Is Nonlinearity Important in Non-Breaking Mountain Waves?

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Introduction

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- Clearest example of nonlinear process is wave breaking, which results in gravity wave drag
- Are nonlinear effects important without wave breaking?

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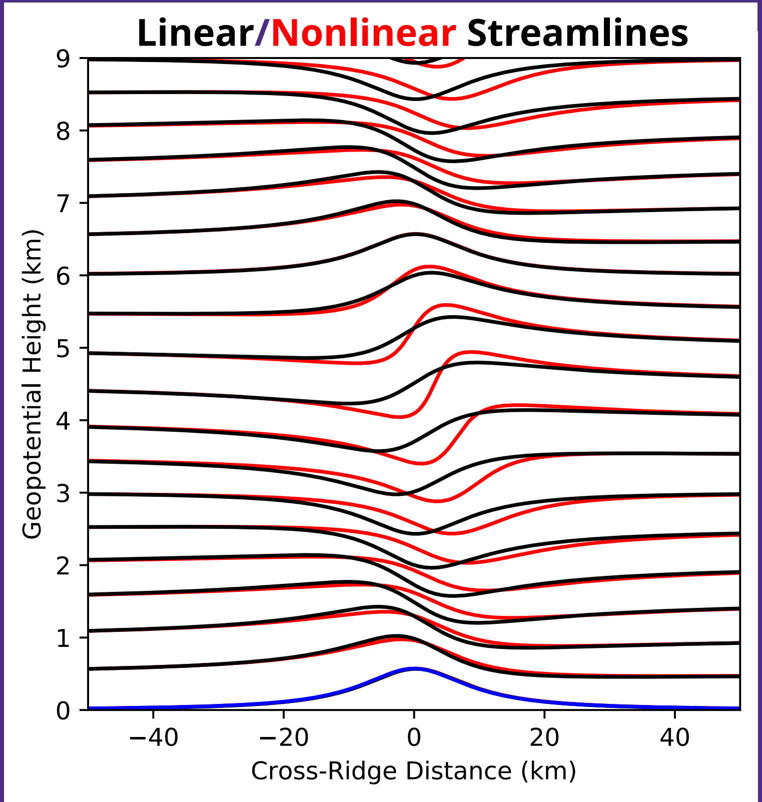
- Results in only minor differences between the linear and nonlinear solutions



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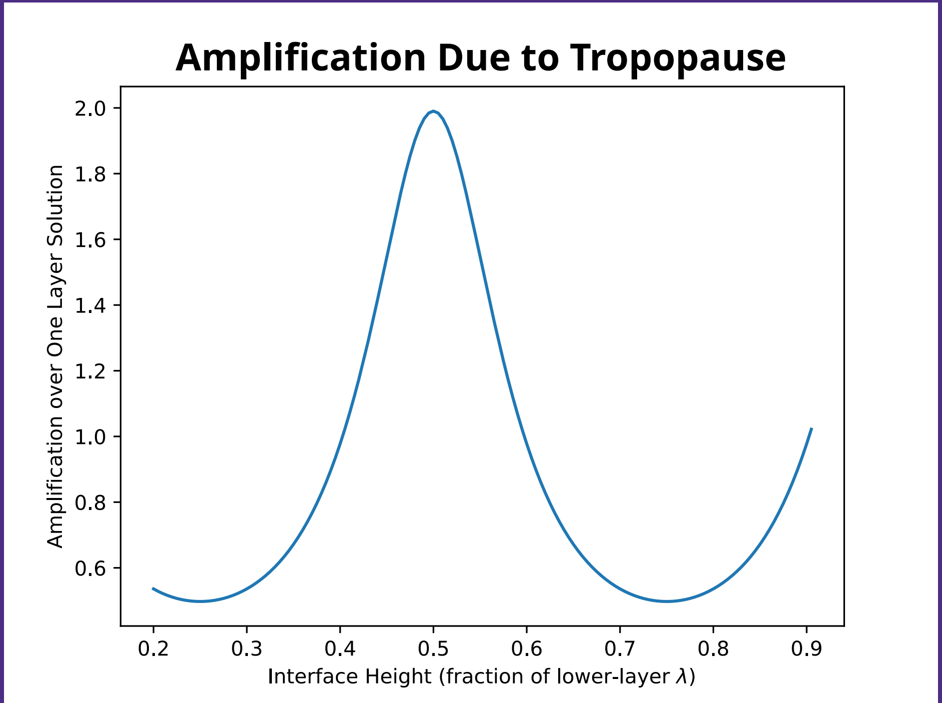

