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A Multistep and Multi-scale Variational Data Assimilation: Spatial Variations of Analysis Error Variance

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1. Introduction

Spectral formulations were derived (Xu et al. 2016, *Tellus A,* X16 hereafter) to compute the analysis error covariance efficiently for multistep and multi-scale variational data assimilation in which broadly distributed coarse-resolution observations are analyzed first and then locally distributed high-resolution observations are analyzed in the second step. However, the computed analysis error variance was constant and thus limited to represent only the spatially averaged error variance. To overcome the limitation, a suite of formulations is constructed in this paper to estimate the spatial variation of analysis error variance and associated spatial variation in analysis error covariance.

2. Spatial variation of analysis error variance

For a single observation at $x = x_m$ in the one-dimensional space of x, the error variance reduction produced by analyzing this observation is

$$\Delta\sigma_m^2(x) = \gamma_b[\sigma_b C_b(x - x_m)]^2, \quad (1)$$

where $\gamma_b = \sigma_b^2/(\sigma_b^2 + \sigma_o^2)$, σ_b^2 (or σ_o^2) is the background (or observation) error variance, $C_b(x)$ is the background error correlation function. The error variance reduction produced by analyzing M coarse-resolution observations, denoted by $\Delta\sigma_M^2(x)$, is bounded above by $\sum_m\Delta\sigma_m^2(x)$. The domain averaged value of $\sum_m\Delta\sigma_m^2(x)$ can be computed by

$$\Delta\sigma_{bs}^2 \equiv \int_D dx \sum_m \Delta\sigma_m^2(x)/D \approx \gamma_b\sigma_b^2 M \int dx C_b^2(x)/D, \quad (2)$$

where D is the domain length and the domain is extended periodically.

For M uniformly distributed coarse-resolution observations, $\Delta\sigma_M^2(x)$ can be estimated by

$$\Delta\sigma_M^2(x) = \sum_m\Delta\sigma_m^2(x) - \Delta\sigma_{bs}^2 + \Delta\sigma_{be}^2, \quad (3)$$

where $\sigma_{be}^2 \equiv \sigma_b^2 - \sigma_e^2$, and σ_e^2 is the domain averaged analysis error variance estimated by the spectral formulation in X16.

For M non-uniformly distributed coarse-resolution observations, γ_b in (1) needs to be modified into

$$\gamma_m = \sigma_b^2/(\sigma_b^2 + \beta_m\sigma_b^2 + \sigma_o^2), \quad (4)$$

where $\beta_m = [C_b^2(\Delta x_{com+}) + C_b^2(\Delta x_{com-}) - 2C_b^2(\Delta x_{co})]/[1 - C_b^2(\Delta x_{co})]$, $\Delta x_{co} \equiv D/M$ is the averaged resolution and Δx_{com+} (or Δx_{com-}) is the spacing of the m^{th} coarse-resolution observation from its right (or left) adjacent observation. The maximum (or minimum) of $\sum_m\Delta\sigma_m^2(x)$, denoted by $\Delta\sigma_{emx}^2$ (or $\Delta\sigma_{emn}^2$), needs to be adjusted to $\Delta\sigma_{mx}^2$ (or $\Delta\sigma_{mn}^2$) – the maximum (or minimum) of $\Delta\sigma_M^2(x)$ computed by (3) but with Δx_{co} decreased to Δx_{omn} (or increased to Δx_{omx}), where Δx_{omn} (or Δx_{omx}) is the minimum (or maximum) spacing between two adjacent observations among all non-uniformly distributed coarse-resolution observations. This gives

$$\Delta\sigma_M^2(x) = [\sum_m\Delta\sigma_m^2(x) - \Delta\sigma_{emn}^2]\rho + \Delta\sigma_{mn}^2, \quad (5)$$

where $\rho = [\Delta\sigma_{mx}^2 - \Delta\sigma_{mn}^2]/[\Delta\sigma_{emx}^2 - \Delta\sigma_{emn}^2]$. The analysis error variance $\sigma_a^2(x)$ is then estimated by $\sigma_a^2(x) \approx \sigma_{a*}^2(x) \equiv \sigma_b^2 - \Delta\sigma_M^2(x)$. (6)

