

Potential vorticity homogenization in an idealized model of the Southern Ocean

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Motivation/questions

What determines the equilibrium slope of buoyancy surfaces in the interior of the Southern Ocean?

How sensitive is the interior buoyancy structure to surface wind stress?

Theory

Buoyancy decomposition:

$$b = \tilde{b}(z) + b'(x, y, z, t) \quad \begin{matrix} N^2(z) = \partial_z \tilde{b} \\ M^2(x, y, z, t) = \partial_y b' = \partial_y b \end{matrix}$$

Quasi-geostrophic potential vorticity gradient:

$$\partial_y q = \beta + \partial_z (f_0 N^{-2} \partial_y b') \quad (1)$$

Slope of buoyancy surfaces:

$$N^{-2} \partial_y b' = M^2 / N^2 = \partial_y b / \partial_z \tilde{b} = -s_b$$

So equation (1) can be written (zonal and depth mean):

$$\overline{\partial_y q} = \beta - f_0 \overline{\partial_z s_b} = \beta \left(1 - \frac{\overline{\partial_z s_b}}{\beta / f_0} \right) = \beta(1 - r)$$

PV homogenization metric: $r(y, t) \equiv \frac{\overline{\partial_z s_b}}{\beta / f_0}$

Under homogenous PV ($r=1$):

$$\left(\frac{\partial s_b}{\partial z} \right) = \frac{\beta}{f_0} = \frac{2\Omega a^{-1} \cos(\phi_0)}{2\Omega \sin(\phi_0)} = a^{-1} \cot(\phi_0)$$

