P.37 Differential Ability of Annual Maximum Series and Peaks-Over-Threshold Series to Detect Trend in Extreme Daily Rainfall - A Simulation Approach

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1. INTRODUCTION

Understanding of regional distribution of extreme precipitation events as a response to global warming is a subject of active research as suggested in IPCC-4. Studies on secular trend of precipitation are well documented (e.g., Groisman et al. 2001), but those studies adopted spatial averaging so that results of extreme precipitation trend are relatively on coarse domain to depict extreme precipitation in finer scale. What makes extreme precipitation trend study difficult is the criterion on extreme precipitation. Groisman et al. (2001) analyzed frequencies of heavy rain climatology with three definitions of heavy rain event. The first is an event of daily precipitation which exceed threshold of 2 inches (heavy) and 4 inches (very heavy). The second is in terms of percentiles of precipitation days through the year, 90% (heavy) and 99% (very heavy). The third is in terms of return period of event, 1-year return period (heavy) and 20-year (very heavy).

National Weather Service (NWS) is in the process of establishing operational frequency database to provide users a precipitation amount (depth) given precipitation duration and average recurrence interval at any geographic location of Conterminous United States (Bonnin et al. 2006) as NOAA Atlas 14. The database is outcome of rigorous quality control on available daily and sub-daily historical precipitation data, and various statistical tests (e.g., heterogeneity test). Unfortunately, *L*-moment method (Hoskings and Wallis, 1997) used in Atlas 14 cannot consider time-dependency ("non-stationary") while some concerns have been raised (e.g., DeGaetano, 2009). The main purpose of this study is to answer to questions: If there is a trend, can we use non-stationary statistical model to reliably detect the trend and estimate its magnitude? What are the relative strength and weakness of extreme value samples to non-stationary model in this respect?

Hence, we engaged in a simulation study of trend detection using non-stationary model to both stationary and non-stationary data. The non-stationary model will estimate trend as zero if data-series is stationary, and estimate the prescribed trend if data are non-stationary. We opted to use publically available R package "ismev" (Coles, 2001) which contain functions that handle time-dependent covariates.

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2. EXTREME VALUES SAMPLING, GENERALIZED EXTREME VALUE (GEV) DISTRIBUTION

Our approach is to fit extreme value samples to a generalized extreme value (GEV) distribution. The GEV distribution function is described as

$$\Pr(X \le x) = \exp\left\{-\left(1+\xi\frac{x-\mu}{\psi}\right)^{-1/\xi}\right\}, \quad (1)$$

where μ is a location parameter, ψ a scale parameter ($\psi > 0$), ξ a shape parameter. The GEV distribution function is defined within the range of x; $x < \mu - \psi/\xi$, for $\psi > 0$, and $x > \mu - \psi/\xi$, for $\psi < 0$. See Coles (2001) for detail. The shape parameter controls skewedness of density of GEV function. The location parameter is approximately a median value of the sample and the scale parameter is related to spread of distribution. The parameters are estimated by maximum likelihood method because of its ability to estimate time-dependent covariate as recommended by Katz et al. (2002).

One of commonly used extreme value sampling is to pick the highest value per year, hence it generate annual maximum series (AMS) whose sample size is identical with the number of years (N_y). It does not include all extreme values because any second highest would be dropped out of N_y samples regardless how frequent extreme events per year. The other procedure is called peaks-over-threshold (POT). With this approach, we chose a threshold value as top 99 percentile and treat observation above the threshold as extreme values. DeGaetano (2009) used partial duration series (PDS) whose sample size is N_y, namely, a subset of POT. We must note both POT and PDS may have multiple extremes in a year which create uncertainties in extreme value modeling as reported by Begueria (2005). Hence, we adopted point process approach which handles multiple extreme values per year after Smith (1989). We are interested in assessing the ability of maximum likelihood method of parameter estimation to AMS and POT whose threshold is determined by top 99 percentile to both stationary and non-stationary time-series with known trend. During the course of experiments with long-term daily rainfall time-series, we found the top 99 percentile value was in the proximity of 2 inches. Other possibility is to look at stability of standard errors of maximum likelihood estimates by various threshold value as briefly discussed in the Section 3.