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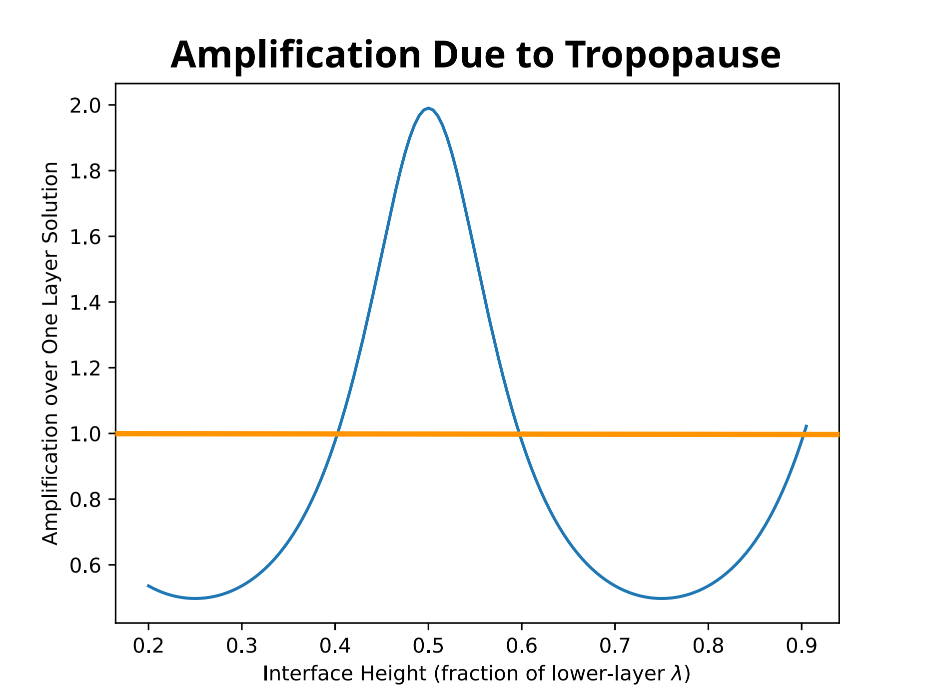
- ***In most GWD parameterizations, this curve would be a constant!***



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- However, semi-analytic methods are only available for constant *N* and *U* layers with infinitesimal transition layers between