3. Spatial variation of analysis error covariance

Using $\sigma_{a*}(x)$ in (6), the previously estimated analysis error covariance matrix $\mathbf{A}_e$ in X16 can be modified into $\mathbf{A}_a$, $\mathbf{A}_b$ or $\mathbf{A}_c$ with its ij^{th} element given by

$$A_{aij} \equiv \sigma_{a*}(x_i)\sigma_{a*}(x_j)C_a(x_i - x_j), \quad (7)$$

$$A_{bij} \equiv \sigma_{a*}^2[(x_i + x_j)/2]C_a(x_i - x_j), \quad (8)$$

or $A_{cij} \equiv A_{eij} + \{\sigma_{a*}^2[(x_i + x_j)/2] - \sigma_e^2\}C_b(x_i - x_j)$, (9)

where $A_{eij} \equiv \sigma_e^2 C_a(x_i - x_j)$ is the ij^{th} element of $\mathbf{A}_e$.

4. Illustrative examples

The improved estimates of analysis error variance and covariance are shown in Figs. 1 and 2, respectively. Using $\mathbf{A}_a$, $\mathbf{A}_b$ or $\mathbf{A}_c$ to replace $\mathbf{A}_e$ for the two-step analysis with 50 iterations in section 3.3 of X16, the analysis error is reduced from 0.150 to 0.142, 0.098 or 0.063 ms^{-1}, while the single step analysis error is 0.365 ms^{-1}.

5. Conclusion

The above constructed formulations can further improve the two-step variational analyses in X16, especially when the coarse-resolution observations become increasingly non-uniform and/or sparse. The formulations can be extended for 2D and 3D analyses.

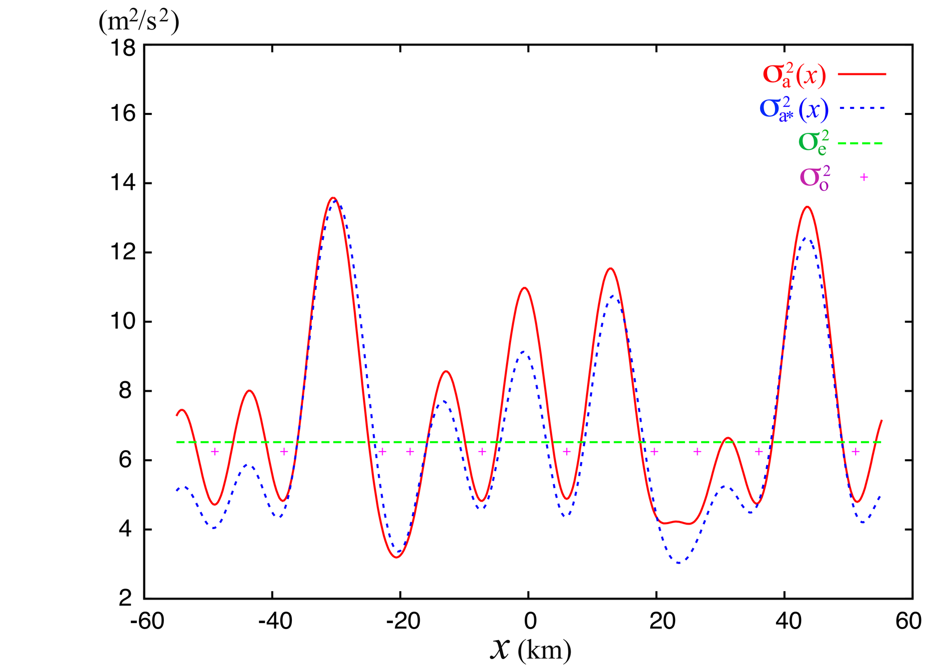

Fig. 1. True $\sigma_a^2(x)$ plotted by red solid curve, and estimated $\sigma_{a*}^2(x)$ in (5) plotted by blue dotted curve. The green dashed line shows the constant σ_e^2 estimated in X16. The purple + signs show σ_o^2 (= 6.25 m^2 s^{-2}) at the locations of M (= 10) non-uniformly distributed coarse-resolution observations. The background error covariance has the double-Gaussian form as in X16 with $C_b(x) = 0.6\exp(-x^2/2L^2) + 0.4\exp(-2x^2/L^2)$], $\sigma_b^2 = 25$ m^2 s^{-2} and $L = 10$ km. The analysis domain length is $D = N\Delta x = 110.4$ km with $\Delta x = 0.24$ km.