The large-scale slope structure is constrained by planetary-geometric parameters

Results: steady state

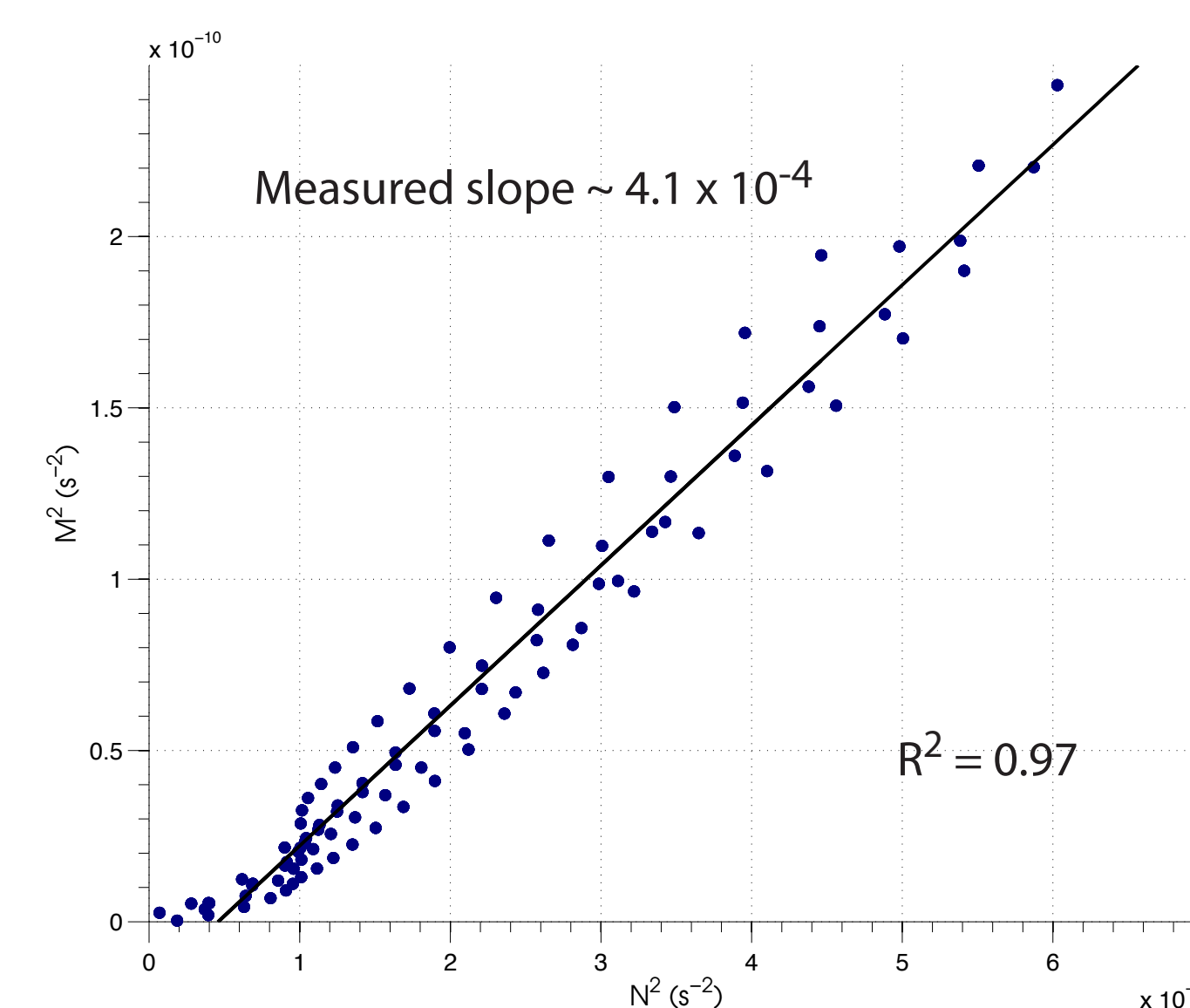


Fig. 1. Horizontal stratification versus vertical stratification in an idealized Southern Ocean model. Each point is calculated from a particular longitude/depth across the channel.

The large-scale slope structure is constrained by planetary geometric parameters

The planetary-geometric constraint is most effective when PV is uniform ($r=1$)

Stronger wind stress increases eddy activity, which homogenizes large-scale PV and makes the planetary-geometric constraint *more* dynamically relevant

Prediction:

$$\frac{\Delta s_b}{\Delta z} = -\frac{1}{H} \frac{M^2}{N^2} = \frac{\beta}{f_0}$$

