In atmospheric research, explicit assumptions are often made about the shapes of velocity or vorticity profiles of axisymmetric vortical flows. Some examples of such “vortices” are the potential vortex, the Rankine-combined vortex, the Burgers–Rott vortex, the Sullivan vortex, the Kuo vortex, the Serrin vortex, the Lamb–Oseen vortex; etc. In many such models, the tangential velocity away from the vortex axis (beyond the RMW) decays as \( r^{-1} \), where \( r \) is the distance from the vortex axis, while the axial vorticity \( \omega_z \) zero or approaching it as \( r \) increases beyond the RMW.

We revisit some experimental and theoretical results at different scales and focus on the velocity/vorticity decay in the “intensification region,” which, in many cases, exhibits a power law decay with \( u_r \propto r^{-\beta} \) and/or \( \omega_r \propto r^{\alpha+1} \) with \( 0 < \beta < 1 \).

This phenomenon seems to be prevalent across many scales, including a bathtub vortex, the vortex chamber, tornados, mesocyclones, and tropical cyclones.

**The Bathtub Vortex** [4]

An analysis due to Klimenko [4] results in a power law for the behavior of the axial vorticity, \( \omega_z \propto r^{-\alpha} \) with \( \alpha = 4/3 \) in the intensification region, implying an \( r^{-1} \) component in the tangential velocity. It is argued that for “realistic vortical flows” \( 4/3 \leq \alpha \leq 3/2 \).

**Tropical Cyclones Data Analysis [8]**

Mallen et al. [8] have analyzed wind speeds in 251 tropical cyclones classified as prehurricanes, minimal, and major hurricanes and came up with the values below for the decay of \( u_r \propto r^{-\beta} \).

<table>
<thead>
<tr>
<th>Category</th>
<th>Min. ( \beta )</th>
<th>Std. ( \beta )</th>
<th>Max. ( \beta )</th>
<th>Range ( \beta )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Prehurricane</td>
<td>0.65</td>
<td>0.14</td>
<td>0.94</td>
<td>0.51 – 0.94</td>
</tr>
<tr>
<td>Minor hurricane</td>
<td>0.88</td>
<td>0.11</td>
<td>0.96</td>
<td>0.81 – 0.96</td>
</tr>
<tr>
<td>Major hurricane</td>
<td>0.11</td>
<td>0.04</td>
<td>0.14</td>
<td>0.04 – 0.14</td>
</tr>
<tr>
<td>All TCS</td>
<td>0.63</td>
<td>0.15</td>
<td>0.71</td>
<td>0.43 – 0.71</td>
</tr>
</tbody>
</table>

Some results for major hurricanes are shown below. On the left, the data from the individual 72 major hurricanes analyzed together with their composite average (\( \alpha = 0.49 \)); on the right, comparison of the average profile with some idealized vortices.

**Tornado Velocity Studies [5, 6, 10, 11]**

Recent radar studies show that typical velocity profiles in tornados exhibit similar power laws. Using Doppler radar data, Wurman et al. have demonstrated \( u_r \propto r^{-0.87} \) for the 1998 Spencer, SD tornado (left), [10], and \( u_r \propto r^{-0.6} \) for the 1995 Dimmitt, TX, tornado (right, [11]).

**Mesocyclone/Tornado Vorticity Study [3]**

Vorticity and pseudovorticity have been analyzed by Cai [3] for five nontornado and tornadic storms (Superior, NE; San Angelo, TX, Hays & Garden City, KS; Kelleville, TX). Pseudovorticity has been found to obey \( \omega_r \propto r^{-\alpha} \), with \( \alpha = 0.42, 0.81, 1.02, 1.50, 1.60 \), respectively, indicating a correlation between the magnitude of the exponent and the intensity of the mesocyclone. Also, a “tendency for the vorticity lines to become steeper as the mesocyclone becomes stronger” is demonstrated for Garden City (tornadic) and Hays (nontornado).

**Authors’ work [1]**

In [1], the authors look for solutions to the Navier–Stokes and Euler equations (with appropriate boundary conditions) in spherical coordinates \((R, \alpha, \theta)\)

\[
\frac{F(x)}{x} = G(x) = \frac{u_r}{r} = \frac{\omega_r}{r}\frac{\sqrt{\nu z}}{\sqrt{R}}
\]

where \( x = \cos \alpha, \ r = \frac{R}{\sin \alpha}, \ b > 0 \). (The Navier-Stokes case with \( b = 1 \) was analyzed in [9].)

Main results:
- The NS equations with constant viscosity do not admit a solution of the above form if \( b \neq 1 \).
- A trivial solution is found for all \( b > 0 \):

\[
F(x) = G(x) = 0, \quad \Omega(x) = \text{const}.
\]
- No nontrivial solutions exist for \( b \geq 2 \).
- No nontrivial stable solutions exist for \( 1 < b < 2 \).
- Explicit general solution is found for \( b = 1 \):

\[
\Omega = \Omega_{const}, \quad F = c x \sqrt{(1-x)}; \quad G = c \left(1-2x\right)\sqrt{1+x}\frac{x}{2}\sqrt{1+x}.
\]
- Stable solutions for \( 0 < b < 1 \) computed numerically.

**References**


**Asymptotic Analysis**

Asymptotic analysis of the Navier–Stokes equations in spherical coordinates, \((R, \alpha, \theta)\),

\[
\frac{F(x)}{x} = G(x) = \frac{u_r}{r} = \frac{\omega_r}{r}\frac{\sqrt{\nu z}}{\sqrt{R}}
\]

where \( x = \cos \alpha, \ r = \frac{R}{\sin \alpha}, \ b > 0 \). (The Navier-Stokes case with \( b = 1 \) was analyzed in [9].)

Main results:
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\]
- Stable solutions for \( 0 < b < 1 \) computed numerically.

**Buldakov, Egorov, Sychev [2]**

Buldakov et al. [2] seek steady-state similarity solutions in the viscous vortex core which match external inviscid similarity solutions with a power-law decay \( u_r \propto r^{1-\beta} \).

The form of the sought solutions is

\[
u_r = \frac{K^+U(n)}{r^{1-\beta}}, \quad u_r = \frac{K^0V(n)}{r^{1-\beta}}, \quad \omega_r = \frac{K^2W(n)}{r^{1-\beta}}
\]

where \( n = (K^+)^2/(K^2)^{-1} \) is a nondimensional parameter. They numerically discover no solutions for \( n > 0.618 \), two solutions for \( 0.5 < n < 0.618 \), and one for \( n = 0.618 \). (The threshold value corresponds to \( \omega_r \propto r^{-0.392} \).)