Time-Resolving Model for Gravity Waves in Non-uniformly Stratified Atmosphere

Chen Wei, Oliver Buhler, Esteban G. Tabak
Courant Institute, NYU

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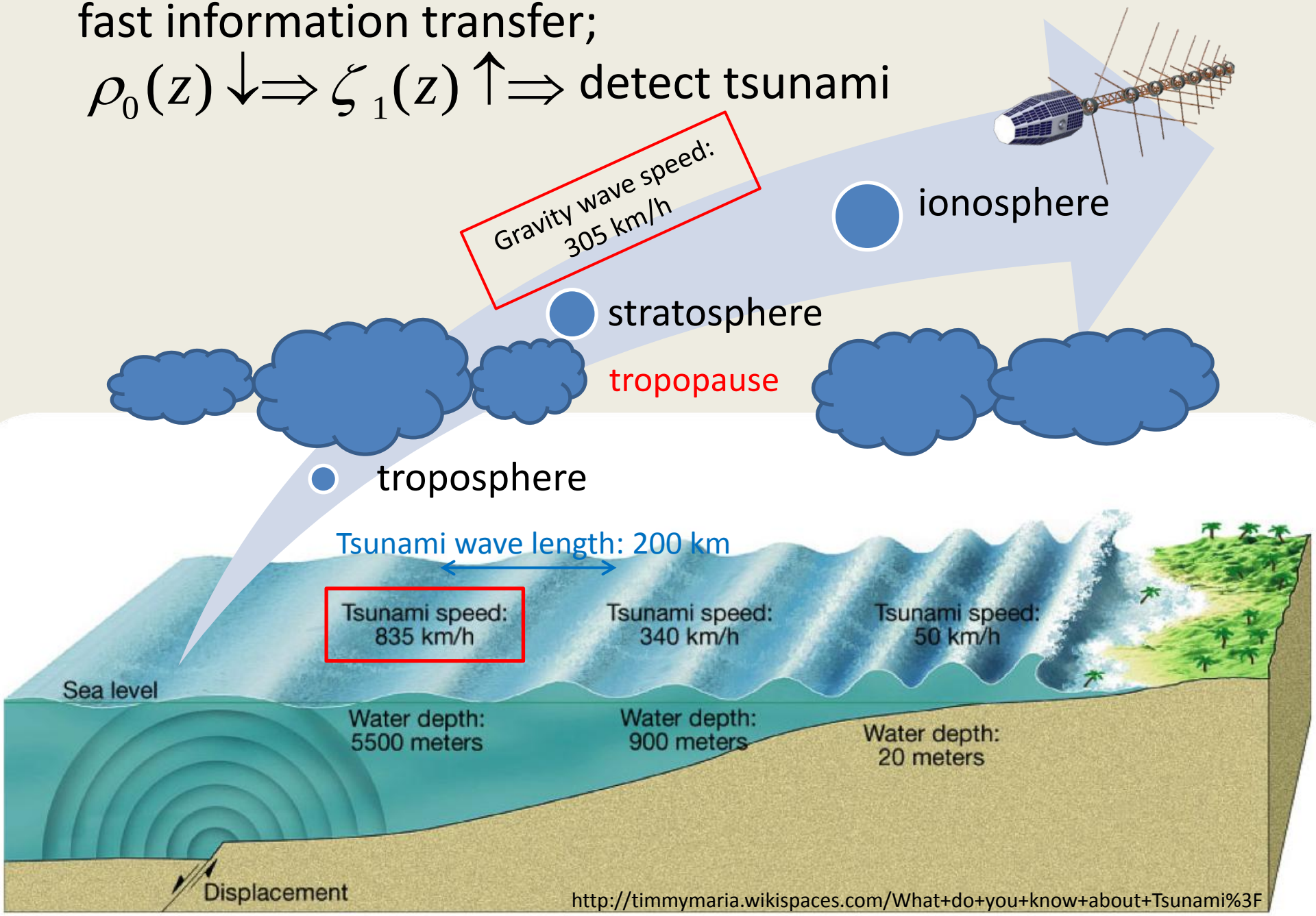
fast information transfer;

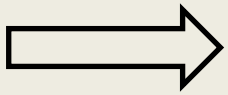
$$\rho_0(z)\downarrow \Rightarrow \zeta_1(z)\uparrow \Rightarrow$$ detect tsunami



http://timmymaria.wikispaces.com/What+do+you+know+about+Tsunami%3F

Restrictions of current gravity wave modeling approach

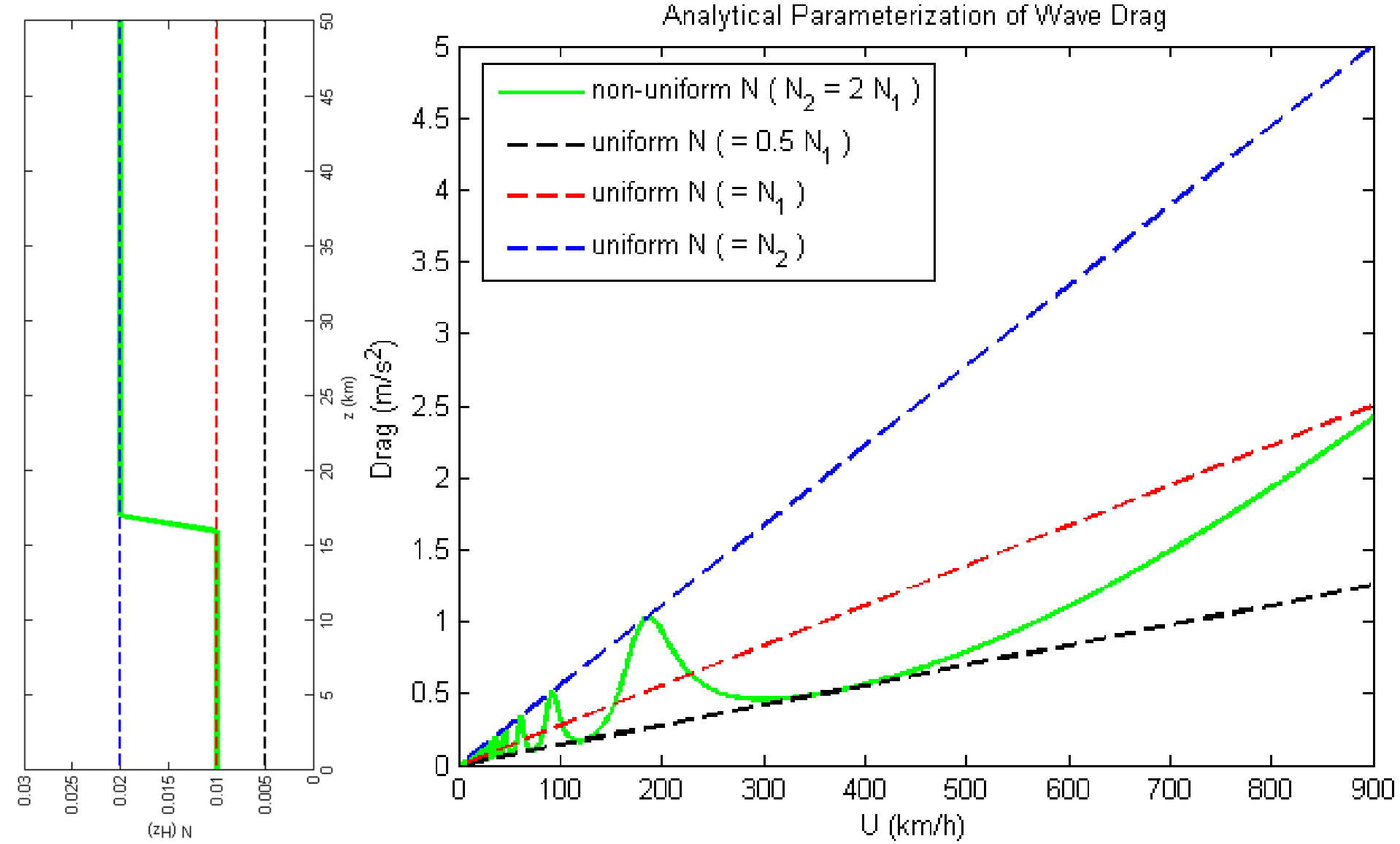
tsunami-induced gravity wave



stationary mountain (=tsunami) lee waves in the frame moving with tsunami

- Stationary solution omits time-dependent details in middle atmosphere.
- Neglects partial back-reflection or assume constant N in the non-uniformly stratified atmosphere.

Why should we take non-uniform stratification into account?