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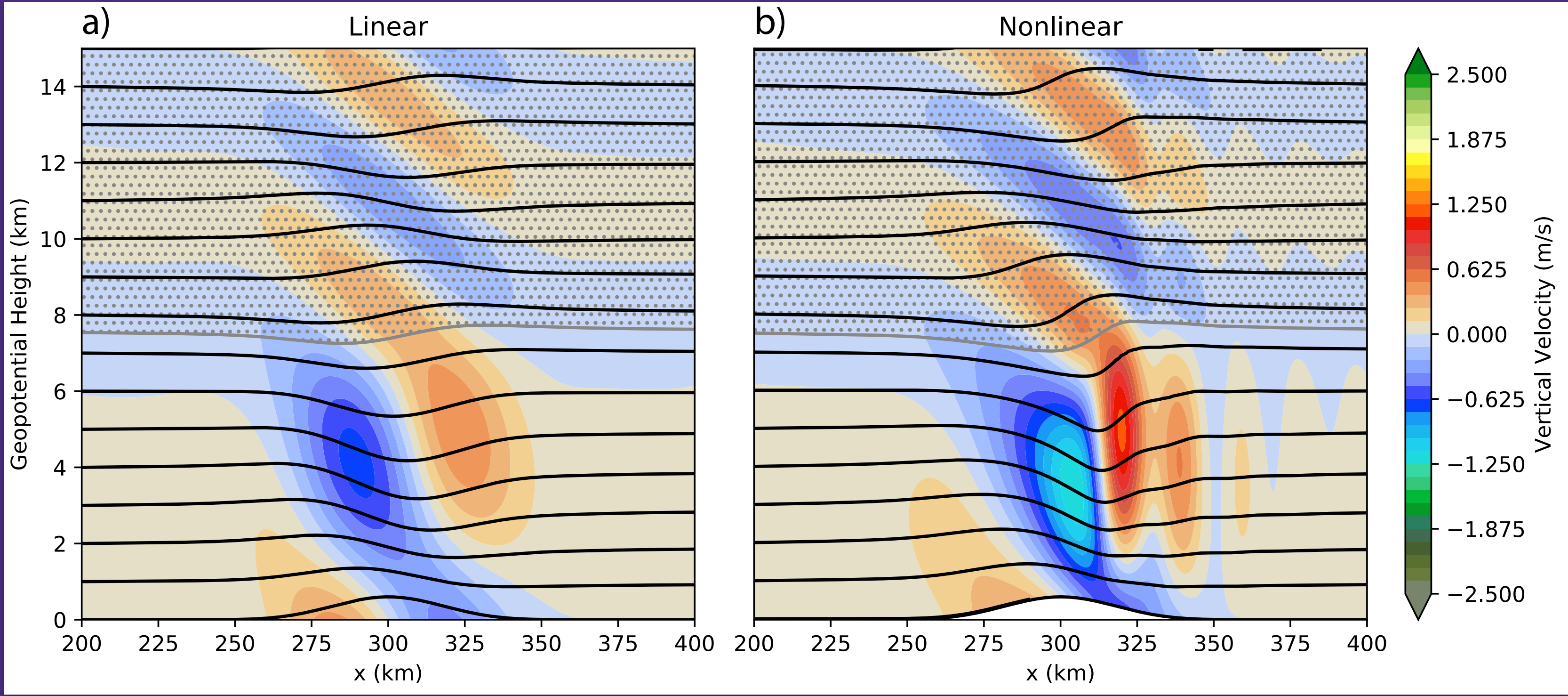
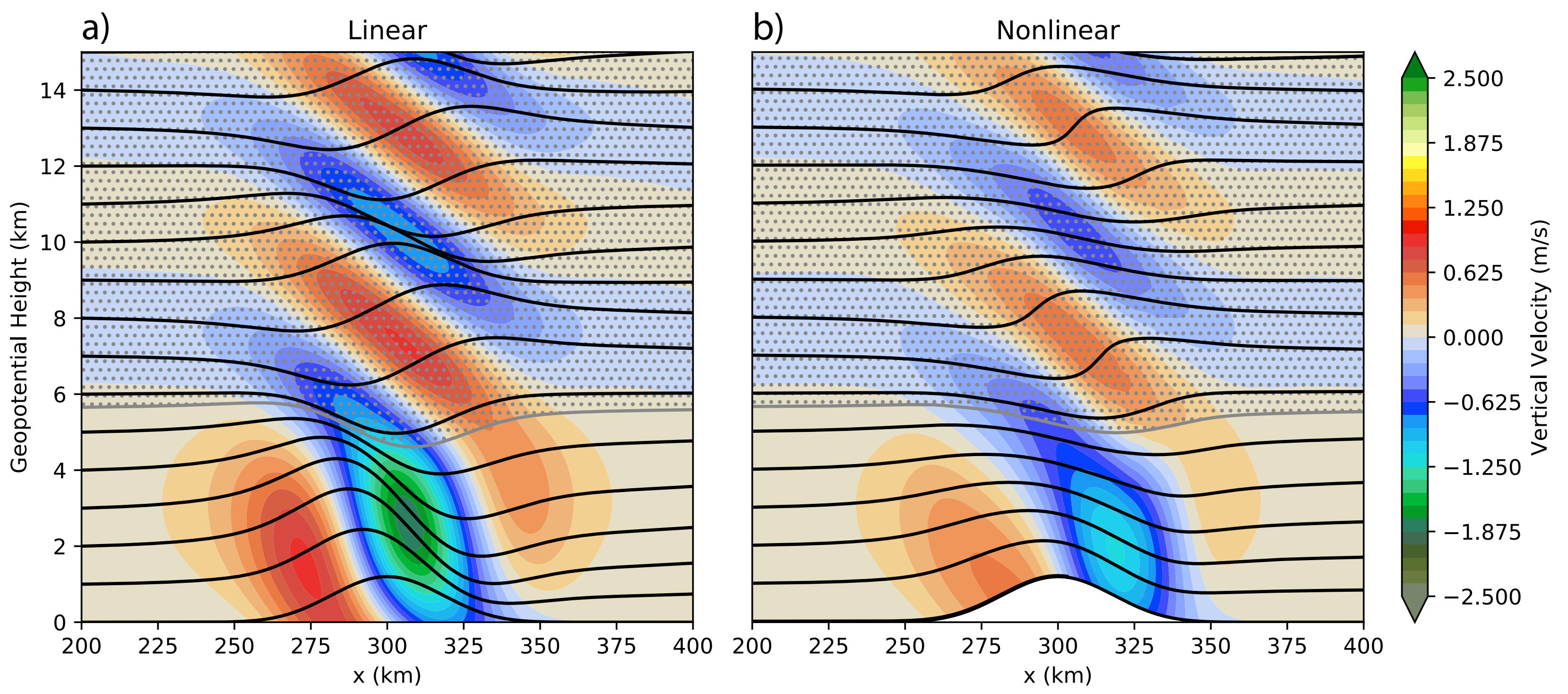
- Take meso12 and linearize advection terms and boundary conditions
- Run both versions of the model in 2D Boussinesq configuration and compare the differences

