Large Scale Structures in Rotationally Constrained Convection

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Motivation
Open-ocean deep convection is turbulent and constrained by rapid rotation:
• natural Rossby number
  \[ Ro^* = (B/\Phi h^2) \approx 0.1 \text{ - } 0.4 \]

Rotationally constrained convection also important in planetary cores and planetary atmospheres.

Non-hydrostatic QG equations
(Julien, et al, 1998)
• Laboratory experiments and numerical simulations of the primitive equations cannot reach parameter regimes seen in nature.
• Non-hydrostatic quasigeostrophic equations
  • Filter out fast inertial waves
  • Filter out thin Ekman layers.
  • Resolve geostrophically balanced convection.
  • Allow us to reach previously unexplored geostrophic turbulent convection.
  • Equations are analogous to traditional QG
  • Pressure gradient and Coriolis forces are balanced to lowest order.
  • Difference: they allow \( O(1) \) vertical motion.

Rayleigh-Bénard simulations
Classical idealization for convection
As \( RaE^{4/3} \) increases, turbulent regime reached

(\( \Theta \) \( \wedge \): cold)
(\( \Theta \) \( \backslash \): hot)

Unexpected result:
large scale structures form in convective geostrophic regime (\( RaE^{4/3} = 100 \))

Horizontal kinetic energy spectra
• baroclinic spectra consistent with forward cascade of 3D turbulence
• barotropic spectra consistent with forward enstrophy cascade and inverse energy cascade with box-scale condensates of 2D turbulence

Barotropic component
Structure formation understood by barotropic-baroclinic decomposition.
Result: well known barotropic vorticity equation for 2D turbulence, now forced by baroclinic eddies

\[
\begin{align*}
\partial_t \zeta + J[\psi, \zeta] - \partial_z w &= \nabla^2 \zeta, \\
\partial_t w + J[\psi, w] + \partial w \psi &= \frac{RaE^{4/3}}{\sigma} \theta + \nabla^2 w, \\
\partial_t \theta + J[\psi, \theta] + w \partial_z \theta &= \frac{1}{\sigma} \nabla^2 \theta, \\
\partial_t \Theta + \partial_z (w \theta) &= \frac{1}{\sigma} \partial_z \nabla^2 \Theta 
\end{align*}
\]

\( \zeta \): vertical vorticity
\( \psi \): streamfunction
\( w \): vertical velocity
\( \theta \): temperature
\( \Theta \): mean temperature
\( E \): Ekman # \( \ll 1 \)
\( Ra = Ra \text{Le} \# \gg 1 \)
\( \sigma = Prandtl \# \approx O(1) \)
\( \overline{\cdot} \): horizontal average

Conclusions
• Geostrophic turbulent convection has unexpected large-scale self-organization.
• Barotropic-baroclinic decomposition of non-hydrostatic QG equations shows barotropic component behaves as forced 2D turbulence.
• Positive feedback loop between barotropic and baroclinic components.
• Baroclinic component: 3D turbulent cascade
• Barotropic component: 2D turbulent cascades.
• Self-organization of geostrophic convection provides new physical mechanism to explain large scale structures in geophysical systems.