Time-resolving model allowing jump in buoyancy frequency

T-G equation: initial/boundary value problem

Laplace transform

ODE: boundary value problem

$$(\partial_t + U\partial_x)^2 \zeta_{zz} + N^2(z)\zeta_{xx} = 0$$

$$\zeta(t=0^-) = \zeta_t(t=0^-) = 0$$

$$\zeta(z=0) = h(x), \zeta(z=\infty) = 0.$$

$$(s + ikU)^2 \hat{\zeta}^T_{zz} - k^2 N^2(z)\hat{\zeta}^T = 0$$

$$\hat{\zeta}^T(z=0) = \hat{h}(k)/s$$

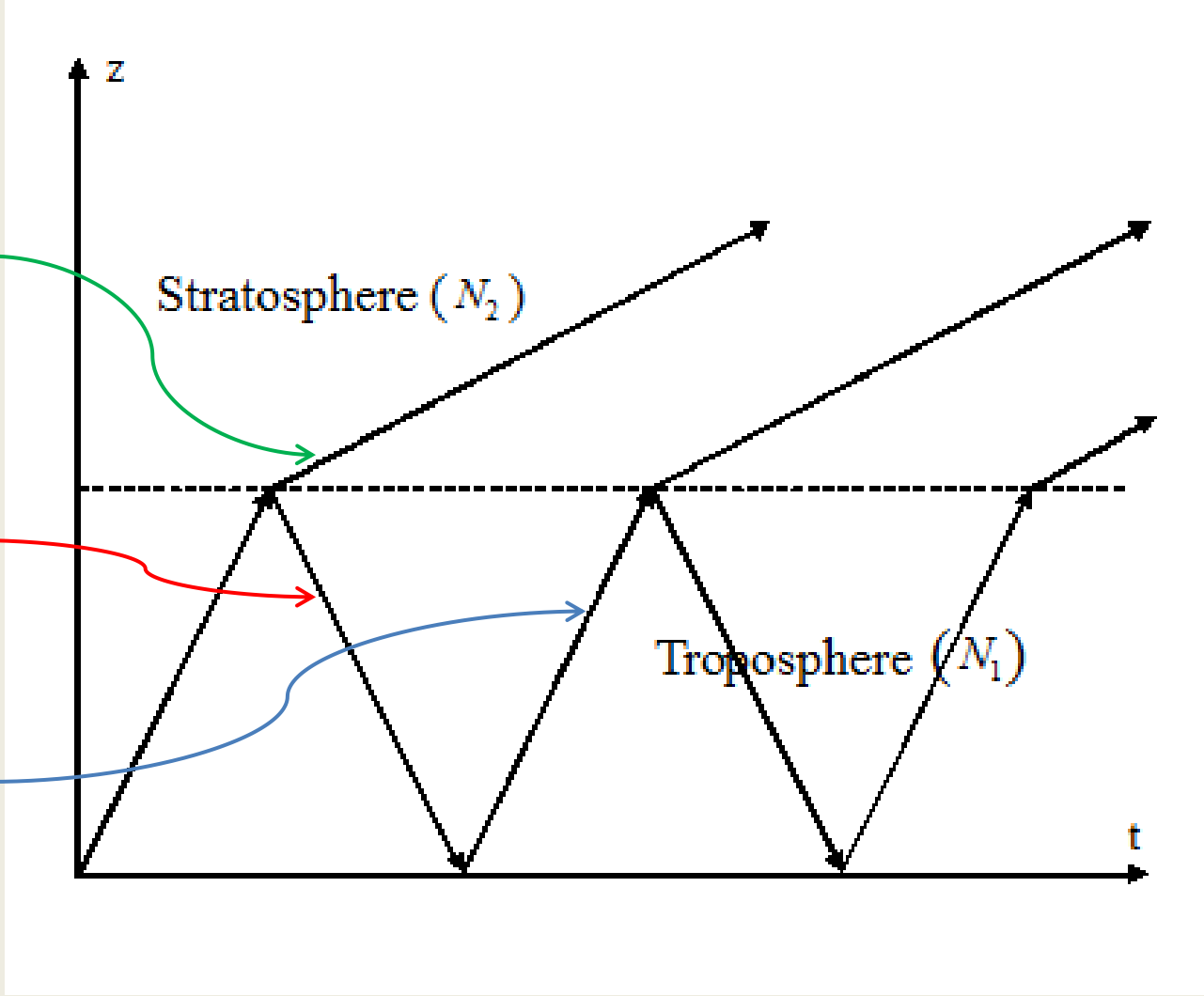
$$\hat{\zeta}^T(z=\infty) = 0$$

Wave-train approximation

+ exp[+i m_2(k) z] term

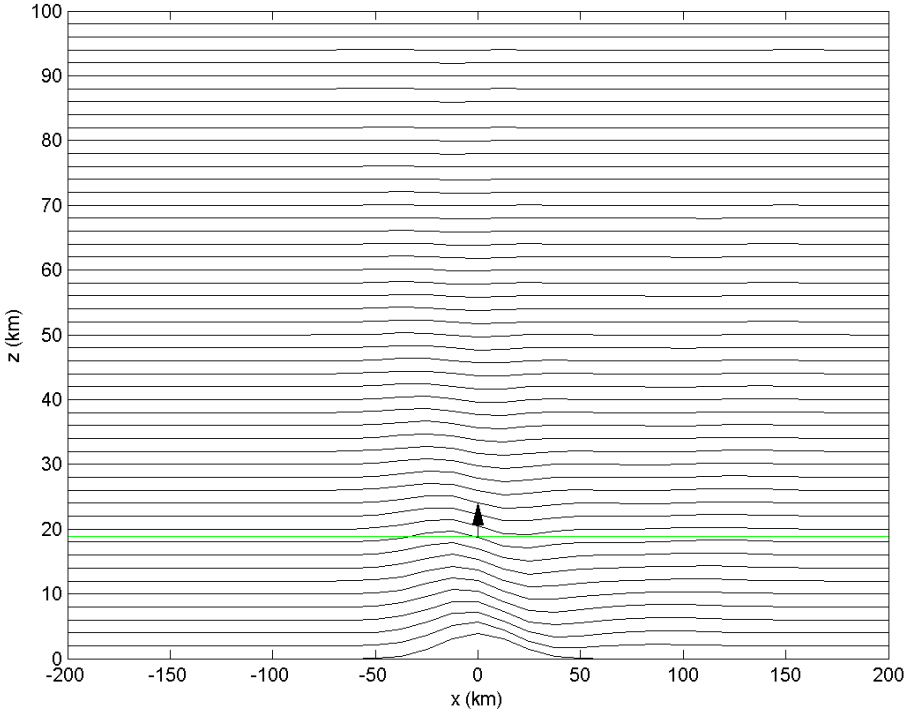
