Reduced-Order Methods for Prediction of Photo-Voltaic Generation Variability due to Fair-Weather Cumuli

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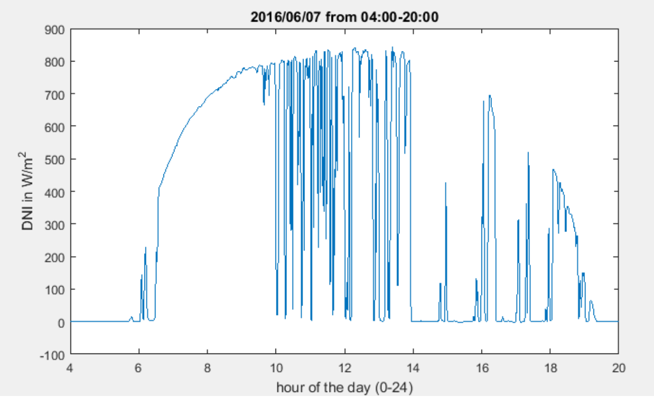
Outline

Solar PV is Variable due to Clouds

Summertime PV can be very productive - up to $1kW \cdot m^{-2}$ at the surface of the earth. Clouds cause drastic variability, though, over periods as short as a few minutes [7].



On/off Insolation causes voltage swings in the neighboring distribution system area.

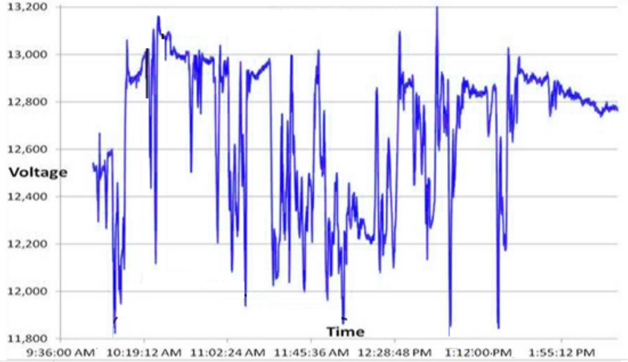
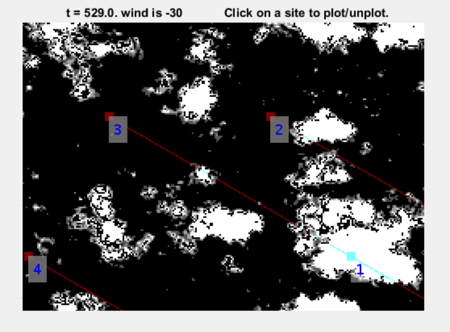


Figure: Illustration from [1].

Simple Advection Model

Clouds move with the wind. A crude estimate of downwind cloudiness can be made by simply translating the existing clouds downwind to $x + \Delta x$ at time $t + \Delta t$, where $\Delta x = u_x \Delta t$.



$\Rightarrow$ This may work well for stable clouds, but not for convective cumulus clouds, which are ever-changing.

Outline

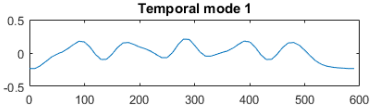
The Proper Orthogonal Decomposition (POD) has been used for many years to model fluid flows [3].

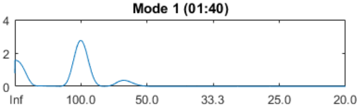
The POD is simply the Singular Value Decomposition (SVD) of the $(space \times time)$ data matrix X.

$$\mathbf{X} = \begin{bmatrix} x_{1,1} \cdots x_{1,M} \\ \vdots \quad\quad \vdots \\ x_{N,1} \cdots x_{N,M} \end{bmatrix} = \mathbf{U\Sigma V}^*$$

- Columns of U are spatial modes
- Columns of V are temporal modes
- Σ is diagonal with descending σ_i

But the temporal modes are arbitrary and cannot be easily predicted forward in time:





Outline

The first POD mode has the highest energy and contains the solar disk.

Please see accompanying video:

`sun_remove_rotate.avi`

(Cloud imagery provided by UCSD [2])
Average cloud velocity is found by Horn & Schunck Optical Flow ([4], [5])

Outline

Fair-Weather Cumuli grow and then subside

- A cloud lifetime begins with a growth phase, followed by a decay phase.
- Prediction of a future state requires knowing where the cloud is in its lifetime.

