Surface Roughness Height Model for Stratified Atmospheric Boundary Layer Interacting with Wind Farms

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Background

Several important features of the interactions between large wind farms and the atmospheric boundary layer (ABL) can be described succinctly using an effective roughness length. Recent Large Eddy Simulation (LES) studies in neutral atmospheric conditions have led to model expressions for the wind farm effective roughness scale that depends upon parameters such as
wind turbine spacing, ground roughness, rotor diameter, hub-height etc.

Objectives

New model equations for the effective surface roughness length in thermally-stratified conditions are derived using single-column analysis. The effect of the rotor wakes is taken into account through an additional wake eddy viscosity and eddy diffusivity coefficients. The model leads to four coupled nonlinear equations for the effective roughness scale, the friction velocities corresponding to the regions below and above the rotor, and the surface heat flux. A number of LES cases are performed to characterize the impact of wind farms on the thermally-stratified ABL in the asymptotic case of very large wind farms under fully-developed steady-state conditions, with prescribed boundary layer height. The results from LES are used to measure horizontally-averaged vertical profiles of velocity, potential temperature, and vertical turbulent momentum and heat flux. These profiles are used to obtain the effective roughness scale, and results are compared with the model.

Effective roughness height for stratified ABL

The starting point is the analysis of Calaf et al. (2010) (see also Frandsen (1992)) who studied the neutral ABL. Assuming a third logarithmic profile within the rotor wake region and an additional wake eddy-viscosity in the wake, the mean streamwise velocity profiles in stratified conditions are:

$$u(z) = \frac{u_{*,lo}}{\kappa}\left\{\ln\left[\left(\frac{z}{z_H}\right)^{1-\beta}\left(\frac{z_H}{z_{0,lo}}\right)\left(\frac{z_H-D/2}{z_H}\right)^{\beta}\right] -(1-\beta)\Psi_M\left(\frac{z}{L_{lo}}\right)-\beta\Psi_M\left(\frac{z_H-D/2}{L_{lo}}\right)\right\} \quad \text{if } z_H-\frac{D}{2}<z<z_H,$$

$$u(z) = \frac{u_{*,hi}}{\kappa}\left\{\ln\left[\left(\frac{z}{z_H}\right)^{1-\beta}\left(\frac{z_H}{z_{0,hi}}\right)\left(\frac{z_H+D/2}{z_H}\right)^{\beta}\right] -(1-\beta)\Psi_M\left(\frac{z}{L_{hi}}\right)-\beta\Psi_M\left(\frac{z_H+D/2}{L_{hi}}\right)\right\} \quad \text{if } z_H<z<z_H+\frac{D}{2}$$

So an equation relating the roughness velocities above and below the wind turbines can be obtained as

$$\frac{u_{*,lo}}{u_{*,hi}} = \frac{\ln\left[\left(\frac{z_H}{z_{0,hi}}\right)\left(\frac{z_H+D/2}{z_H}\right)^{\beta}\right]-(1-\beta)\Psi_M\left(\frac{z_H}{L_{hi}}\right)-\beta\Psi_M\left(\frac{z_H+D/2}{L_{hi}}\right)}{\ln\left[\left(\frac{z_H}{z_{0,lo}}\right)\left(\frac{z_H-D/2}{z_H}\right)^{\beta}\right]-(1-\beta)\Psi_M\left(\frac{z_H}{L_{lo}}\right)-\beta\Psi_M\left(\frac{z_H-D/2}{L_{lo}}\right)}$$

A second equation for the two roughness velocities is obtained by using a horizontal averaging of the momentum equation; the final result is

$$u_{*,hi}^2 = u_{*,lo}^2 + \frac{1}{2}c_{ft}u(z_H)^2$$

A third equation is obtained by relating the roughness velocity above the rotors with the geostrophic velocity using

$$U_G = \frac{u_{*,hi}}{\kappa}\left[\ln\left(\frac{H_G}{z_{0,hi}}\right)-\Psi_M\left(\frac{H_G}{L_{hi}}\right)\right]$$