$$M^2 = \beta H |f_0|^{-1} N^2$$

$$\text{Slope} = \beta H |f_0|^{-1}$$

$$\text{Slope} \approx 4.7 \times 10^{-4}$$

Model setup

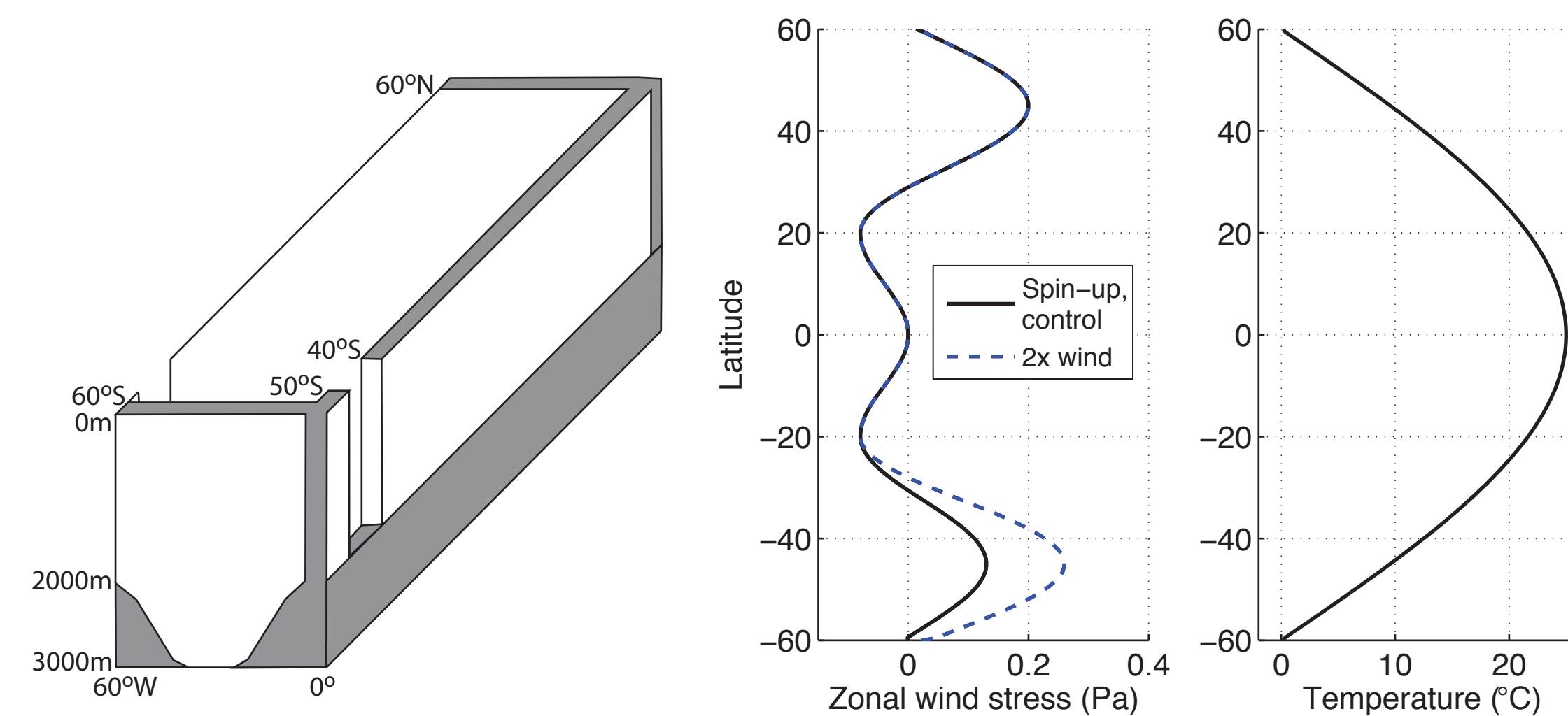


Fig. 2. Model geometry with circumpolar channel (left), surface wind forcing (middle), and surface temperature profile (right). The domain has 42 vertical levels and an eddy-permitting horizontal resolution ($1/6^\circ \times 1/6^\circ$).

Results: variability

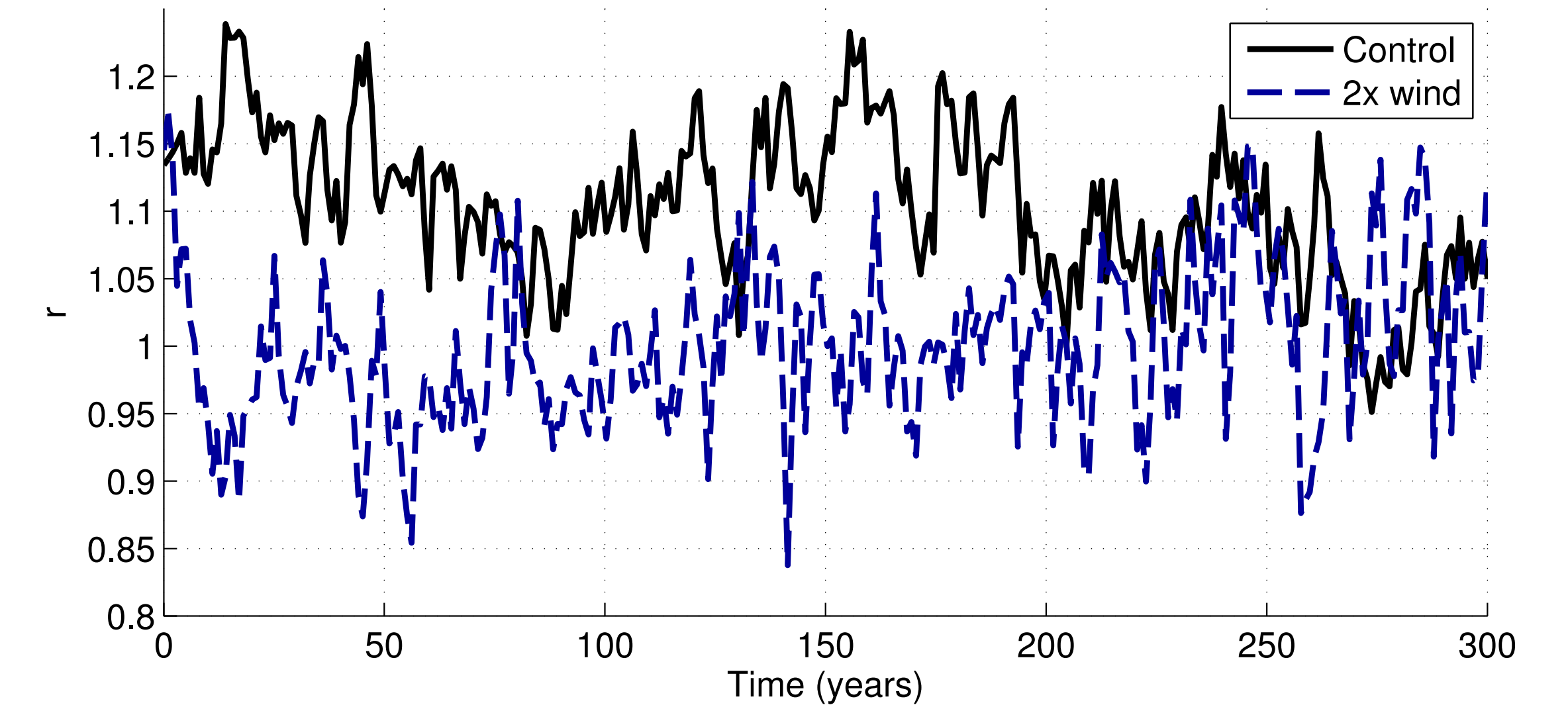


Fig. 3. Mean value of r (the PV homogenization metric) in the circumpolar channel for the control (black, solid) and strong wind (blue, dashed) cases.

