Time-Resolving Model for Gravity Waves in Non-uniformly Stratified Atmosphere

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fast information transfer;
\[ \rho_0(z) \downarrow \Rightarrow \zeta_1(z) \uparrow \Rightarrow \text{detect tsunami} \]

Gravity wave speed: 305 km/h

Tsunami wave length: 200 km

Tsunami speed:
- 835 km/h
- 340 km/h
- 50 km/h

Water depth:
- 5500 meters
- 900 meters
- 20 meters

http://timmymaria.wikispaces.com/What+do+you+know+about+Tsunami%3F
Restrictions of current gravity wave modeling approach

- Stationary solution omits time-dependent details in middle atmosphere.
- Neglects partial back-reflection or assume constant N in the non-uniformly stratified atmosphere.
Why should we take non-uniform stratification into account?
Time-resolving model allowing jump in buoyancy frequency

**T-G equation:**

Initial/boundary value problem

\[
(\partial_t + U \partial_x)^2 \zeta_{zz} + N^2(z)\zeta_{xx} = 0
\]

\[
\zeta(t = 0^-) = \zeta_t(t = 0^-) = 0
\]

\[
\zeta(z = 0) = h(x), \zeta(z = \infty) = 0.
\]

**Laplace transform**

ODE: boundary value problem

\[
(s + ikU)^2 \hat{\zeta}_T^{zz} - k^2 N^2(z)\hat{\zeta}_T = 0
\]

\[
\hat{\zeta}_T(z = 0) = \hat{h}(k) / s
\]

\[
\hat{\zeta}_T(z = \infty) = 0
\]
Wave-train approximation

\[ + \exp[i \, m_2(k) \, z] \text{ term} \]

\[ + \exp[-i \, m_1(k) \, z] \text{ term} \]

\[ + \exp[i \, m_1(k) \, z] \text{ term} \]
Uniformly stratified atmosphere

Exact solution, t=5min

Wave-train approximation, t=5min
Uniformly stratified atmosphere

Exact solution, t=10min

Wave-train approximation, t=10min
Uniformly stratified atmosphere

Exact solution, $t=15\text{min}$

Wave-train approximation, $t=15\text{min}$
Uniformly stratified atmosphere

Exact solution, $t=20\text{min}$

Wave-train approximation, $t=20\text{min}$
Uniformly stratified atmosphere

Exact solution, $t=25\text{min}$

Wave-train approximation, $t=25\text{min}$
Non-Uniformly stratified atmosphere
(N2=2*N1)

Exact solution, t=5min

Wave-train approximation, t=5min
Non-Uniformly stratified atmosphere
(N2=2*N1)

Exact solution, t=10min

Wave-train approximation, t=10min
Non-Uniformly stratified atmosphere

(N2=2*N1)

Exact solution, t=15min

Wave-train approximation, t=15min
Non-Uniformly stratified atmosphere (N2=2*N1)

Exact solution, t=20min

Wave-train approximation, t=20min
Non-Uniformly stratified atmosphere
(N2=2*N1)

Exact solution, t=25min

Wave-train approximation, t=25min
Summary

• Develop a time-resolving model based on Laplace transform while allowing jump in stratification
• Construct a wave-train approximation including reflections and transmissions
• Recover the gravity wave propagation scheme in the middle and low atmosphere
Thank you for listening 😊