Then the effective roughness length is obtained as

$$z_{0,hi} = z_H\left(\frac{z_H+D/2}{z_H}\right)^{\beta} \times e^{\left(\Omega(L_{hi})-\left\{\frac{c_{ft}}{2\kappa^2}+\left\{\ln\left[\left(\frac{z_H}{z_{0,lo}}\right)\left(\frac{z_H-D/2}{z_H}\right)^{\beta}\right]+\Omega(L_{lo})\right\}^{-2}\right\}^{-1/2}\right)}$$

Unlike the neutral conditions, the thermal stratification is involved in this model. An equation for the temperature difference can be obtained by employing the profile of temperature with an additional eddy diffusivity in the wake region. Such analysis provides:

$$\theta_s-\theta_\infty = \frac{Pr_t q_s}{\kappa u_{*,lo}}\left\{\ln\left[\frac{H_G}{z_H}\left(1+\frac{D}{2z_H}\right)^{\beta}\right]+\frac{u_{*,hi}}{u_{*,lo}}\ln\left[\frac{z_H}{z_{0,s}}\left(1-\frac{D}{2z_H}\right)^{\beta}\right] -\beta\Psi_H\left(\frac{2z_H+D}{2L_{hi}}\right)-\beta\Psi_H\left(\frac{2z_H-D}{2L_{lo}}\right)-\Psi_H\left(\frac{z_H}{L_{hi}}\right)-\Psi_H\left(\frac{H_G}{L_{hi}}\right)-\Psi_H\left(\frac{z_H}{L_{lo}}\right)\right\}$$

which closes the model.

Comparison to LES results

LES of atmospheric boundary layer interacting with infinitely large wind farms has been carried out in both stable and unstable conditions.