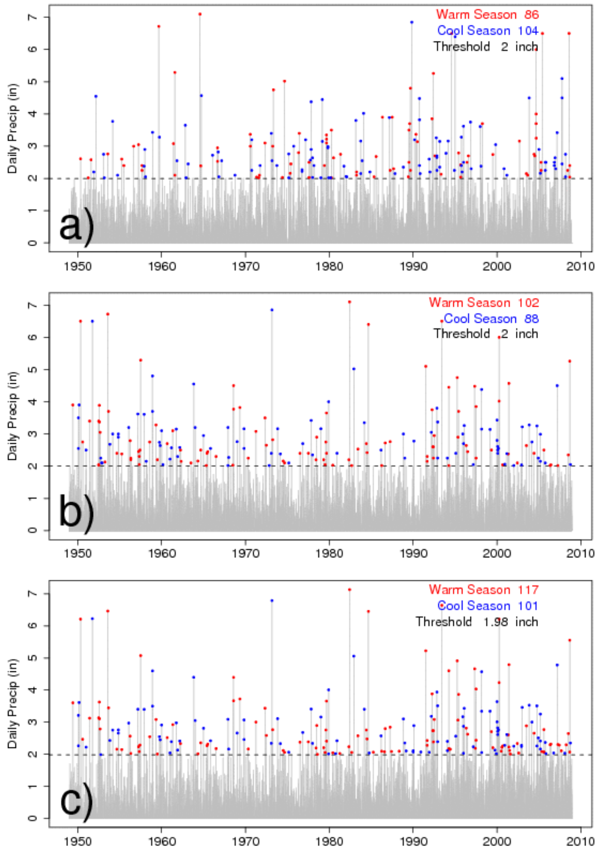

***Figure 1** Daily precipitation time-series of COOP station (380972) during 1949-2008 (a), resampled time-series to assure stationarity (b), time-series with added linear trend of 0.01 inch/year (c). Numbers of extreme values exceeding threshold value (upper 99 percentile) are marked by season; red by warm season (Apr. – Sep.) and blue by cool season. Horizontal line locates threshold value used in this example.*

3. EXPERIMENTAL SETUP

Step 1. We generate a stationary 60-yr daily rainfall time-series by re-sampling from an existing COOP station during 1949 – 2008. In each realization after resampling, stationarity is tested. If linear trend of occurrence of extremes values is significant, we repeat the resampling until stationarity is assured. Figure 1a is the original time-series of daily precipitation of COOP station at 380972 (Branchville, South Carolina). Figure 1b is resampled time-series. Missing values are set to zero to make it complete series of size 21915.

Step 2. We apply a prescribed trend of 0.01 in/year such that beginning day has 0.3 inch less than stationary time-series and ending day has 0.3 inch more. Any negative value as addition of trend is forced to zero. Figure 1c is time-series with trend.

Step 3. We create a series of missing values based on randomized binomial distribution with probability 0.85 so that 15% of samples are encoded as missing values to the outputs of Step 1 and 2.

Step 4. We generate four AMS corresponding to four data-series in Step 1-3, namely stationary time-series of complete record and with 15% missing values, and non-stationary time-series of complete record and with 15% missing values.

Step 5. We generate four POT series by using 99 percentile as a threshold value.

Step 6. We apply maximum likelihood estimate of three GEV parameters to both AMS and POT series. The maximum likelihood estimation is a numerical method that maximized negative log likelihood function, so there may exist multiple maxima or instability that numerical solution converges away from true solution. Figure 2 exemplifies sensitivity of maximum likelihood estimation to increased threshold value of one of stationary realizations.