Please see accompanying video:

`09_16_2016.avi`

The Dynamic Mode Decomposition (DMD) is built on the POD, but finds complex exponential temporal modes [6].

$$\mathbf{X} = (space \times time)$$

$$\mathbf{X} = \begin{bmatrix} x_{1,1} \cdots x_{1,M} \\ \vdots \quad\quad \vdots \\ x_{N,1} \cdots x_{N,M} \end{bmatrix}$$

$$\mathbf{X_1} = \begin{bmatrix} x_{1,1} \cdots x_{1,M-1} \\ \vdots \quad\quad \vdots \\ x_{N,1} \cdots x_{N,M-1} \end{bmatrix}$$

$$\mathbf{X_2} = \begin{bmatrix} x_{1,2} \cdots x_{1,M} \\ \vdots \quad\quad \vdots \\ x_{N,2} \cdots x_{N,M} \end{bmatrix}$$

A first-order difference eqn maps X_1 to the state at the next time step, X_2:

$$\begin{aligned} X_2 &= AX_1 \\ &= AU\Sigma V^*. \end{aligned}$$

where $U\Sigma V^*$ is the SVD of X_1. Define:

$$\tilde{A} := U^* A U = U^* X_2 V \Sigma^{-1}.$$

$\tilde{A}$ is the projection of the transformation A onto the (spatial) POD modes U of the data.

Since Σ can be truncated (along with U and V), we may construct a rank-r model of the $N \times N$ transformation matrix:

$$\tilde{A}_r := U_r^* A U_r = U_r^* X_2 V_r \Sigma_r^{-1}.$$

DMD Discovers Underlying Exponential Time Dynamics

An eigendecomposition of $\tilde{A}_r$ exposes the exponential temporal modes:

$$\tilde{A}_r = WDW^{-1}$$

Let $\Lambda = diag(D)$. Then:

$$\Omega = [\omega_1, \ldots, \omega_r]^T = \frac{1}{\Delta t} log(\Lambda).$$

The DMD modes (in the original N-dimensional space) are given by the columns of:

$$\Phi = X_2 V_r \Sigma_r^{-1} W.$$

(or just $\Phi = UW$).

The reduced-order model X_{dmd} is thus given by

$$X_{dmd} = \Phi e^{\Omega t} b$$

where the DMD mode weights are found from the initial condition x_0:

$$b = \Phi^{+} x_0.$$

X_{dmd} is thus a sum of arbitrary (cloud-shaped) spatial modes which evolve exponentially and sinusoidally over time.

Some clouds are decaying and some are growing

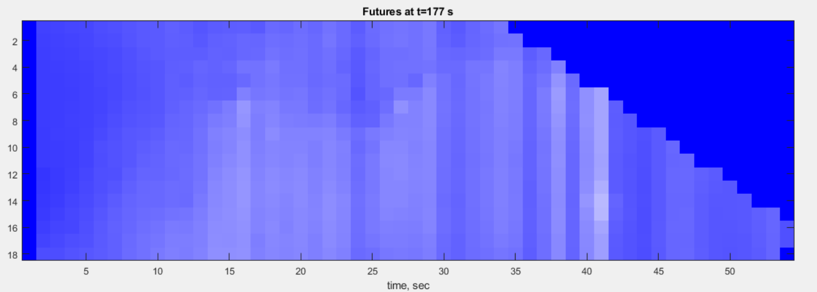
If we do a single-mode DMD, the resulting mode will be either exponentially decaying ($\omega_1 < 1$) or growing ($\omega_1 \geq 1$)

Please see accompanying video:

`rotated.avi`

Single-Mode DMD for each Pixel Column of the Rotated Image

- Uniform velocity, right to left
- Each pixel column has a known advection time to sun
- Compute $(C \times 3)$ DMD for each column
- Find X_{dmd} for that column t_c sec in the future



Each column above is a prediction of conditions at the edge of the solar disk at the indicated time into the future.

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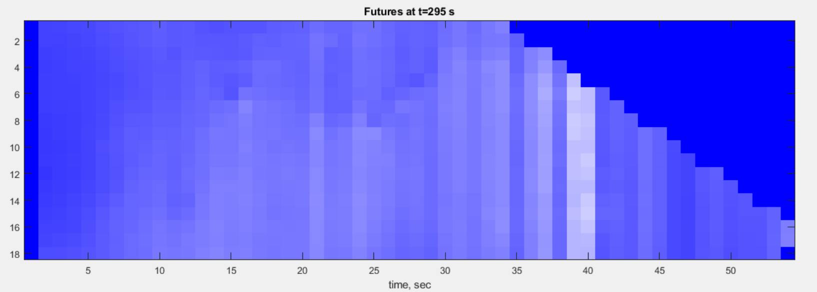

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Outline

Conclusions and Improvements

- Clouds evolve on time scales shorter than our prediction horizon.
- For good predictions, we must account for cloud growth and decay.
- Multi-mode DMD can make better predictions.
- Modeling advecting *vortices* (instead of clouds) may allow prediction beyond one cloud lifetime.

Thank you!

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