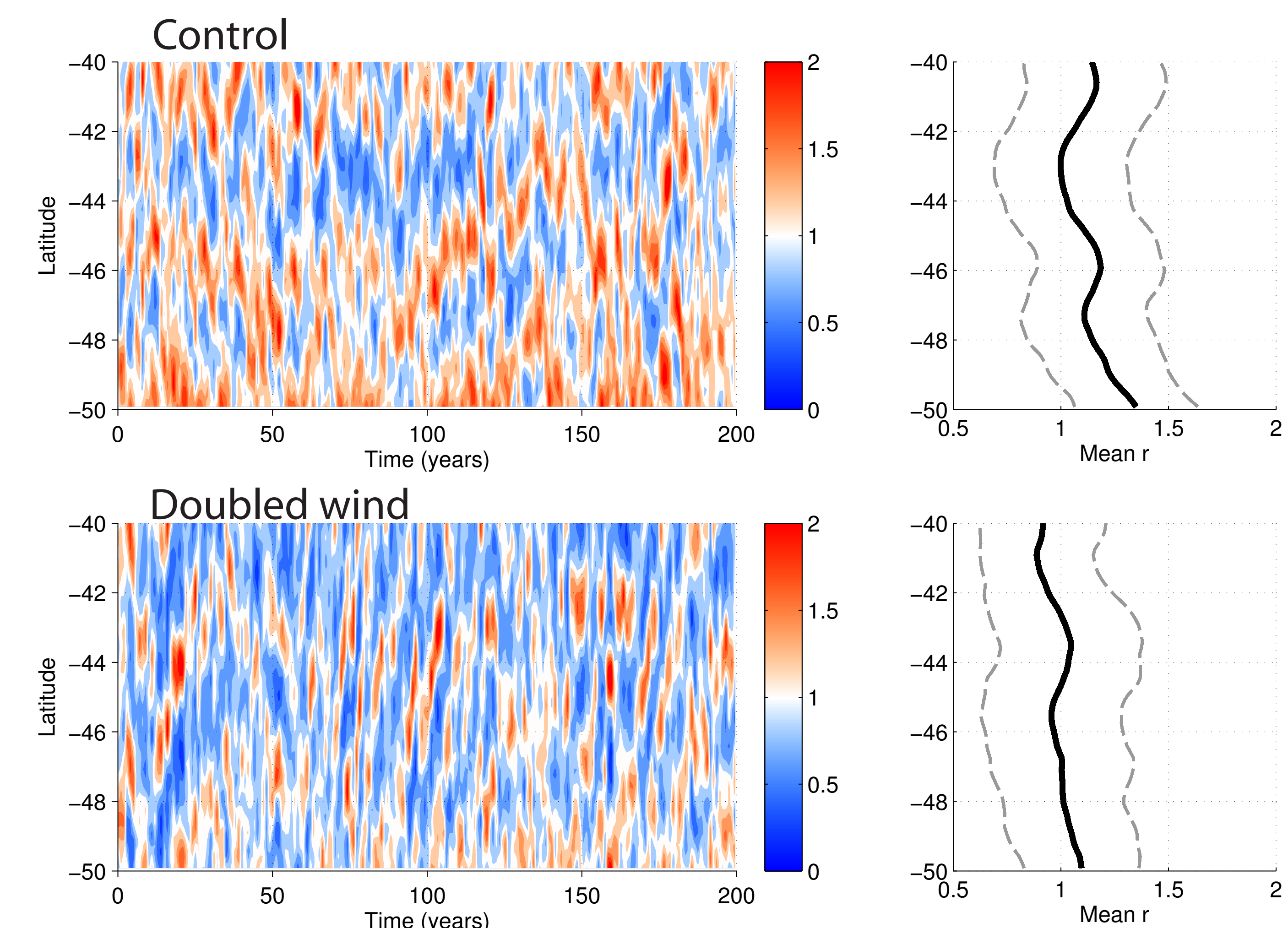


Fig. 4. Hovmöller diagrams (left column) and long-term zonal mean (right column) of r for both the weak wind (top row) and strong wind (bottom row) cases.

$r > 1$ stratification dominant (negative PV gradient)
 $r = 1$ stratification/planetary balance (zero PV gradient)
 $r < 1$ planetary constraint dominant (positive PV gradient)