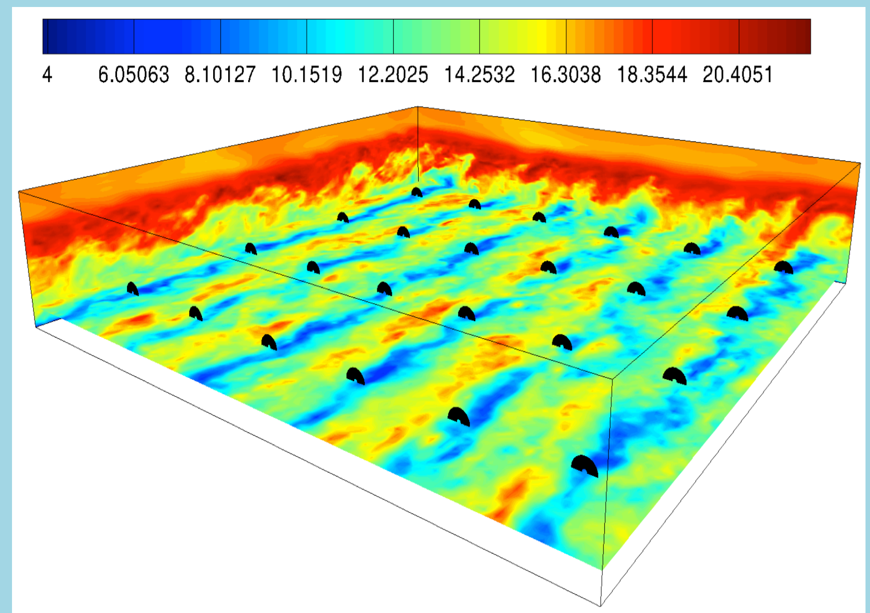

Figure 1: Streamwise velocity contours in stable conditions

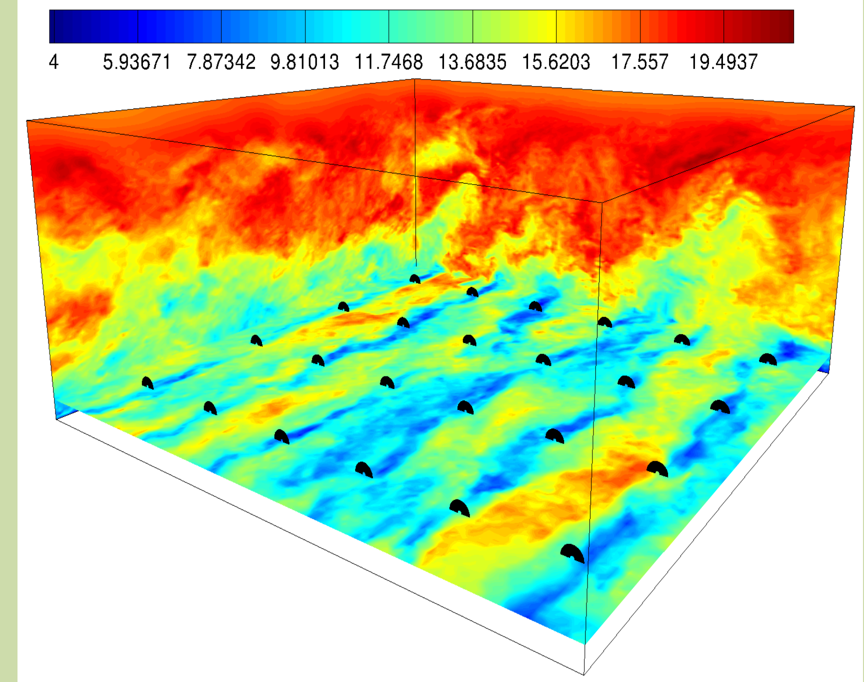

Figure 2: Streamwise velocity contours in unstable conditions

Horizontally-averaged profiles of streamwise velocity are used to extract the effective surface roughness height under various conditions. This is done by starting with the velocity profile (in stable conditions) written in the form:

$$\frac{u}{u_*} = \frac{1}{\kappa}\left[\log(z)-\Psi_M\left(\frac{z}{L}\right)\right]-\frac{1}{k}\log(z_0)$$

where we identify a new variable

$$s = 1/\kappa\left[\log(z)-\Psi_M(z/L)\right].$$

As a result, a linear behavior is obtained according to

$$\frac{u(s)}{u_*} = s - s_{0,hi}$$

where

$$s_{0,hi} = 1/\kappa\ln(z_{0,hi})$$

can be determined from the intercept, while the corresponding surface roughness height can be found as $z_{0,hi} = \exp(\kappa s_{0,hi})$.
The linear behavior is expected to occur above the rotors (fig. 3).
The same approach is applied to unstable conditions.

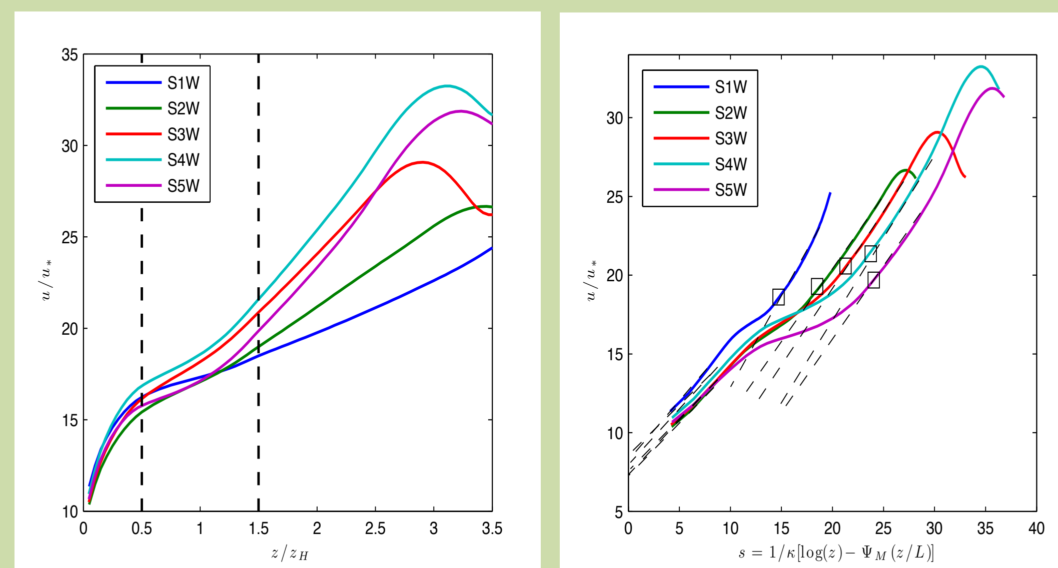

Figure 3: Streamwise velocity profiles as a function of z (left) and s (right), for different stratifications, different rotor spacing and different coefficient of thrust (stable case). Squares on the right hand side represent the maximum height of the rotor.