Two-Layer Nonlinear Amplification

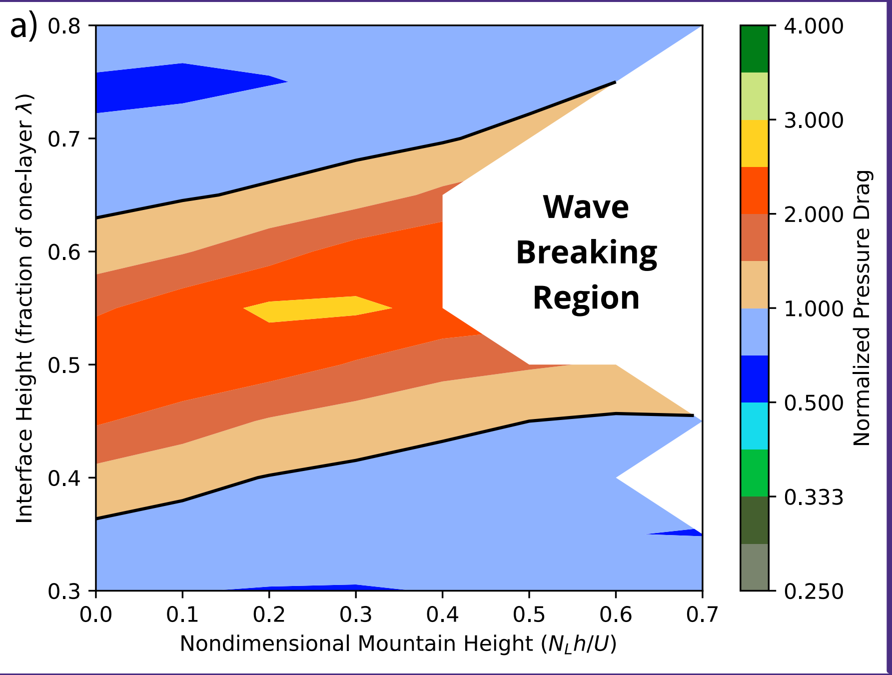


Two-Layer Nonlinear Deamplification



Two-Layer Constant Winds

Normalized Pressure Drag



Maximum: 2.59

Minimum: 0.64

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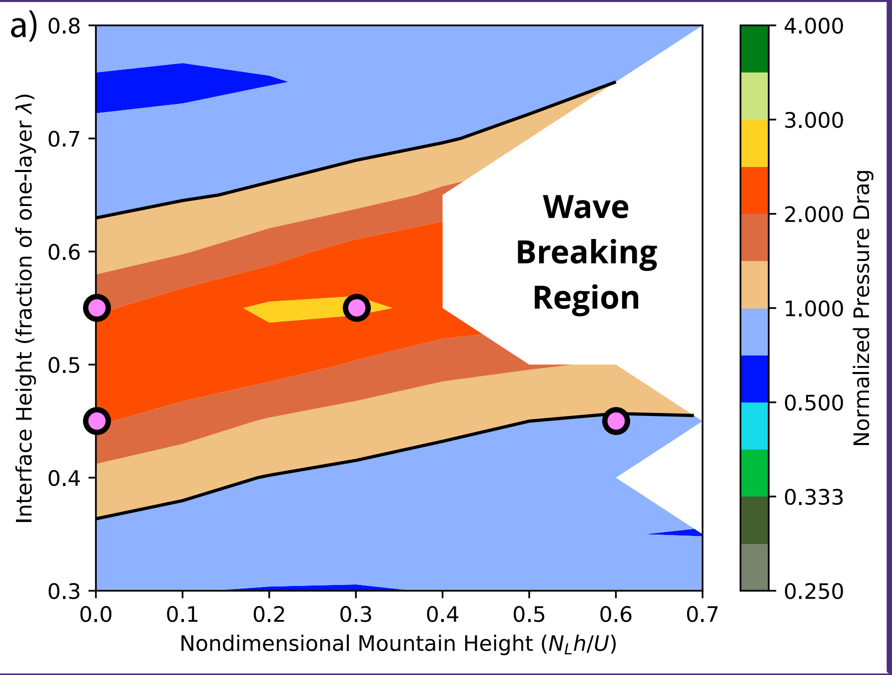

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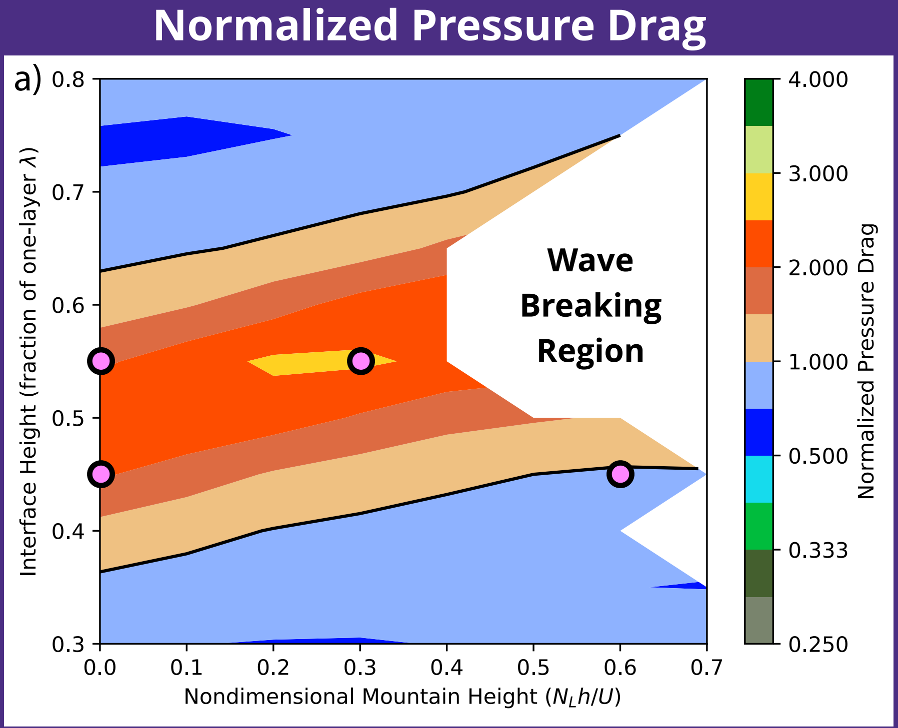