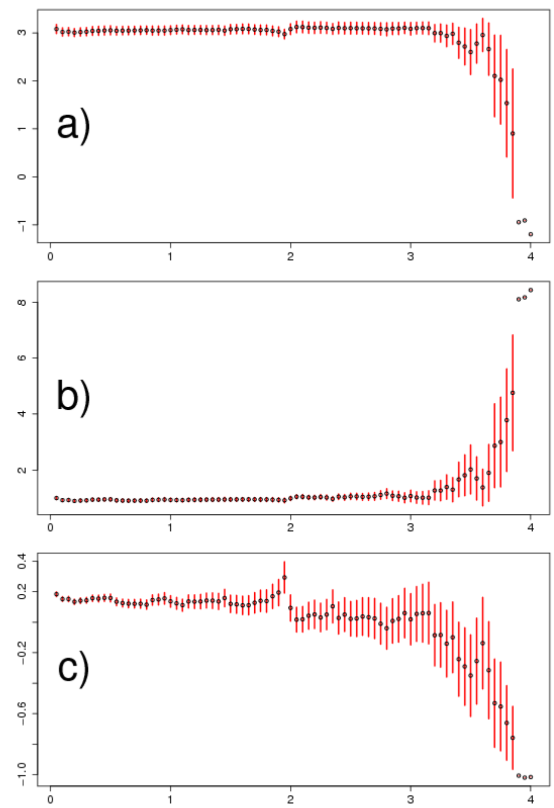

***Figure 2** A bias-variance trade-off diagram of three GEV parameters estimated by maximum likelihood method for visual inspection of stability with respect to threshold values (x-axis). Three parameters, location (a), scale (b) and shape(c) become unstable as threshold value increases. Vertical bars are +/- one standard error.*

Step 7. We repeat Step 3-6 with non-stationary model linear trend covariate to location parameter only;

$$\mu(t) = \mu_o + \beta t, \quad (2)$$

where β is a trend in location parameter, and t is a covariate linear in time from 1949 – 2008.

Therefore, in each realization, we obtained 4 GEV parameters (location, trend in location, scale and shape) in 16 combinations. Sixteen combinations are from two-level factor in sampling method, data series, completeness, and GEV model. Sampling method factors are AMS vs POT. Data series are stationarity vs non-stationarity. Completeness factor is non-missing observations vs 15% missing values. GEV model factor is stationary model vs non-stationary model with time-dependent covariate. We repeated 1000 times above Step 1-7 with a new re-sampling procedure.

4. RESULTS

Among 4 GEV parameters, we focus the result of 1000 realization of trend in location parameter with non-stationary GEV model.

a. Non-stationary GEV modeling

Figure 3 show results of 8 estimated trend parameters by applying non-stationary GEV model with time dependent covariate on location parameter (see Eq. 2). The experiment is to assess the use of non-stationary GEV model regardless of data-series is stationary or not. Figure 4 is box plot of the same estimates used in Figure 3. Some observations are made;

- The frequency of estimated trend with POT series is much narrower than those of AMS, namely, confidence on maximum likelihood estimate from POT samples higher than that from AMS. Larger sample size in POT than AMS is attributed to this result.

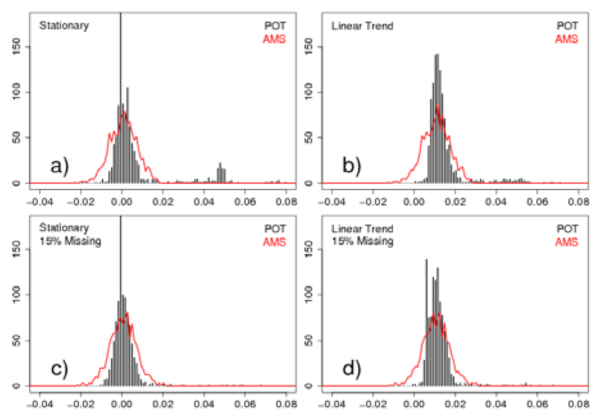

Figure 3 *Histograms of trend estimates from 1000 realizations (red lines for AMS, dark bars are for POT, and x-axis is trend in rain rate in inch/year). Left panels are results of maximum likelihood estimate of trend to stationary time-series. Right panels are trend estimate with non-stationary time-series. Top panels are with complete data set, bottom panels are results with 15% of missing observations. The rain-rate (x-axis) is truncated to capture main feature, but outliers are shown in Figure 4.*

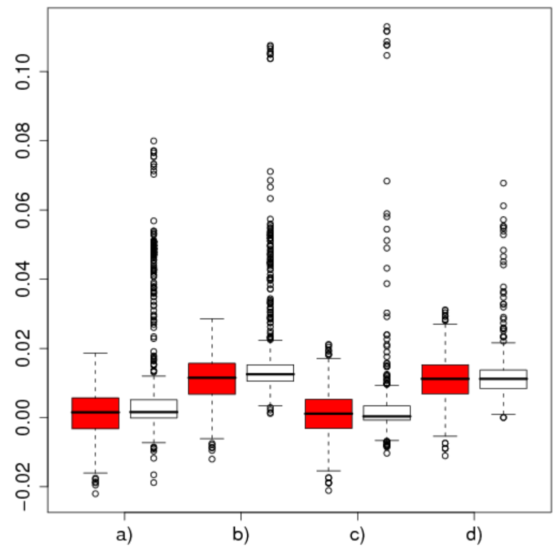

Figure 4 *Box-plot display of four cases which correspond to Figure 3, vertical axis stands for trend of location parameter (inch/year). Large number of outliers of trend estimate by ML method in POT series is observed in comparison with those of AMS.*