+ exp[-i m_1(k) z] term

+ exp[+i m_1(k) z] term

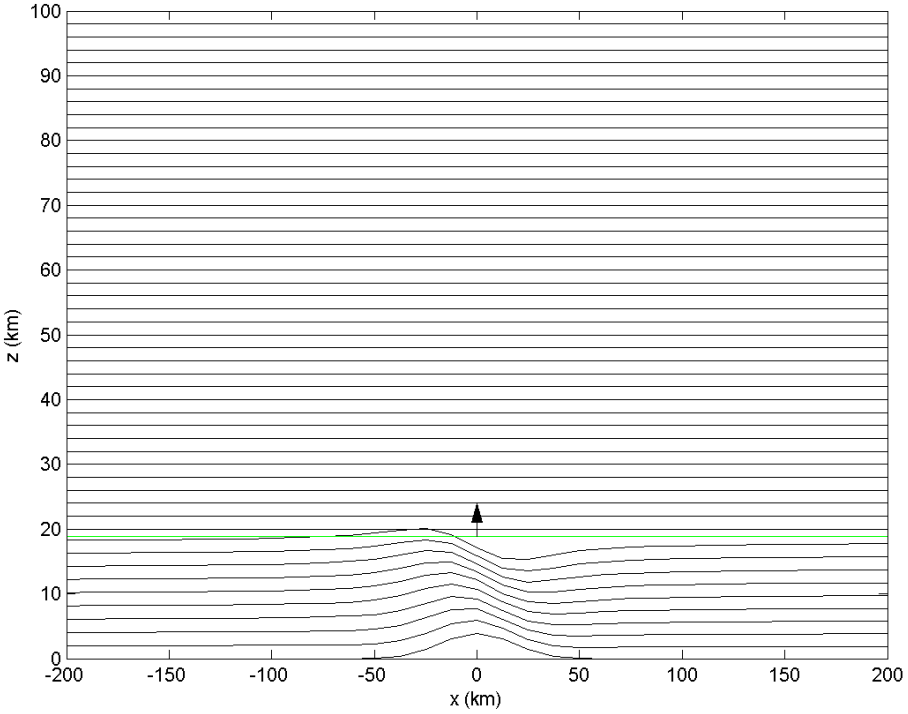


Uniformly stratified atmosphere

Exact solution, t=5min

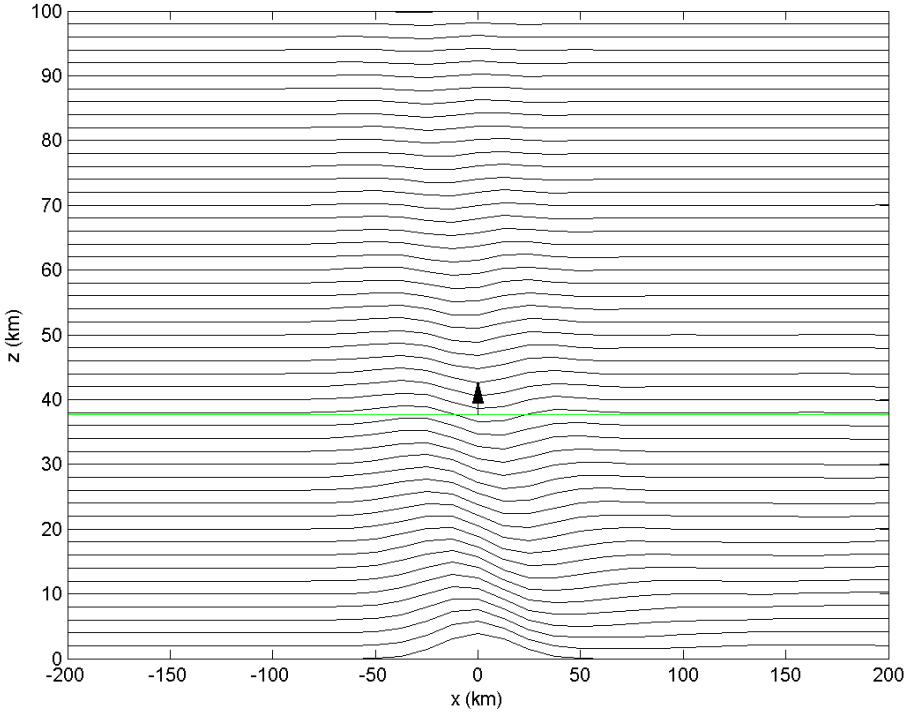


Wave-train approximation, t=5min



Uniformly stratified atmosphere

Exact solution, t=10min



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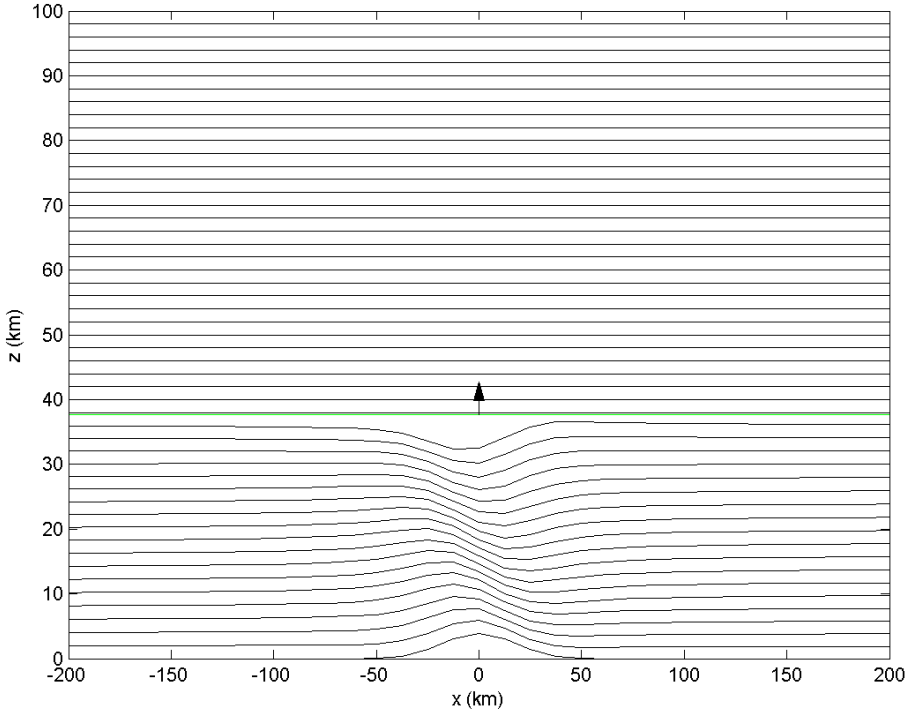
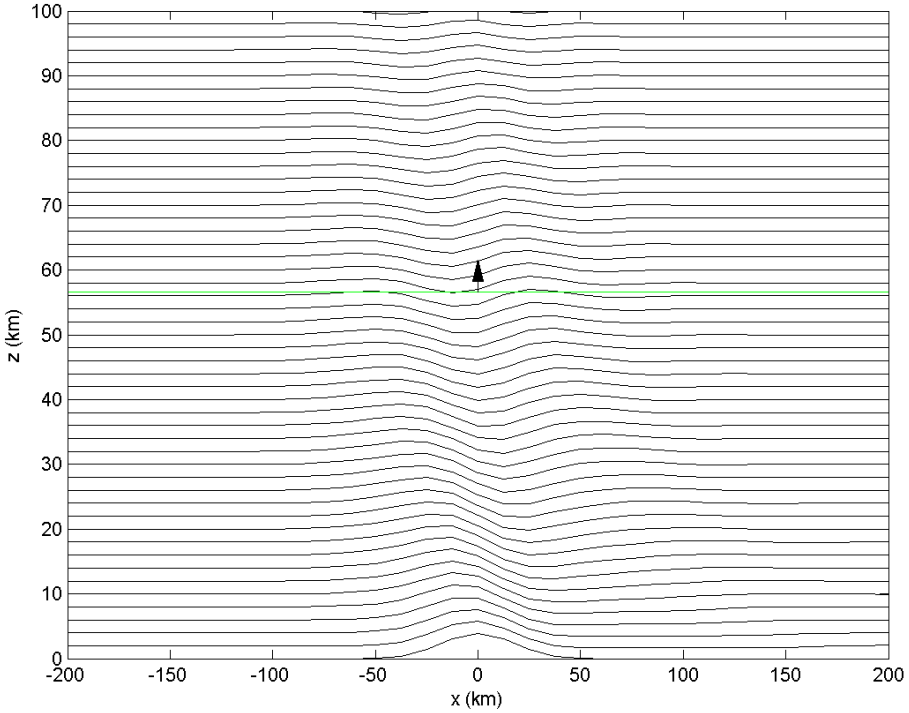


Uniformly stratified atmosphere

Exact solution, t=15min



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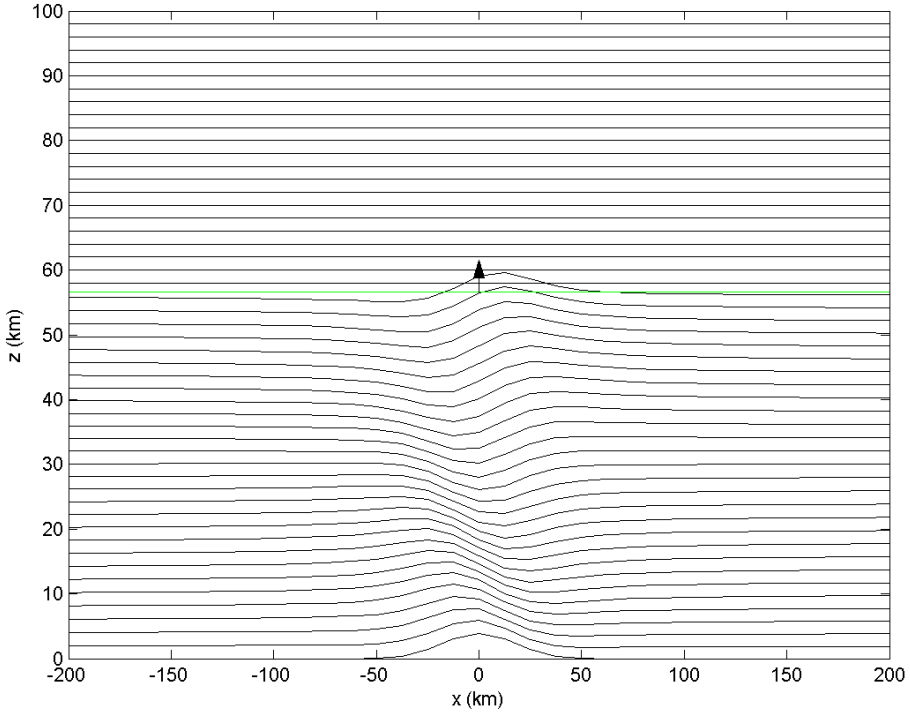