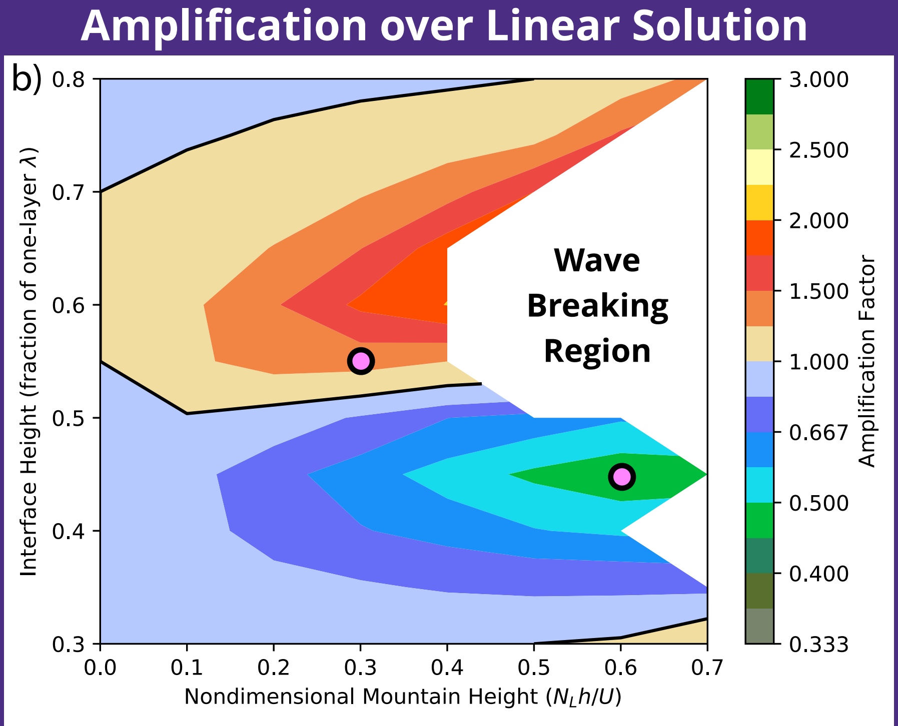

Finite-Depth Tropopause

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$(N = 0.02)$

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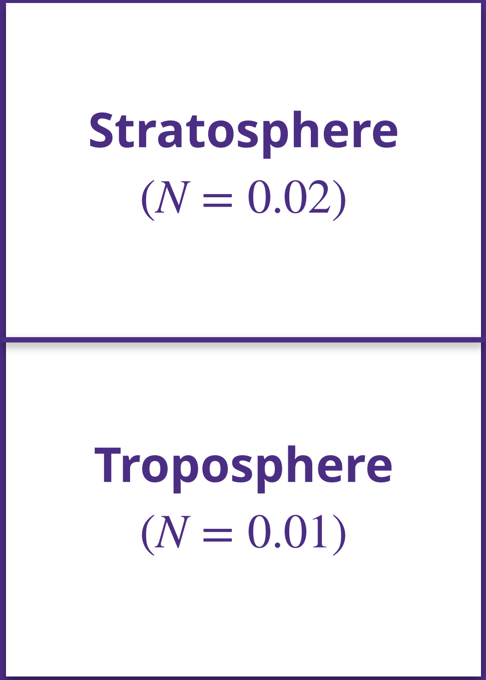

