Large Scale Structures in Rotationally Constrained Convection

Antonio Rubio, Keith Julien, and Jeffrey B. Weiss
University of Colorado, Boulder
antonio.rubio@colorado.edu, keith.julien@colorado.edu, jeffrey.weiss@colorado.edu

Motivation

Open-ocean deep convection is turbulent and constrained by rapid rotation:

- natural Rossby number

  $Ro^* = (B/f^3h^2) \sim 0.1 - 0.4$

Rotationally constrained convection also important in planetary cores and planetary atmospheres

Non-hydrostatic QG equations

(Julien, et al, 1998)

- Laboratory experiments and numerical simulations of the primitive equations cannot reach parameter regimes seen in nature.
- Non-hydrostatic quasigeostrophic equations
  - Filter out fast inertial waves
  - Filter out thin Ekman layers.
  - Resolve geostrophically balanced convection
  - Allow us to reach previously unexplored geostrophic turbulent convection.
- Equations are analogous to traditional QG
  - Pressure gradient and Coriolis forces are balanced to lowest order
- Difference: they allow O(1) vertical motion.

$$\partial_t\zeta + J[\psi,\zeta] - \partial_Z w = \nabla_\perp^2\zeta,$$

$$\partial_t w + J[\psi,w] + \partial_Z\psi = \frac{RaE^{4/3}}{\sigma}\theta + \nabla_\perp^2 w,$$

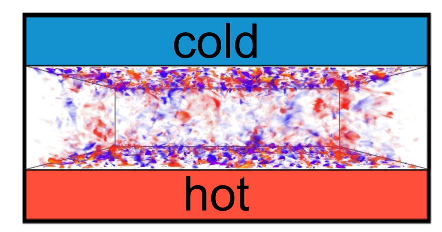
$$\partial_t\theta + J[\psi,\theta] + w\partial_Z\overline{\Theta} = \frac{1}{\sigma}\nabla_\perp^2\theta,$$

$$\partial_\tau\overline{\Theta} + \partial_Z\left(\overline{w\theta}\right) = \frac{1}{\sigma}\partial_Z{}^2\overline{\Theta}$$

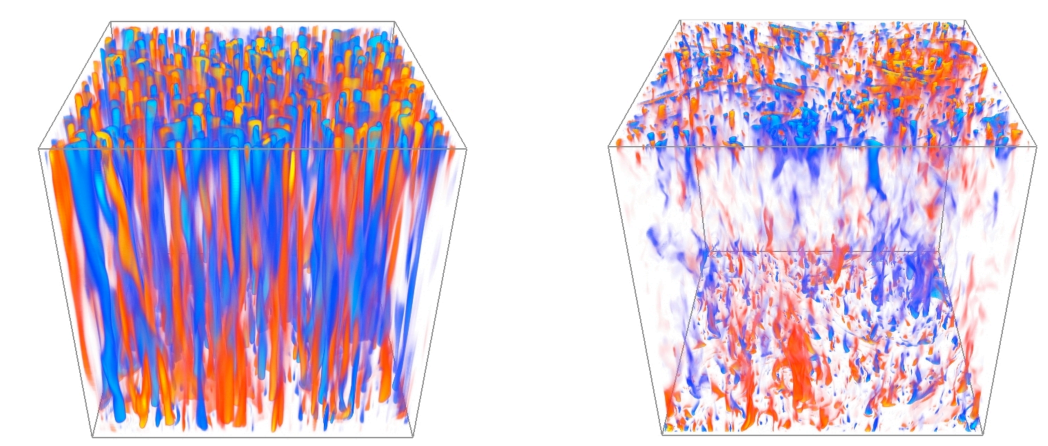
ζ: vertical vorticity	ψ: streamfunction
w: vertical velocity	θ: temperature
Θ: mean temperature	E = Ekman # ≪1
Ra = Rayleigh # ≫1	σ = Prandtl # ~ O(1)
overbar = horizontal average	

