0$:

$$F(x) = G(x) \equiv 0, \qquad \Omega(x) \equiv \text{const.}$$

- No nontrivial solutions exist for $b \ge 2$.
- No nontrivial stable solutions exist for $1 < b < 2$.
- Explicit general solution is found for $b = 1$

$$\Omega \equiv C_\omega, \quad F = c\sqrt{x(1-x)}; \quad G = c\frac{(1-2x)\sqrt{1+x}}{2\sqrt{x}}.$$

- Stable solutions for $0 < b < 1$ computed numerically:

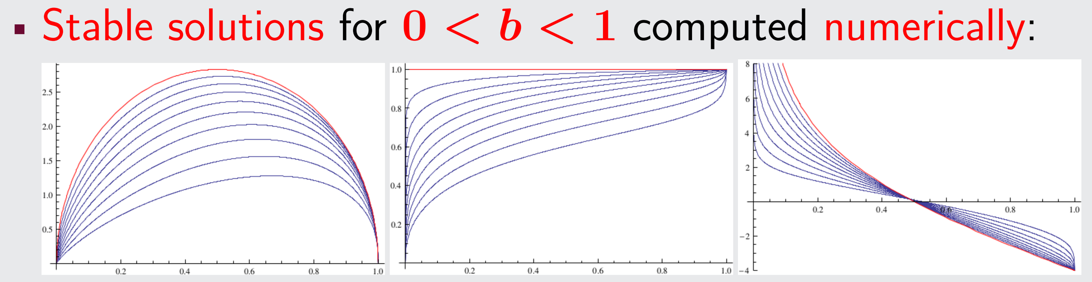

Figure 7: F, Ω, G for $b = 0.1, \dots, 0.9$ (blue) and $b = 1$ (red).

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