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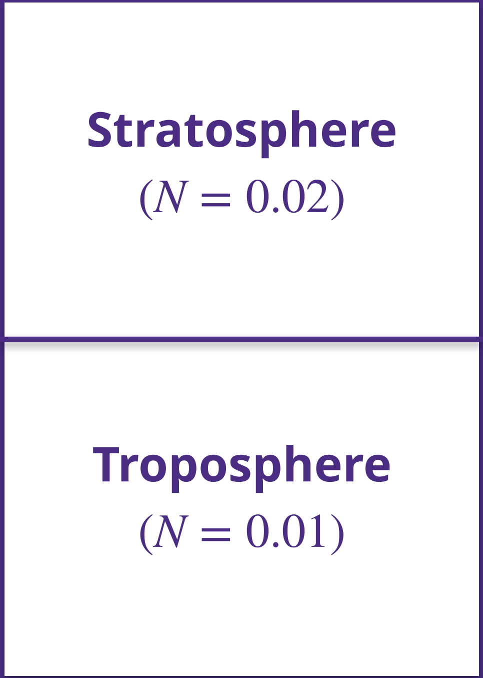
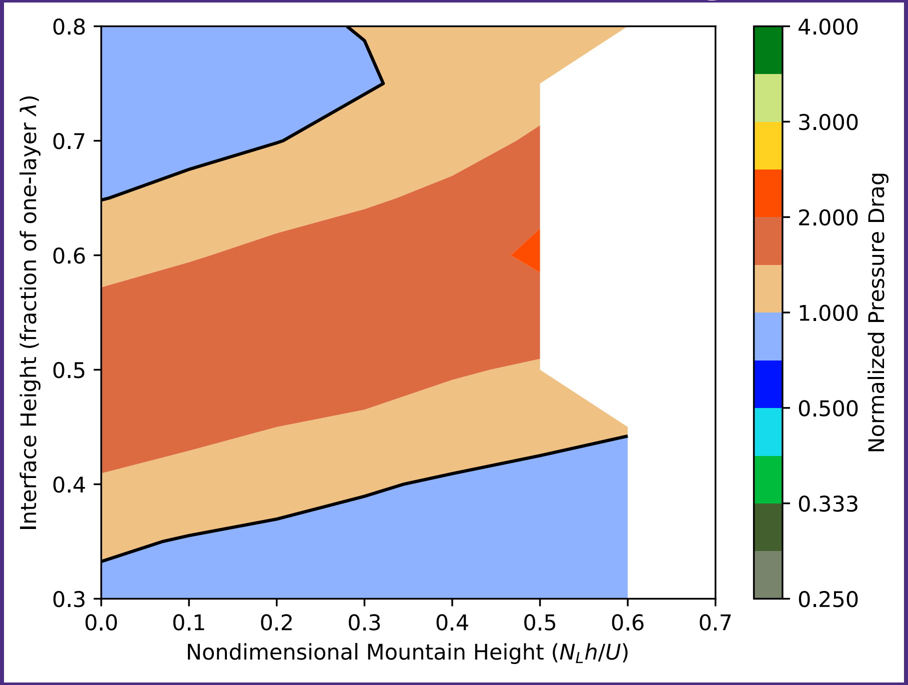
- How sensitive are the previous finite-amplitude results to a smoother transition between layers?

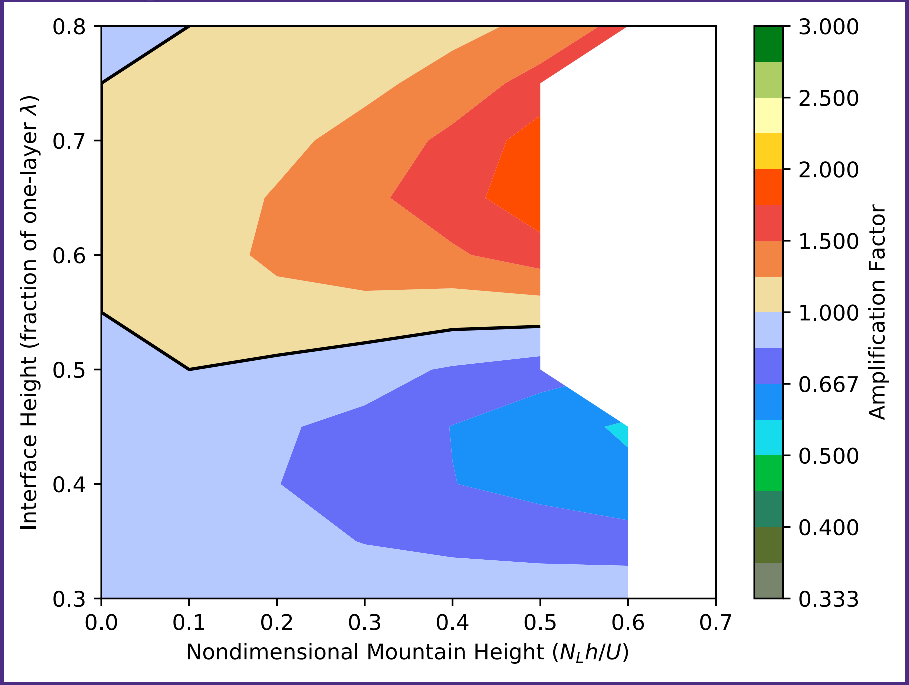


2 km Thick Tropopause Transition

Normalized Pressure Drag



Amplification over Linear Solution



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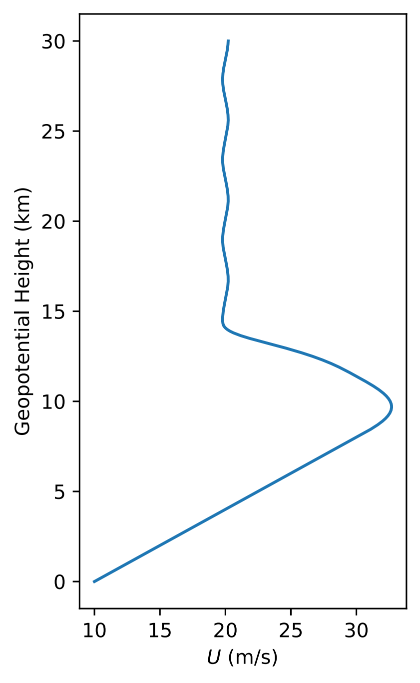
- Clearly the basic state wind speed U is an important component of this equation
- How important are nonlinear processes in a background state with a more realistic profile of U?

