Background
Several important features of the interactions between large wind farms and the atmospheric boundary layer (ABL) can be described succinctly using an effective roughness length. Recent Large Eddy Simulation (LES) studies in neutral atmospheric conditions have led to model expressions for the wind farm effective roughness scale that depends upon parameters such as wind turbine spacing, ground roughness, rotor diameter, hub-height etc.

Objectives
New model equations for the effective surface roughness length in thermally-stratified conditions are derived using single-column analysis. The effect of the rotor wakes is taken into account through an additional wake eddy viscosity and eddy diffusivity coefficients. The model leads to four coupled equations for the two roughness velocities above and below the geostrophic velocity using thick boundary layer height. Results are compared to LES results consisting of horizontally-averaged velocity profiles as a function of z (left) and different coefficient of thrust (unstable case). Squares on the right hand side represent the maximum height of the rotor.

Effective roughness height for stratified ABL
The starting point is the analysis of Calaf et al. (2010) (see also Frandsen (1992)) who studied the neutral ABL. Assuming a third logarithmic profile within the rotor wake region and an ABL effective roughness scale, and results are compared with the model.

Comparison to LES results
LES of atmospheric boundary layer interacting with infinitely large wind farms has been carried out in both stable and unstable conditions.

Conclusions
-A one-dimensional column model was used to introduce a theoretical prediction for the effective roughness length in the presence of a large wind farm, in stratified conditions.
-Comparison to neutral conditions, the analysis is more demanding since the Monin-Obukhov similarity profiles involve stability functions.
-LES results consisting of horizontally-averaged velocity profiles were used to compare with the one-dimensional column model.
-The agreement was good for stable conditions, and acceptable for unstable conditions.

References

Table 1: Comparison between surface roughness lengths calculated from LES and using the model equations for stable conditions.

Table 2: Comparison between surface roughness lengths calculated from LES and using the model equations for stable conditions.

Figure 1: Streamwise velocity contours in stable conditions

Figure 2: Streamwise velocity contours in unstable conditions

Table 1:

<table>
<thead>
<tr>
<th>Case</th>
<th>(s_{0,h} ) LES (m)</th>
<th>(s_{0,h} ) model (m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(U_1)</td>
<td>0.599</td>
<td>0.750</td>
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<tr>
<td>(U_2)</td>
<td>0.656</td>
<td>0.895</td>
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<td>(U_3)</td>
<td>1.003</td>
<td>0.971</td>
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<td>(U_4)</td>
<td>2.121</td>
<td>1.570</td>
</tr>
<tr>
<td>(U_5)</td>
<td>6.966</td>
<td>2.373</td>
</tr>
</tbody>
</table>

Figure 4:
Streamwise velocity profiles as a function of z (left) and s (right), for different stratifications, different rotor spacings and different coefficient of thrust (unstable case). Squares on the right hand side represent the maximum height of the rotor.