Rayleigh-Bénard simulations

Classical idealization for convection

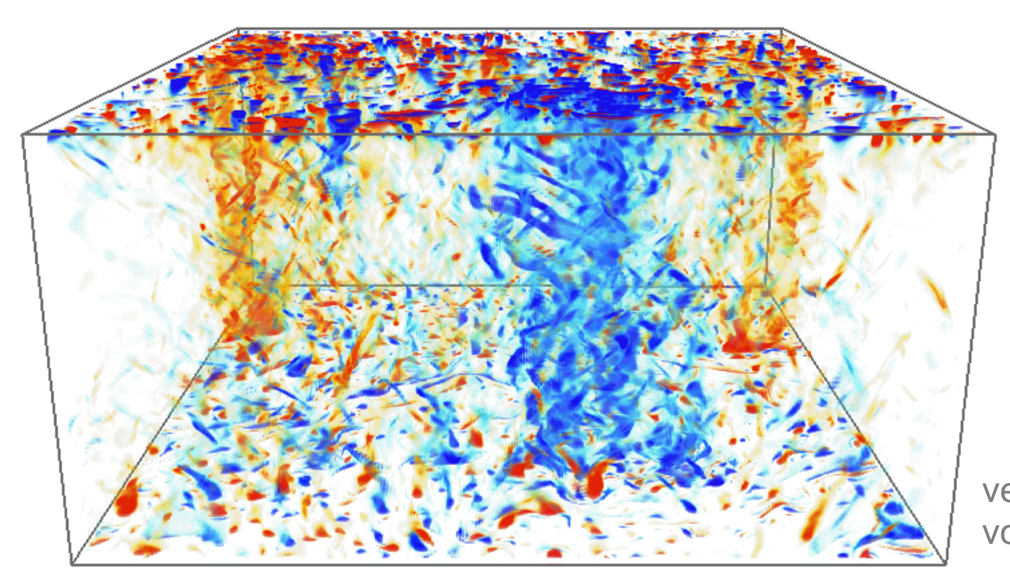


As $RaE^{4/3}$ increases turbulent regime reached



$RaE^{4/3} = 40, \sigma = 1$ $RaE^{4/3} = 100, \sigma = 1$

Unexpected result:
large scale structures form in convective geostrophic regime ($RaE^{4/3} = 100$)



Barotropic component

Structure formation understood by barotropic-baroclinic decomposition.

Result: well known barotropic vorticity equation for 2d turbulence, now forced by baroclinic eddies

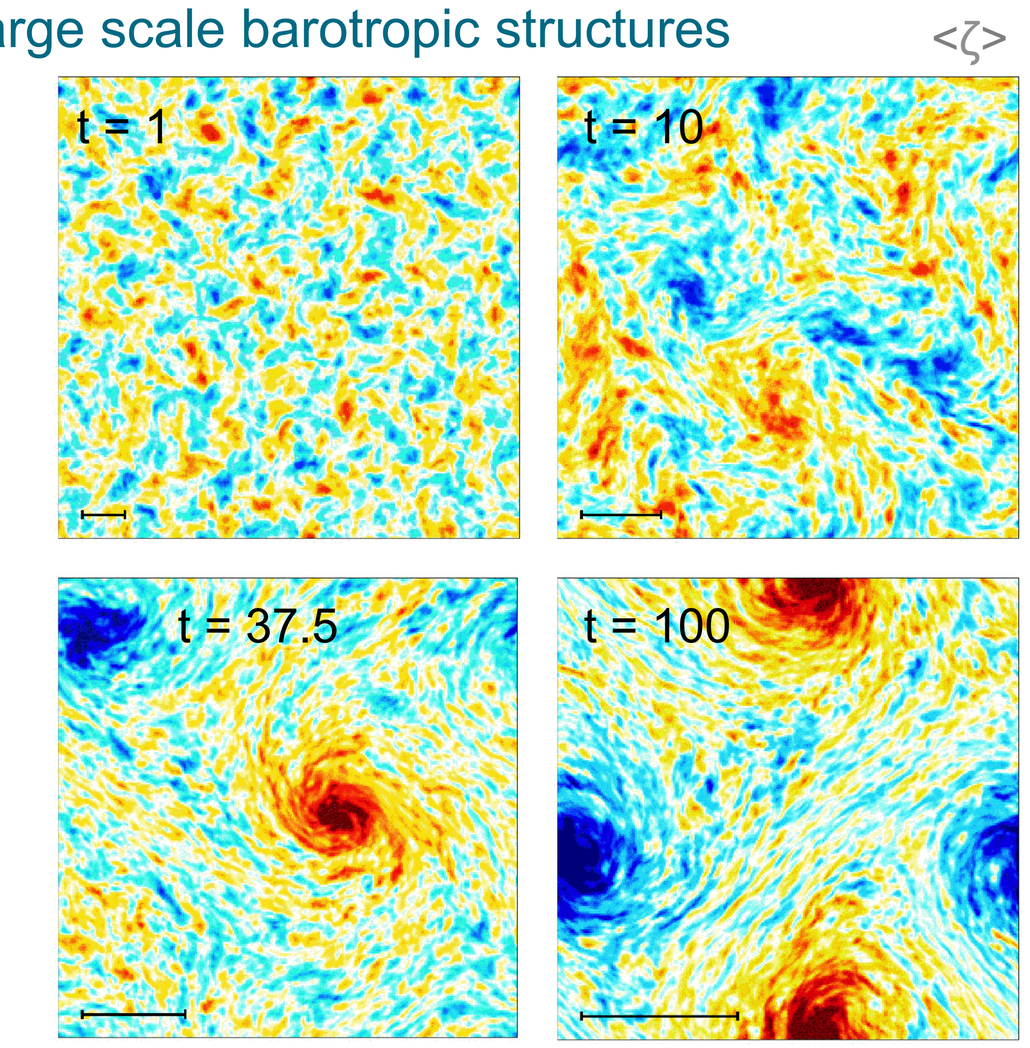
$$\partial_t\langle\zeta\rangle + J[\langle\psi\rangle, \langle\zeta\rangle] = -\langle J[\psi',\zeta']\rangle + \nabla_\perp^2\langle\zeta\rangle$$