Realistic Shear Profile

- Constant shear from 10 m s^{-1} to 30 m s^{-1} in the troposphere
- Relaxes back to ~20 m s^{-1} in the stratosphere

Wind Speed



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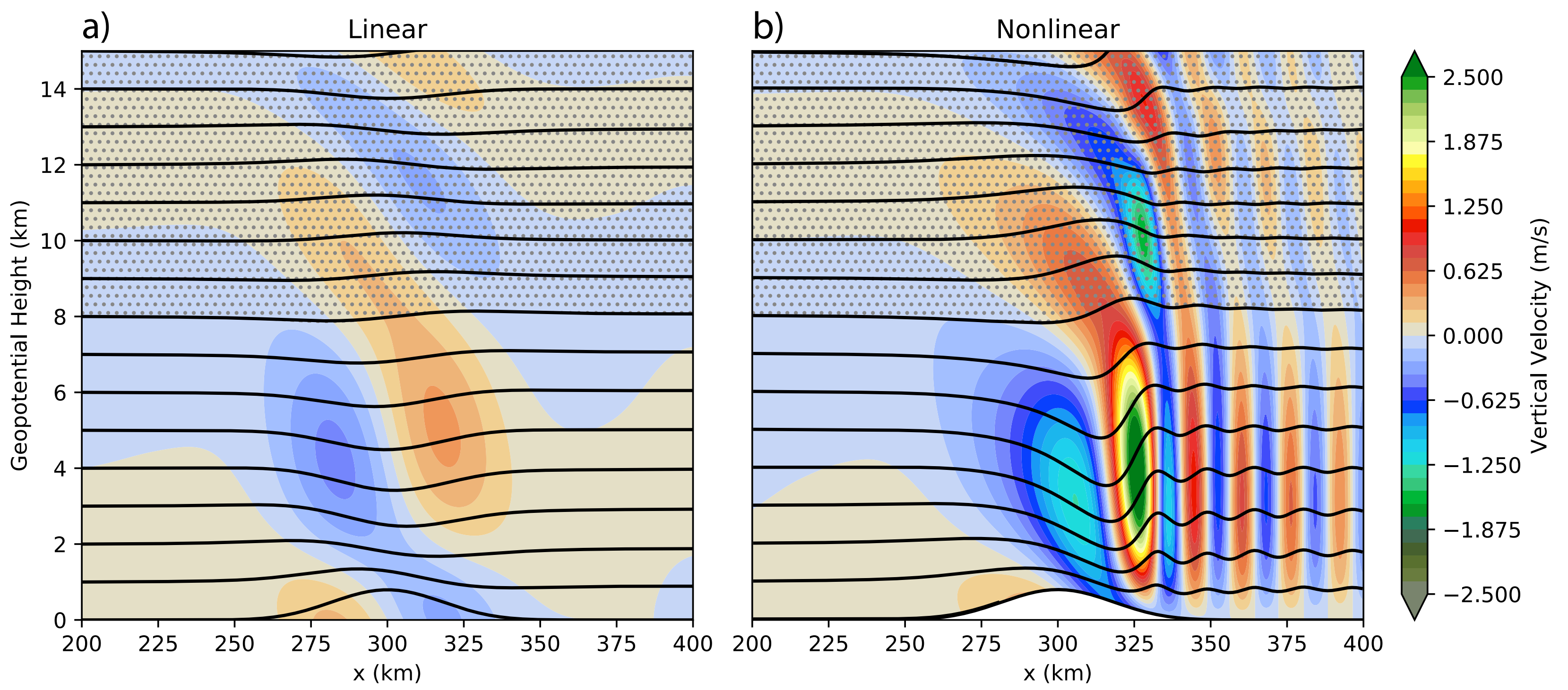

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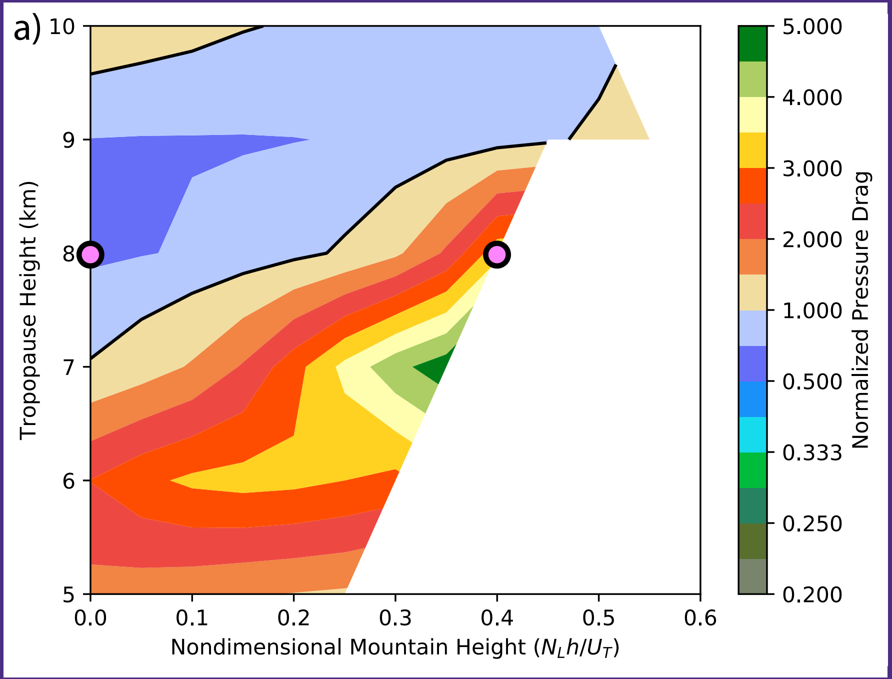