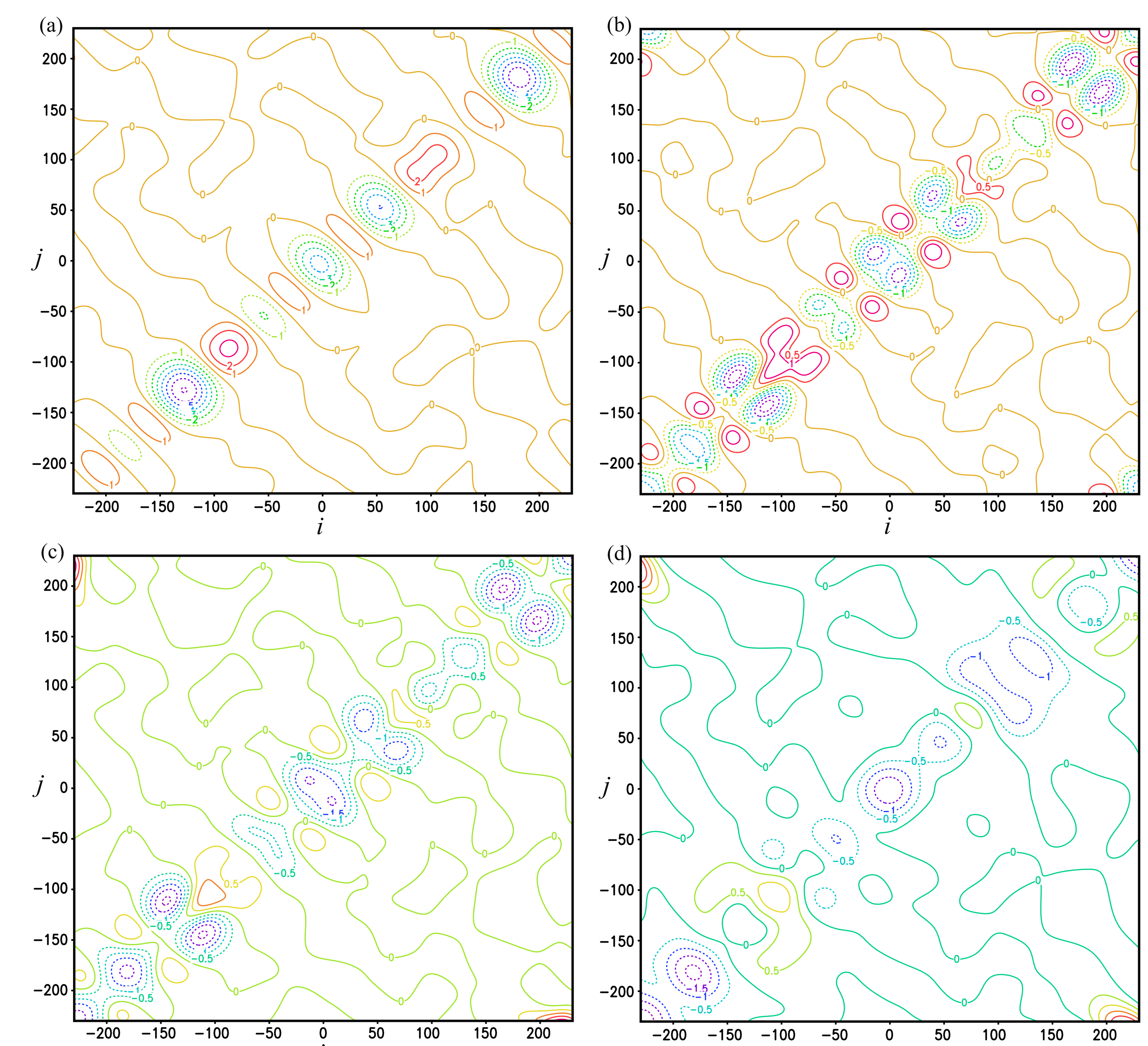

Fig. 2. (a) Deviation of $\mathbf{A}_e$ from true $\mathbf{A}$ plotted by color contours every 1 m^2 s^{-2} for the case of non-uniformly distributed coarse-resolution observations in Fig. 1. Deviations of $\mathbf{A}_a$, $\mathbf{A}_b$ and $\mathbf{A}_c$ from $\mathbf{A}$ are plotted by color contours every 0.5 m^2 s^{-2} in panels (b), (c) and (d), respectively, where $\mathbf{A}_a$, $\mathbf{A}_b$ and $\mathbf{A}_c$ are the successively improved estimates of $\mathbf{A}$ by the newly constructed formulations in (7)-(9).