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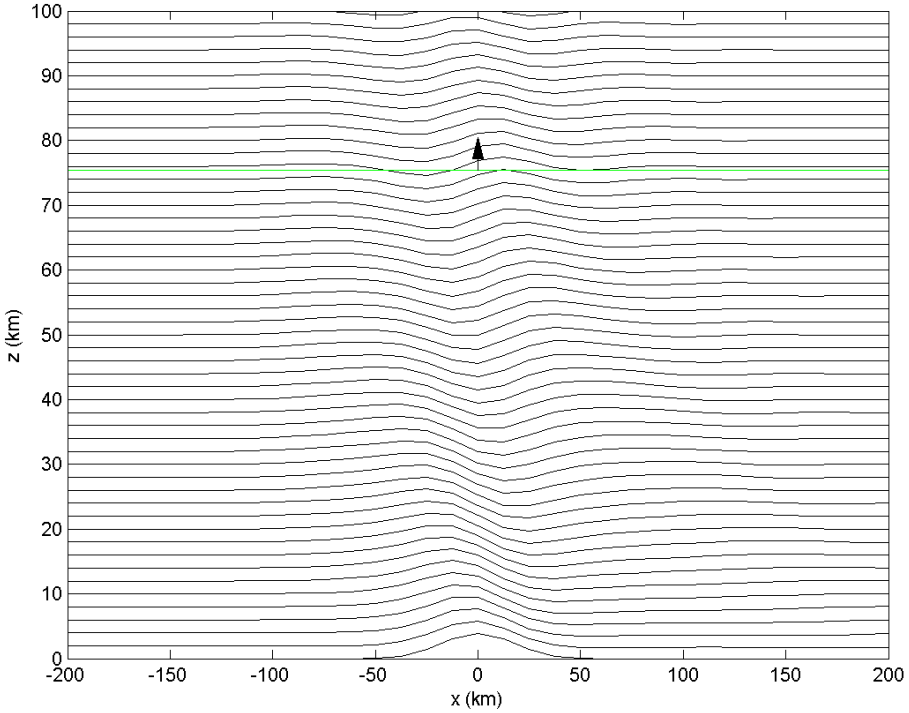
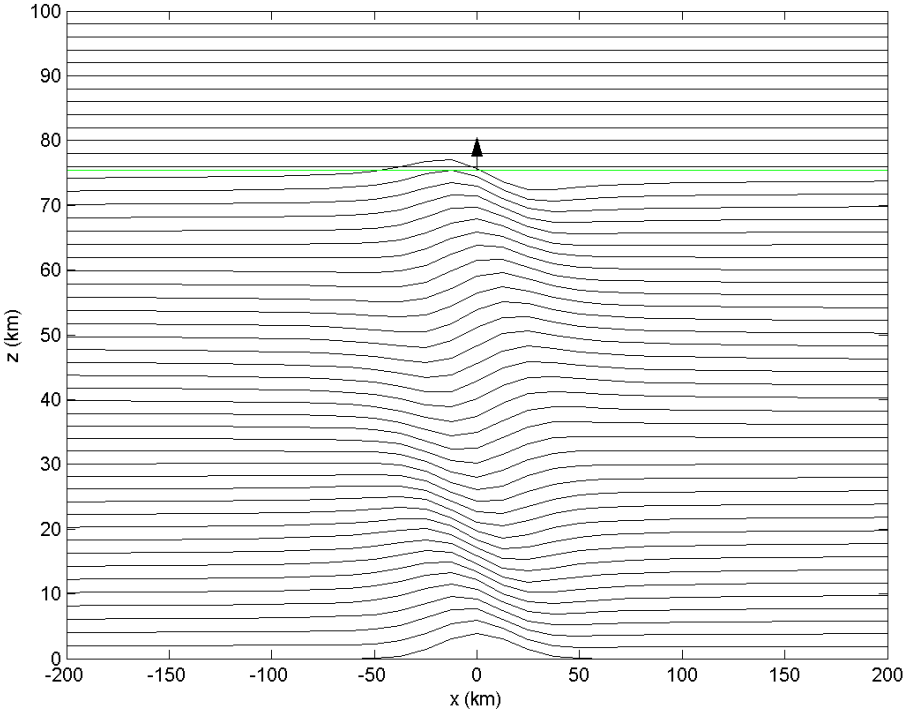
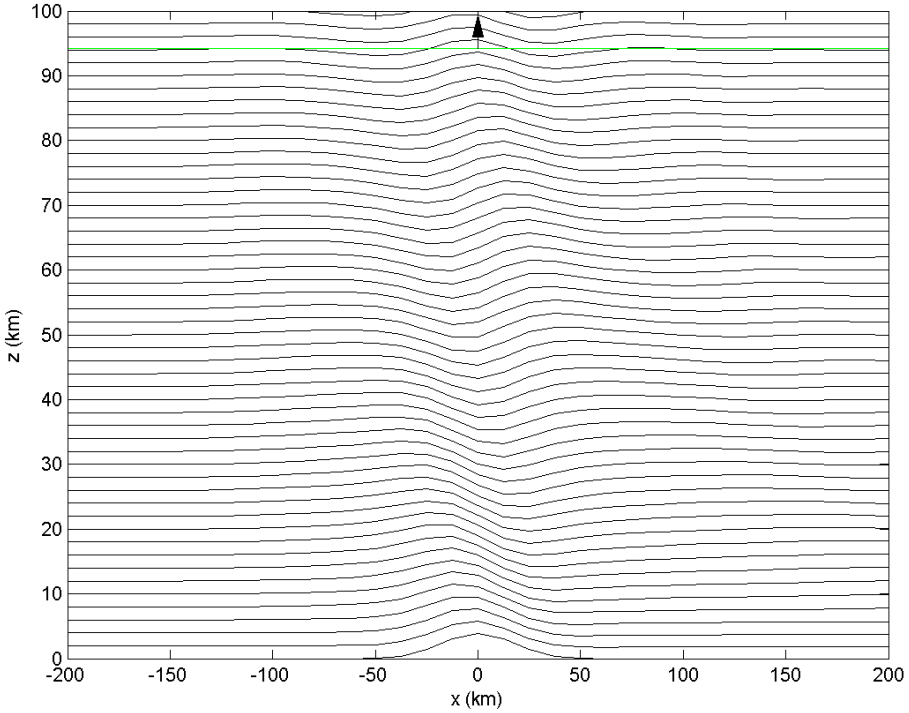