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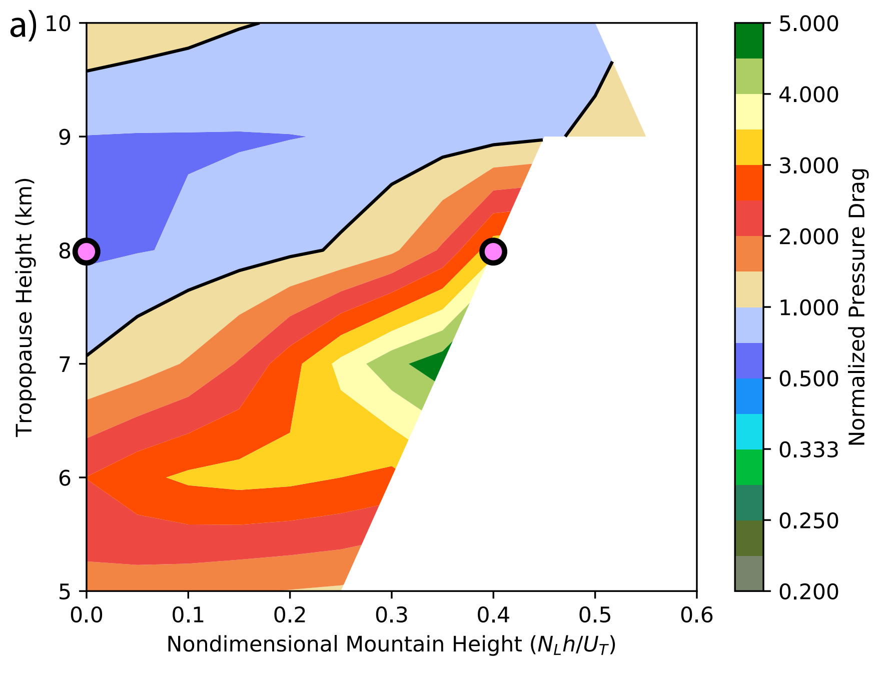

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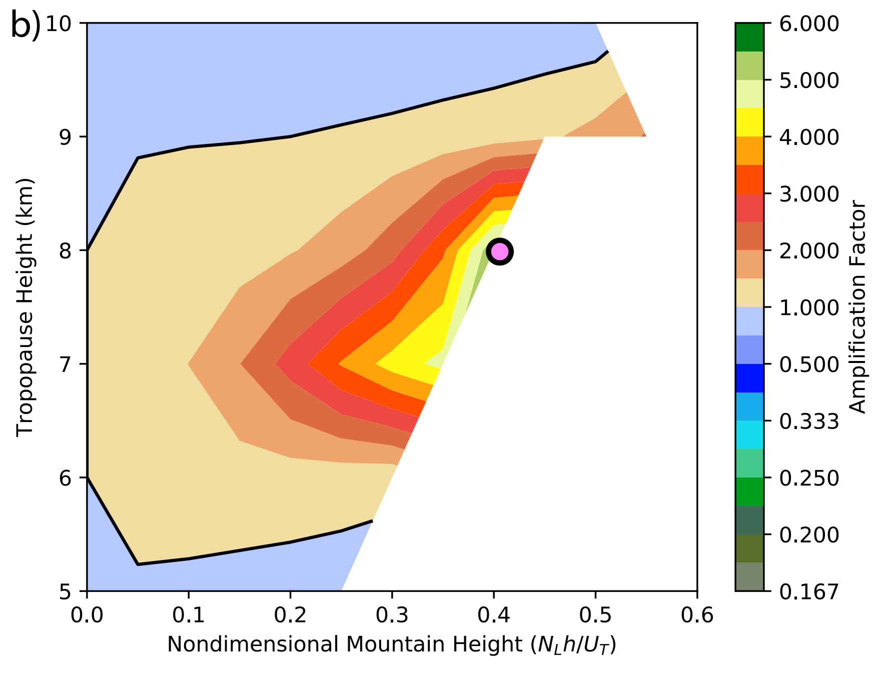

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- > WKB theory fails at the tropopause
- > A parameterization could underestimate the true drag by a factor of 5!
- > Even a parameterization that accounts for the tropopause using linear theory would have significant error due to finite-amplitude effects
- > Associated difficulty with parameterization is one more reason that increasing resolution to explicitly resolve more of the wave spectrum is an important goal