<·> = vertical average = barotropic component
·' = vertical fluctuation = baroclinic component

- J[<ψ>,<ζ>]: barotropic-barotropic interaction as in 2d turbulence
- <J[ψ',ζ'] >: baroclinic-baroclinic forcing

Growth of structures

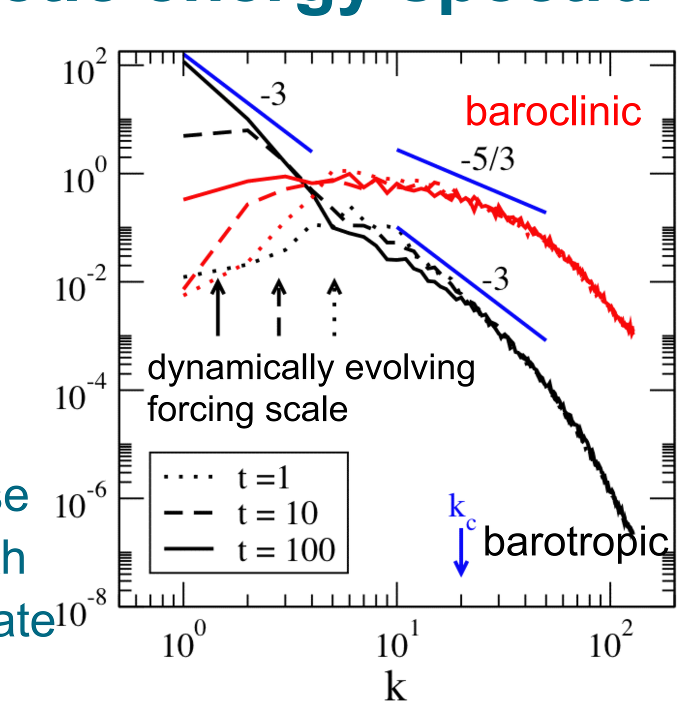
As in 2d turbulence, inverse cascade forms large scale barotropic structures



bars = ½ wavelength of baroclinic forcing scale

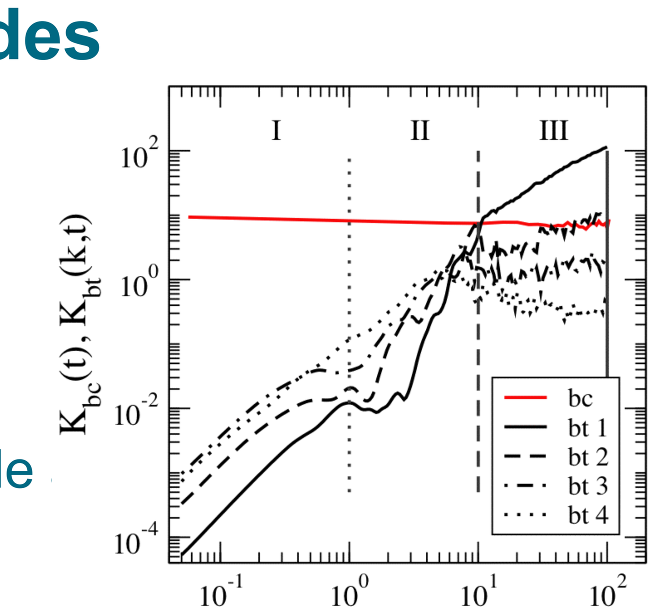
Horizontal kinetic energy spectra

- baroclinic spectra consistent with forward cascade of 3d turbulence
- barotropic spectra consistent with forward enstrophy cascade and inverse energy cascade with box-scale condensate of 2d turbulence