Wave-train approximation, t=20min

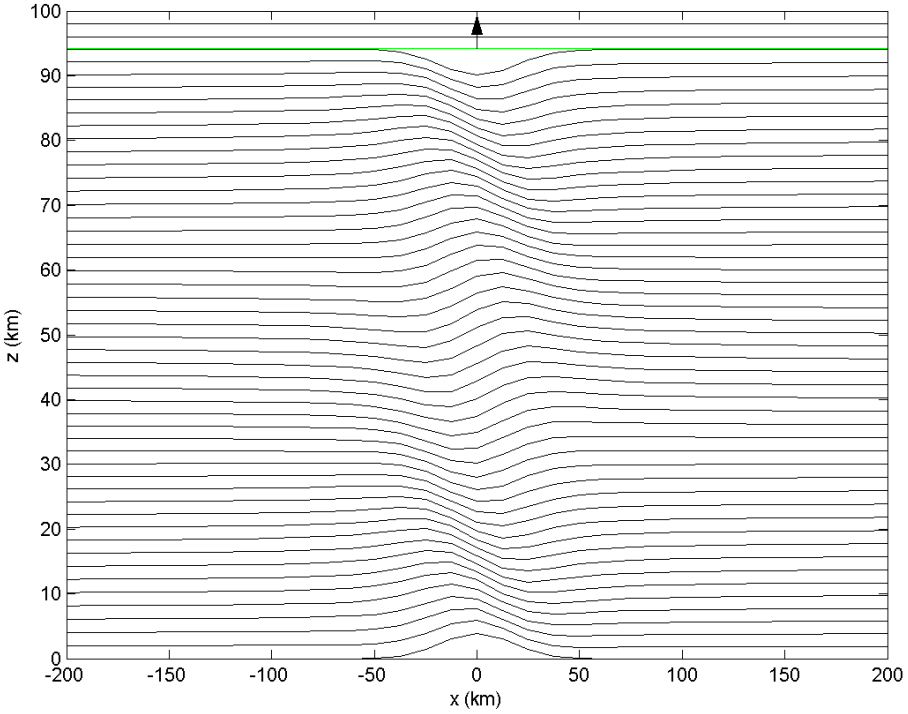


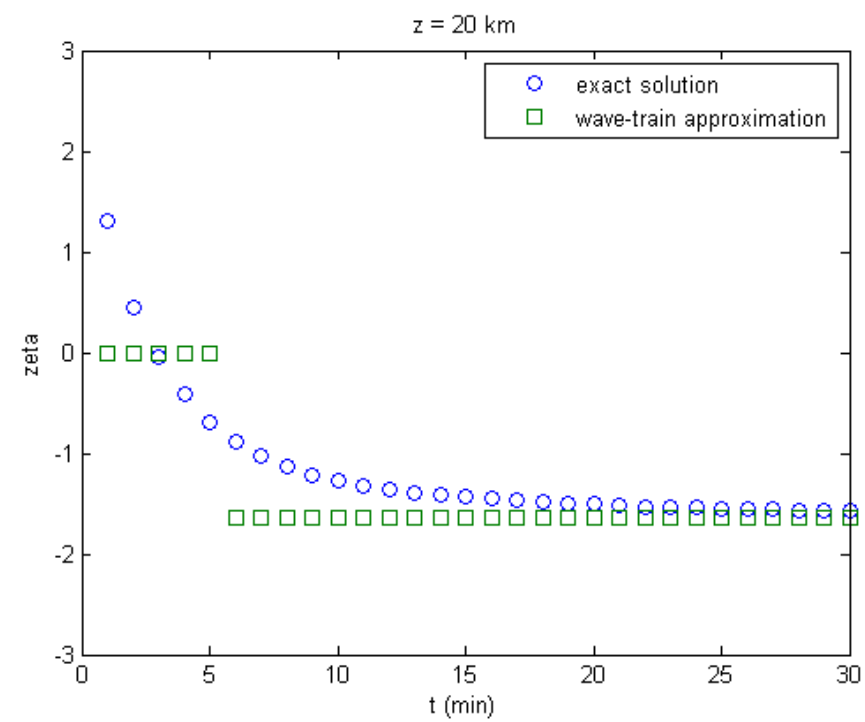
Uniformly stratified atmosphere

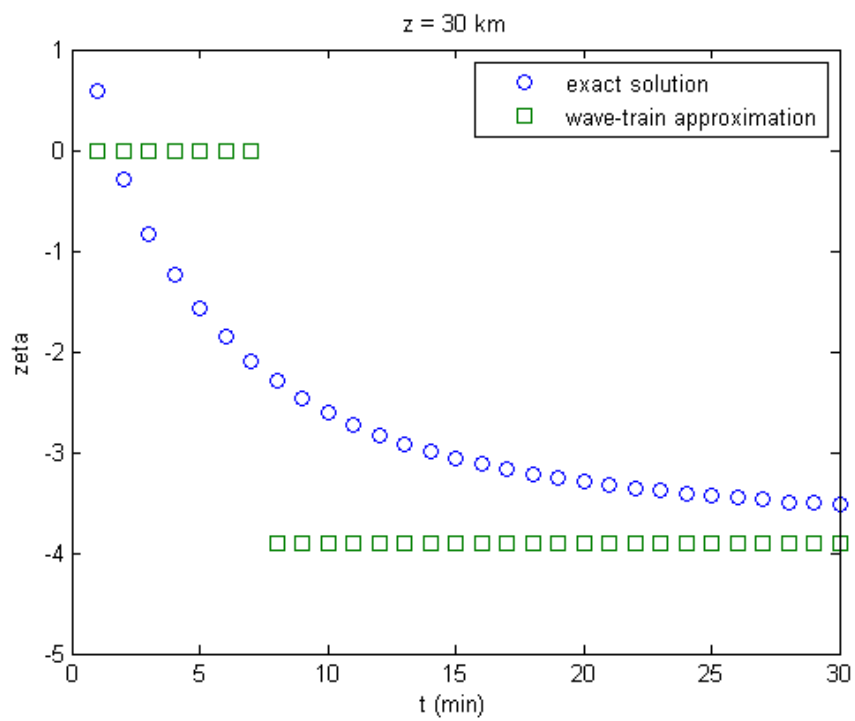
Exact solution, t=25min

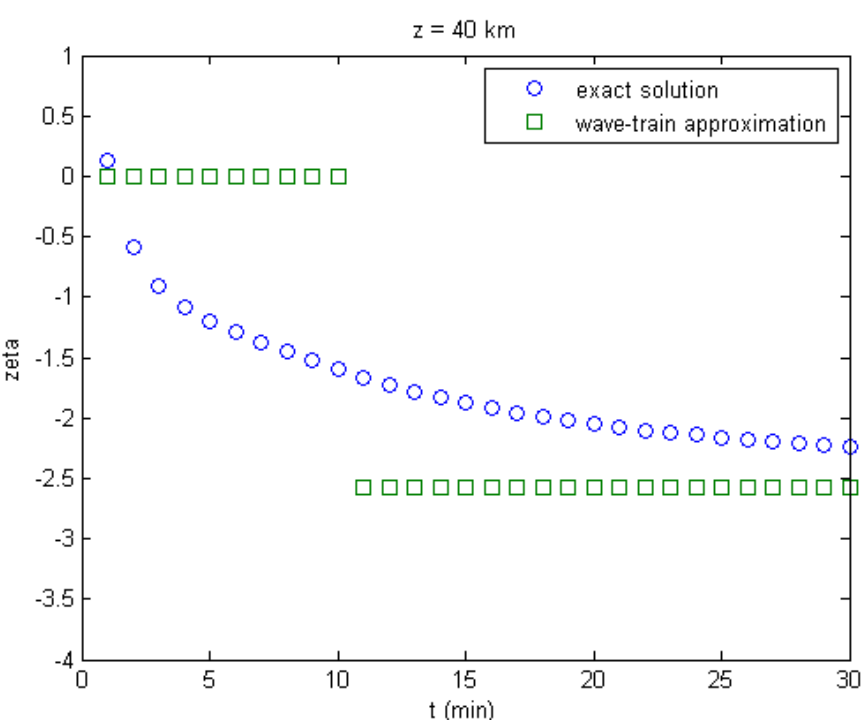


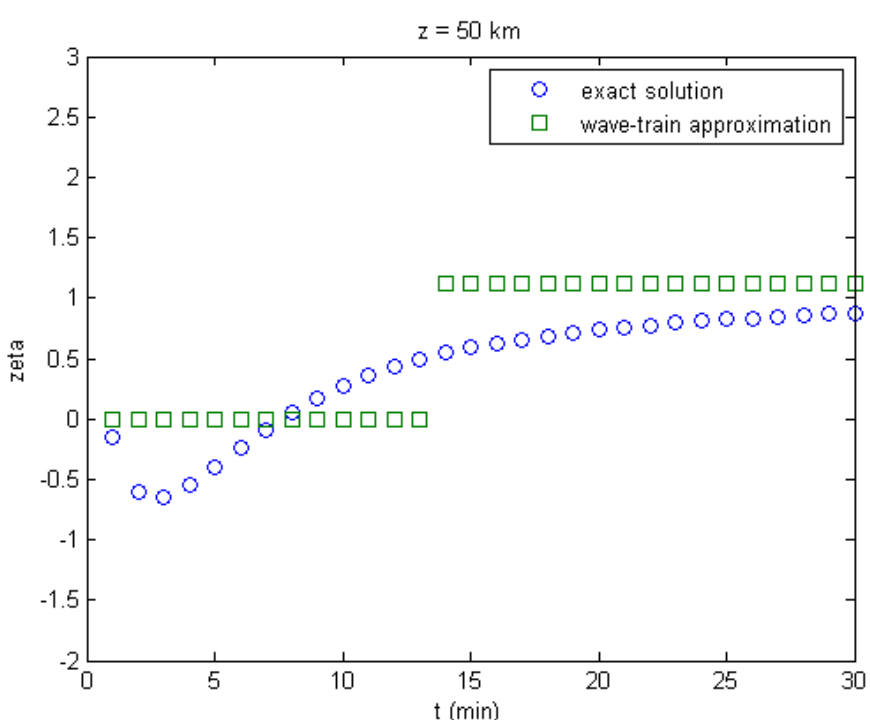
Wave-train approximation, t=25min




z = 20 km
exact solution
wave-train approximation
zeta
t (min)


z = 30 km
exact solution
wave-train approximation
zeta
t (min)


z = 40 km
exact solution
wave-train approximation
zeta
t (min)


z = 50 km
exact solution
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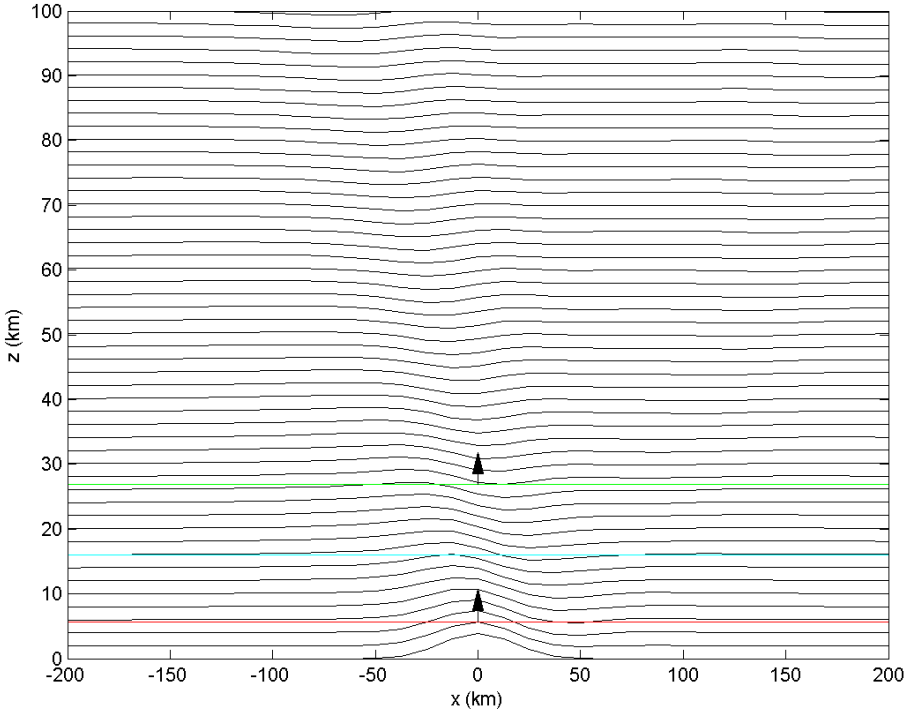
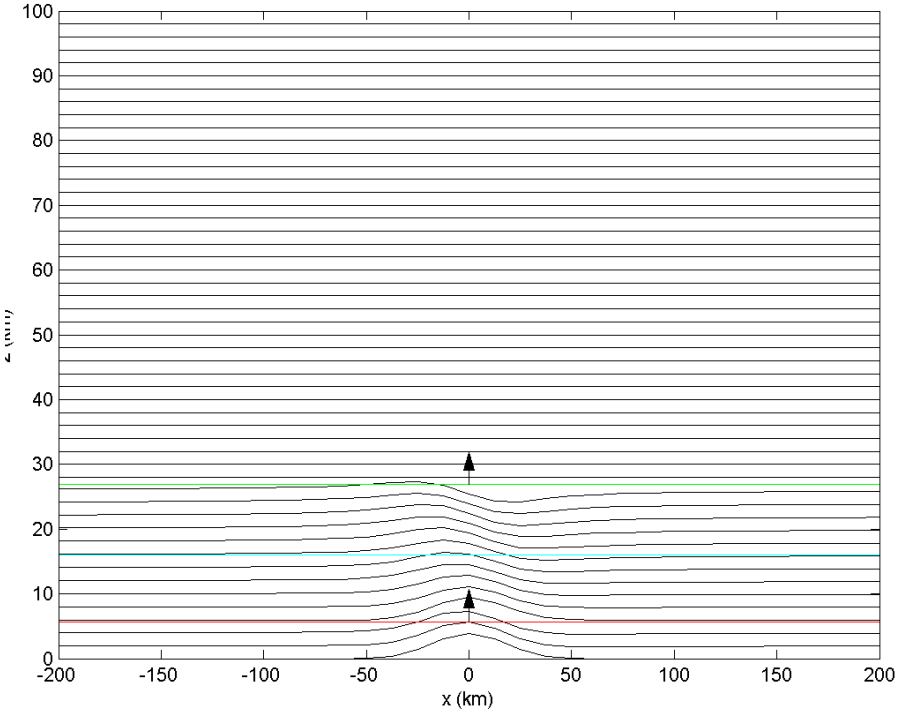
Non-Uniformly stratified atmosphere (N2=2*N1)