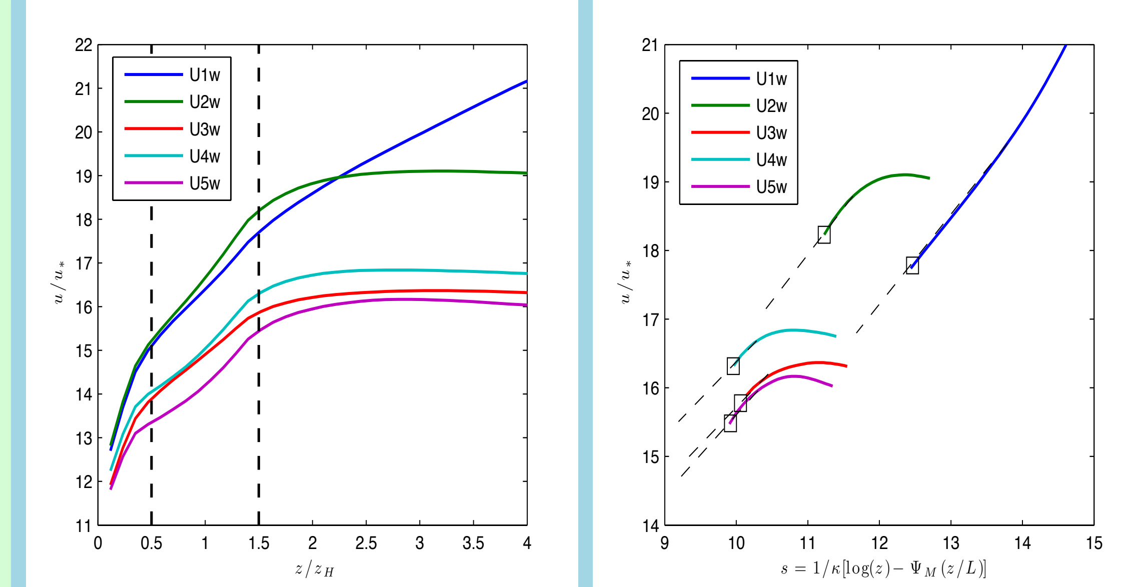

Figure 4: Streamwise velocity profiles as a function of z (left) and s (right), for different stratifications, different rotor spacing and different coefficient of thrust (unstable case). Squares on the right hand side represent the maximum height of the rotor.

Case	$z_{0,hi}$ LES (m)	$z_{0,hi}$ model (m)
S1w	0.509	0.750
S2w	0.656	0.895
S3w	1.003	0.971
S4w	2.121	1.570
S5w	2.686	2.373

Table 1: Comparison between surface roughness lengths calculated from LES and using the model equations for stable conditions.

Case	$z_{0,hi}$ LES	$z_{0,hi}$ theory
U1w	0.654	0.673
U2w	0.747	0.901
U3w	0.175	0.941
U4w	0.521	1.507
U5w	1.476	2.300

Table 1: Comparison between surface roughness lengths calculated from LES and using the model equations for stable conditions.

Conclusions

-A one-dimensional column model was used to introduce a theoretical prediction for the effective roughness length in the presence of a large wind farm, in stratified conditions.
-Compared to neutral conditions, the analysis is more demanding since the Monin-Obukhov similarity profiles involve stability functions.
-LES results consisting of horizontally-averaged velocity profiles were used to compare with the one-dimensional column model.
-The agreement was good for stable conditions, and acceptable for unstable conditions.

References

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Frandsen S (1992) On the wind speed reduction in the center of large clusters of wind turbines. J. Wind Eng. Ind. Aerodyn 39:251