- But, estimates from POT series display secondary peaks far off the correct value, zero for stationary data and 0.01 for non-stationary data. We speculate the cause of this is related to numerical optimization routine and initial values in maximizing likelihood function.
- None of estimated trend values with POT series are negative in either complete series or with missing values (see case b and d).
- The 15% missing values with random occurrence are not negatively affecting in trend estimate with non-stationary GEV model.

b. Comparative metric of trend estimate

We computed median and inter-quartile range of trend estimates corresponding to Figure 3. Table 1 is 2 x 2 table of metric as results of stationary data corresponding to cases a) and c) in Figure 3 and 4, table 2 is from non-stationary data which corresponds to cases b) and d) in Figure 3 and 4. Unit is in inch/year. Table 3 is metrics of trend estimates from POT which are outside of estimates from AMS.

Table 1. Medians and inter-quartile ranges of trend estimates from GEV model fit to stationary data-series. Inter-quartile range values are in the parentheses. Unit is in inch/year.

	Complete	**Incomplete**
AMS	0.0015 (0.0089)	0.00115 (0.00839)

POT	0.00156 (0.00526)	0.00033 (0.00407)

Table 2. Medians and inter-quartile ranges of trend estimates from GEV model fit to non-stationary data-series. Inter-quartile range values are in the parentheses. Unit is in inch/year.

	Complete	**Incomplete**
AMS	0.01151 (0.00893)	0.01117 (0.00836)
POT	0.01252 (0.00474)	0.01121 (0.00537)

Table 3. Medians and inter-quartile ranges of trimmed trend estimates of POT. Trimmed estimates are outside of boundaries of AMS estimates. First row indicates percentage of trimmed estimates from POT.

	Complete	**Incomplete**
Trimmed (%)	12.3	1.8
Stationary	0.00081 (0.003995)	0.0003 (0.00389)
Non-stationary	0.0122 (0.004055)	0.011125 (0.0052825)

5. SUMMARY AND FUTURE WORKS

The experiment presented here is a series of "simple" tests to detect trend estimation of extreme rainfall in long-term time-series. This requires a method which can include time-dependent covariate in the GEV modeling, namely, non-stationary GEV model. We have chosen point process approach which fits POT samples with maximum likelihood method. Hence, direct evaluation with *L*-moment is not in the scope of this study. We stated simple test, because simulated data-series with linear trend is not a typical non-stationary data-series. Also, we decided upper 99 percentile as threshold value to avoid definition problem of "extreme" precipitation even though estimate critically depend on threshold value as well as data record length. Although many assumptions are made for simplistic experiment, the answer to the question "Can we estimate trend in extreme rainfalls?" is yes. The non-stationary GEV modeling handles stationary data-series, while stationary modeling cannot estimate trend in non-stationary data-series. We found differential abilities in trend detection from AMS and POT, but they are complementary as summarized below;

- Estimates from AMS are stable and less sensitive to incompleteness and to non-stationarity of data. Namely, sizes of four red-colored boxes in Fig. 4 are almost the same. Such robustness against missing values is attributed to lower chance of annual maximum being missing value.
- Estimates from POT series exhibited many outliers unfortunately. It is not known that if point process approach to POT series is less effective to heavily clustered realizations or time-dependent covariate is not scaled and centered at mid-point of record period. However, inter-quartile range is much narrower than those of AMS as ascertained by pairs of boxes (red and white) in Fig 3 and compactness of POT histograms in Fig. 4.
- Trend estimate from AMS can be treated as a prior of parameter distribution of POT which constrains outliers.

We propose future works that will exploit current results, and reflect more realistic situations of real-world;

1) Test of different types of non-stationary model, cyclic or log-linear in time to all parameters within the framework of point process representation of GEV modeling.
2) Generate non-random missing values such as week-long or month-long missing value series. It is highly sensitive to modeling result if any extreme value is missed.
3) Develop a method to restrict outliers of parameters from POT series either by de-clustering POT series or by bounding parameters from AMS series.
4) Compute n-year return levels with confidence intervals similar to Figure 2.
5) Consider a spatial distribution of trend within a homogeneous region identified in NOAA Atlas 14.

6. ACKNOWLEDGMENT

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