Exact solution, t=5min



Wave-train approximation, t=5min



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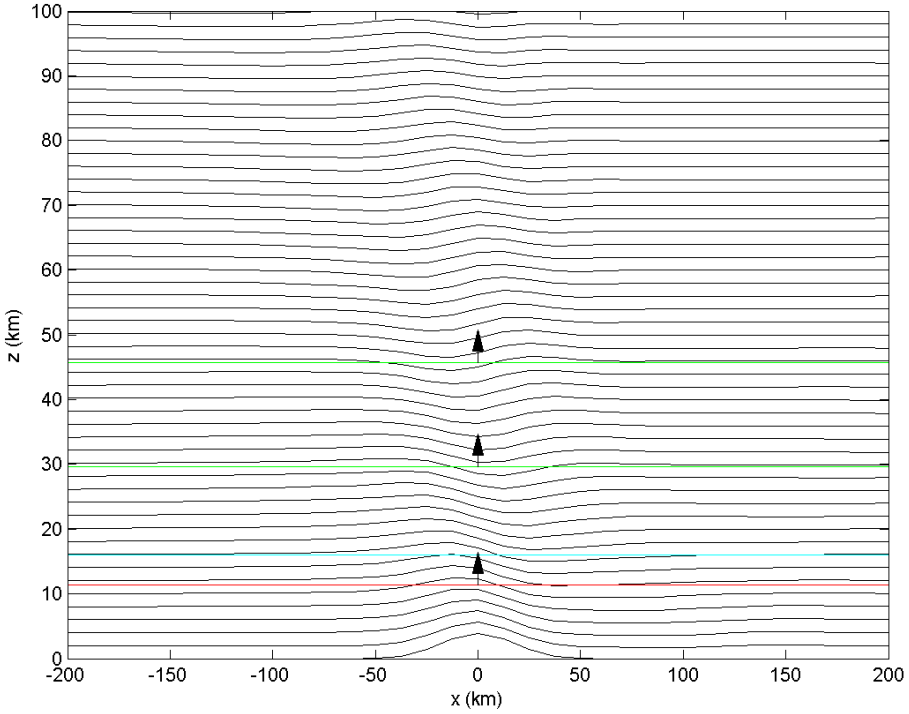
Exact solution, t=10min



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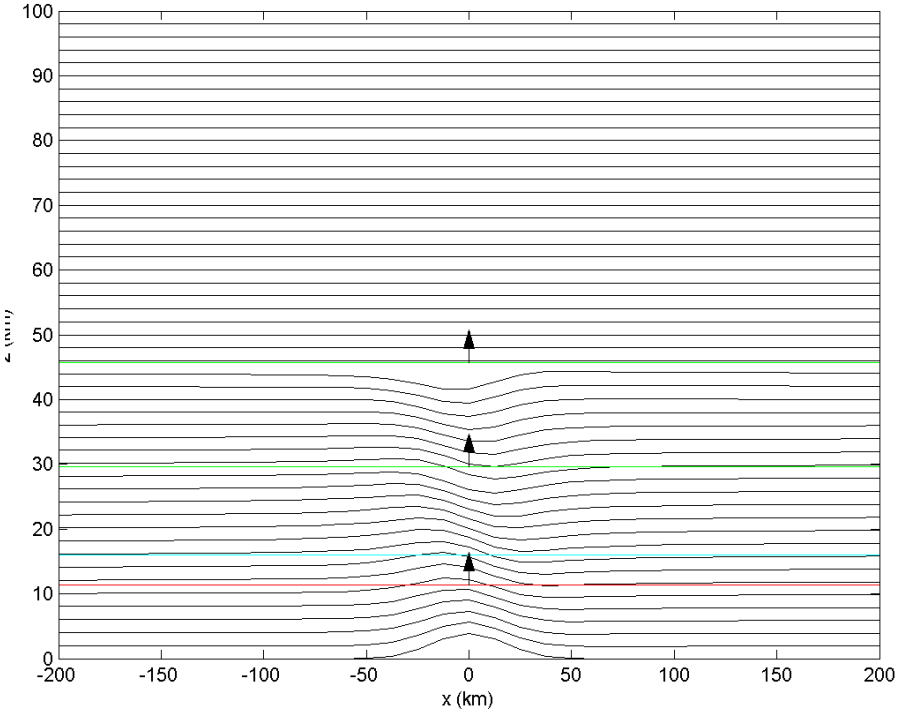

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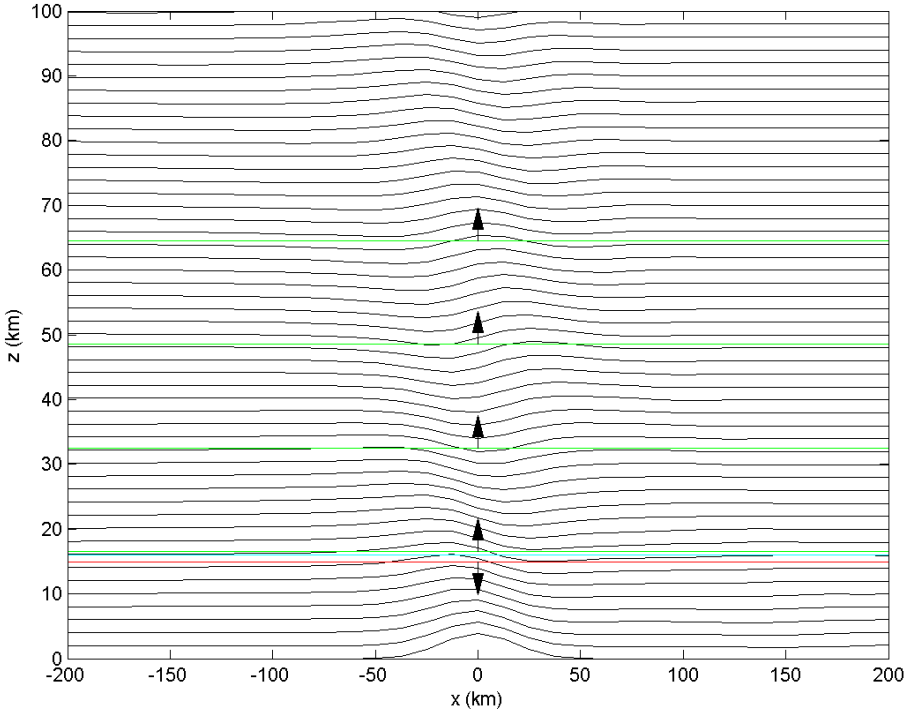