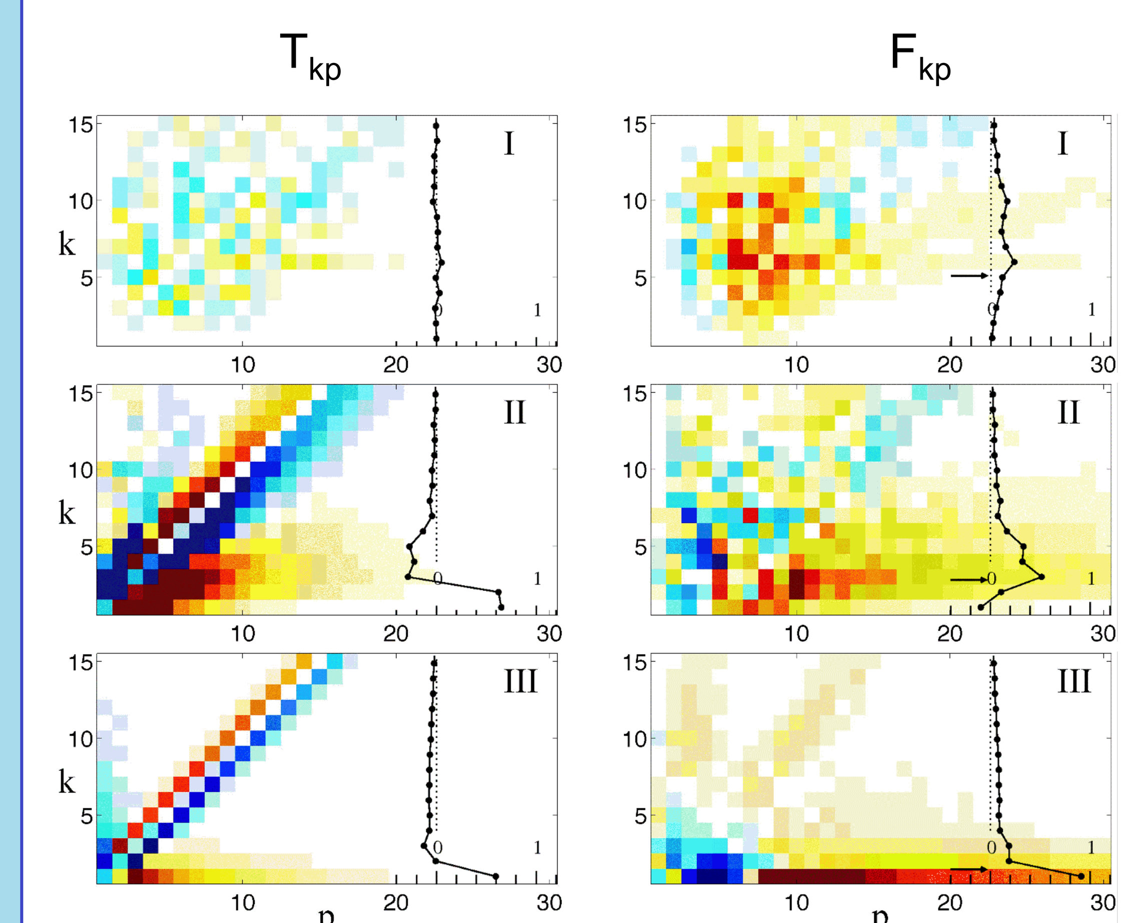
Barotropic modes

- baroclinic kinetic energy ~constant
- barotropic energy grows by several orders of magnitude large scales



Transfer functions

Barotropic kinetic energy in mode k is forced by barotropic-barotropic interactions T_{kp} and baroclinic-baroclinic interactions F_{kp}



color scale: blue negative, red positive

- barotropic inverse cascade
- baroclinic forcing from broad range of scales
- baroclinic forcing scale grows:
  - positive feedback between baroclinic and barotropic components

Conclusions

- Geostrophic turbulent convection has unexpected large-scale self-organization.
- Barotropic-baroclinic decomposition of non-hydrostatic QG equations shows barotropic component behaves as forced 2d turbulence.
- Positive feedback loop between barotropic and baroclinic components.
- Baroclinic component: 3d turbulent cascade
- Barotropic component: 2d turbulent cascades
- Self-organization of geostrophic convection provides new physical mechanism to explain large scale structures in geophysical systems.