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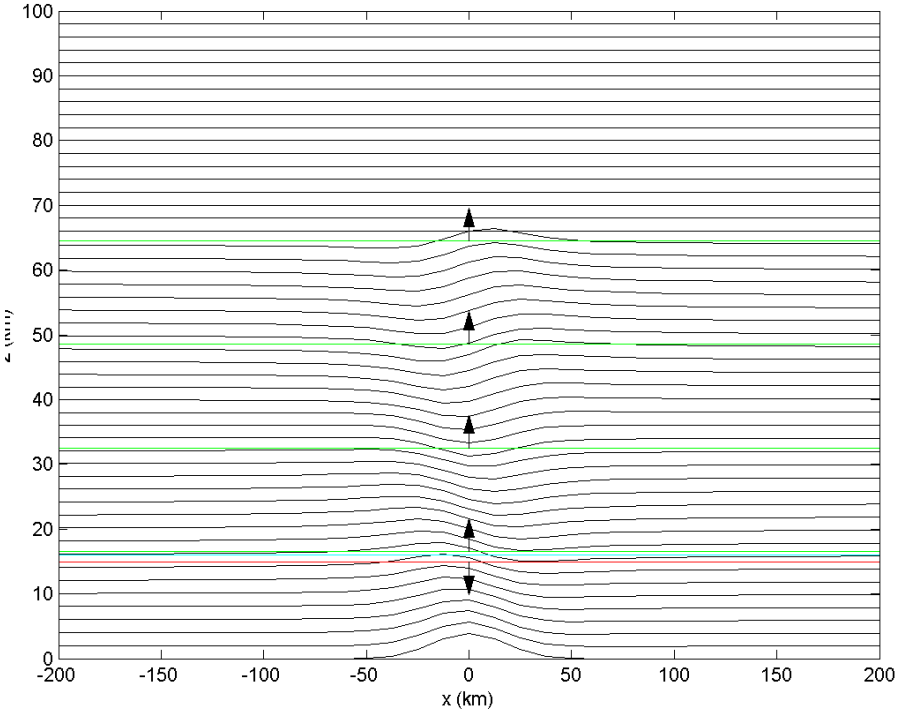

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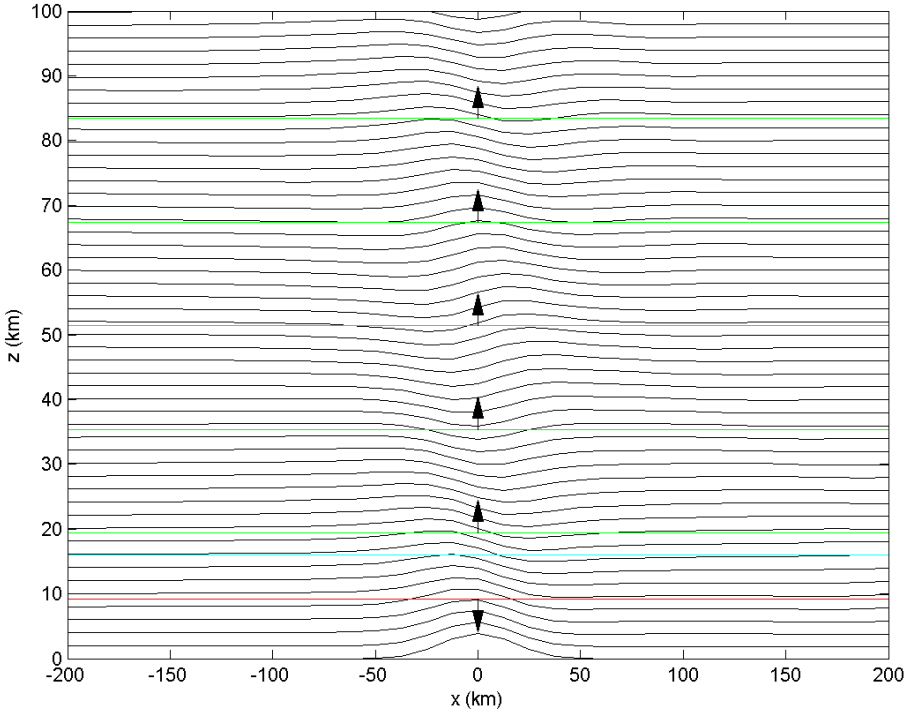

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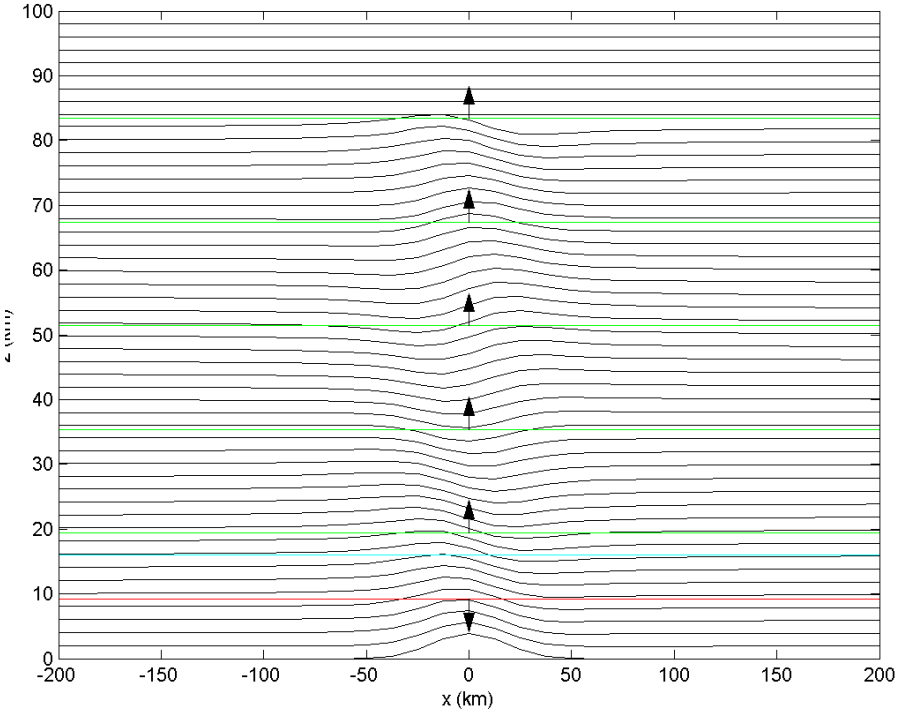

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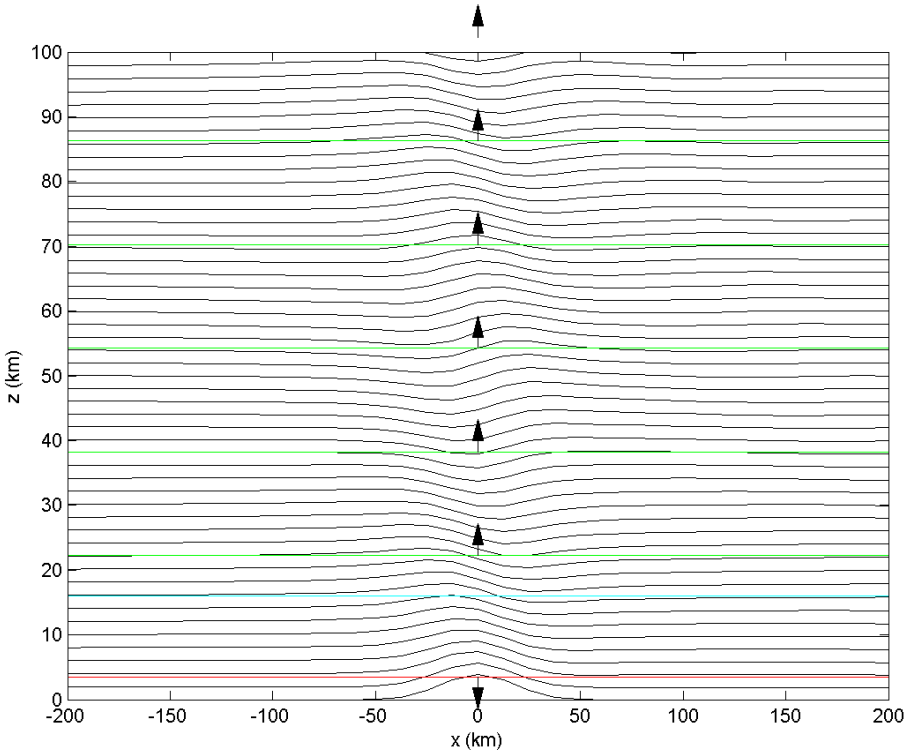

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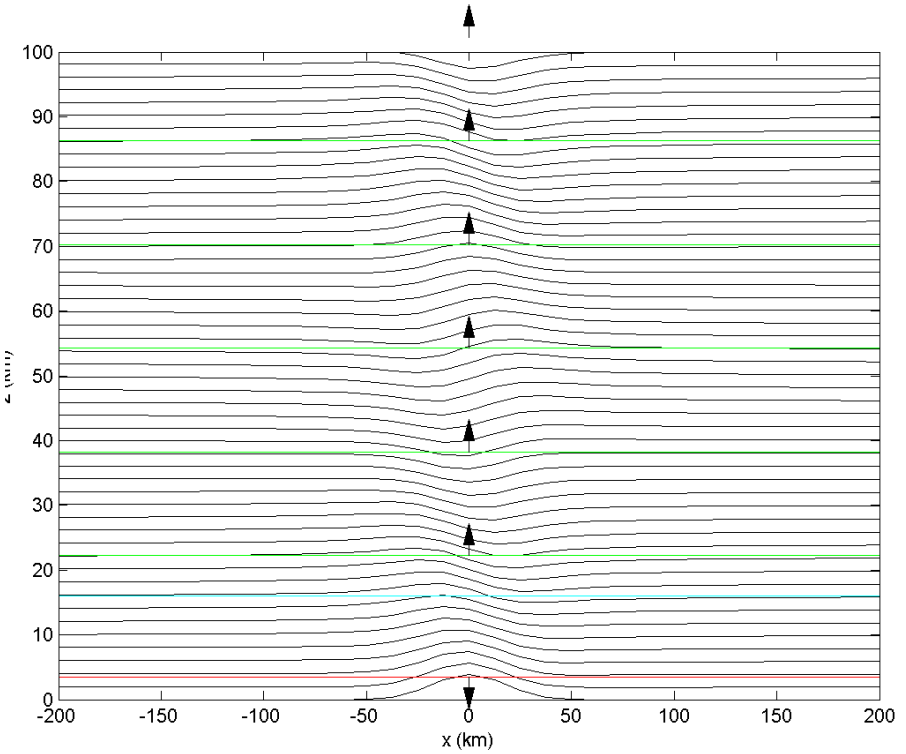

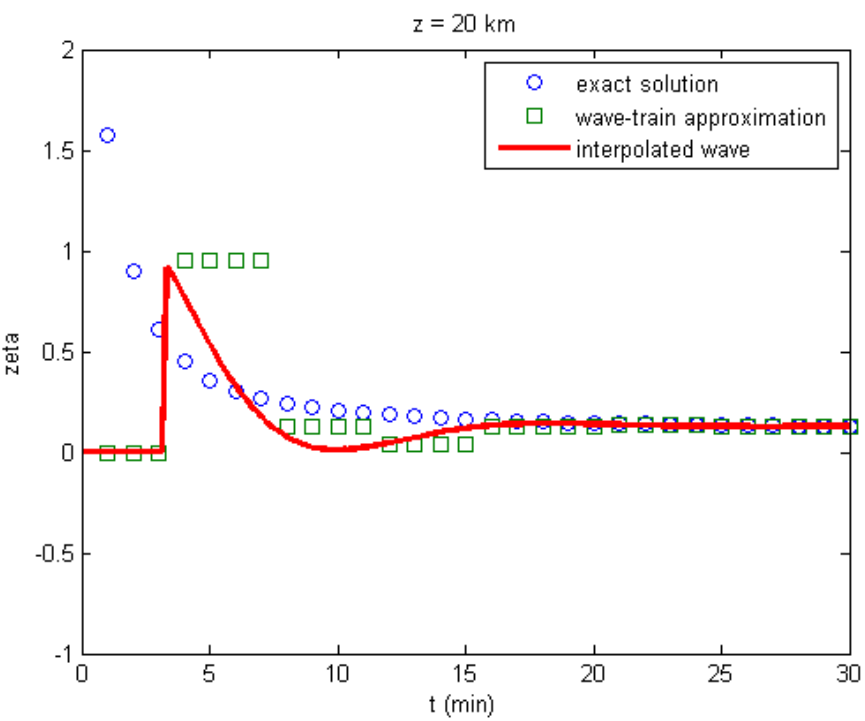
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interpolated wave
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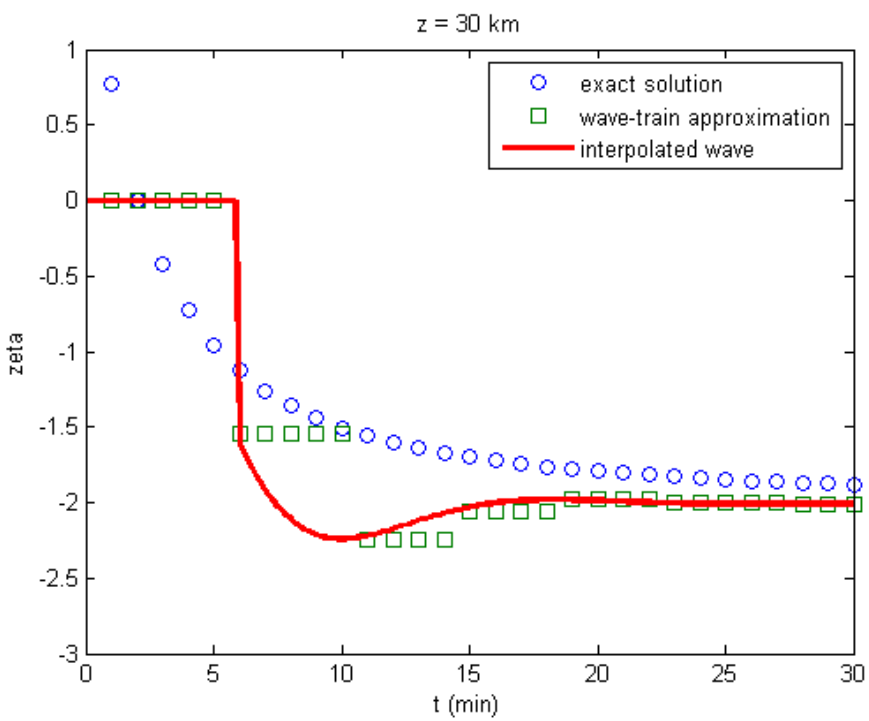
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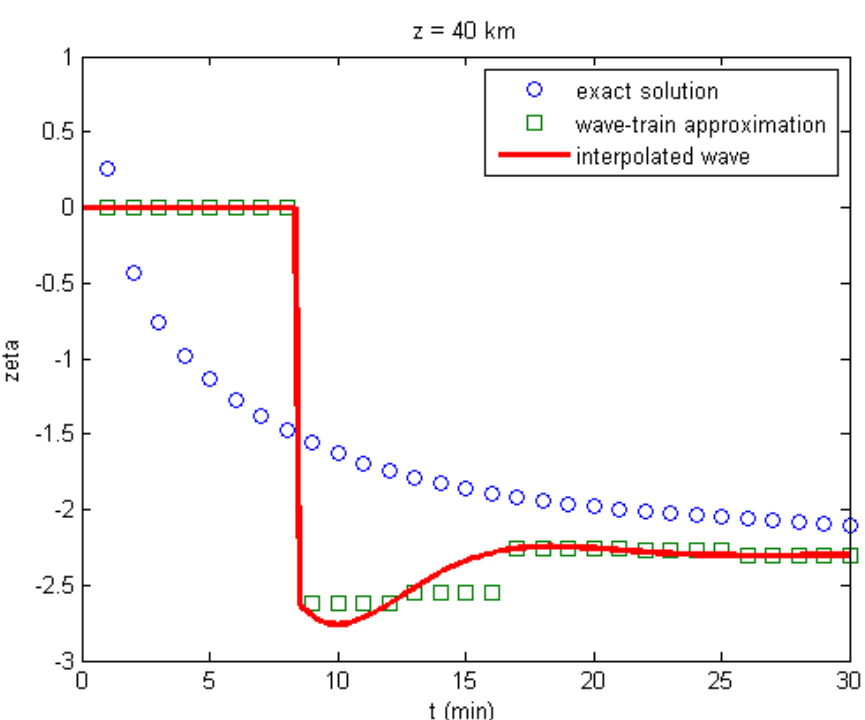
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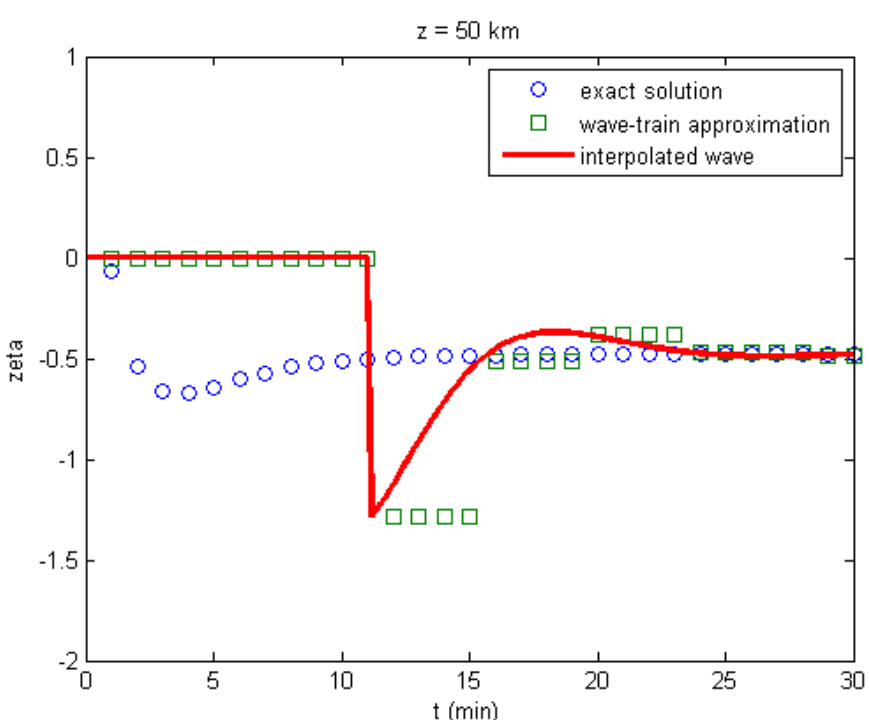
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Summary

- Develop a time-resolving model based on Laplace transform while allowing jump in stratification
- Construct a wave-train approximation including reflections and transmissions
- Recover the gravity wave propagation scheme in the middle and low atmosphere

Thank you for listening ☺