One Problem: Two Methods? Factor Separation in the Atmospheric Sciences

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One Problem, Two Communities → Two Approaches

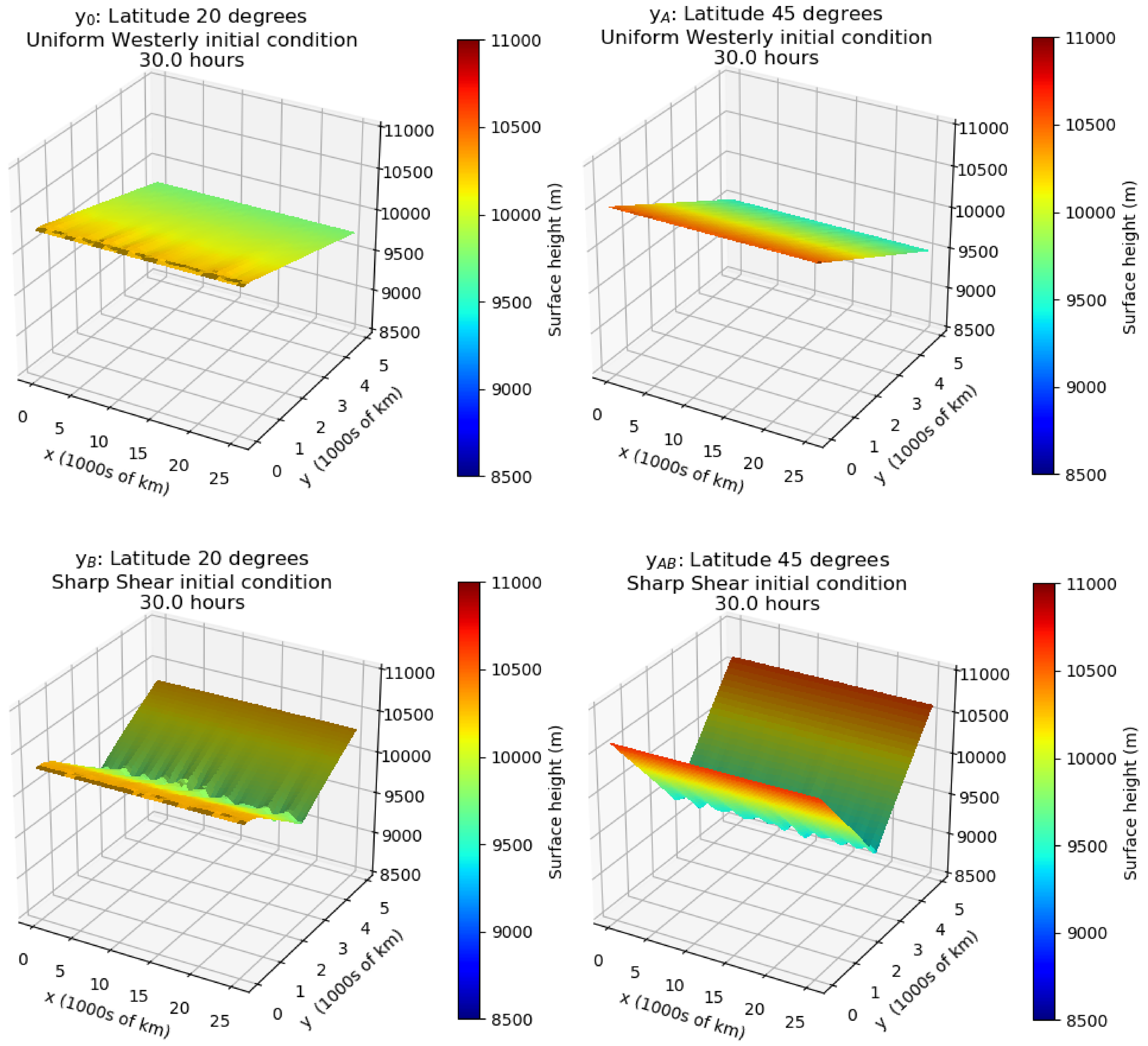
- NWP codes model the atmosphere by properly representing the interactions of many factors; however, this also makes these codes quite complex. Properly sampling the codes is the only computationally tractable means of identifying an optimal configuration.
- The gold standard for sampling a "black box" function subject to a set of k fixed parameters or "factors" each with two levels is called a full factorial or 2^k design. Analyzing a 2^k experiment necessarily involves attributing the results to not only the individual factors but to the interactions between factors.
- Within the atmospheric sciences community, one can use a method called "Factor Separation" (Stein and Alpert 1993) to determine the pure contribution due to any one factor and the pure interaction contributions in a full factorial. We use the term "contribution" to refer to factor estimates produced using Stein and Alpert's method.
- In the Design of Experiments community, the same full factorial results are analyzed using the method of analysis of variance (ANOVA) (Scheffé 1959) to estimate the effects attributable to each of the factors and their interactions as well as assessing their statistical significance. We use the term "effects" to refer to factor estimates produced using ANOVA.
- In this project, we compared and contrasted the contributions obtained using the Stein-Alpert (SA) contributions with the effects obtained with the Design of Experiments (DOE) method and demonstrate these results using a shallow water equations model.

Using Both Methods on the Shallow Water Equations

- We demonstrate SA and DOE using a Shallow Water Equations Model coded in Python by Paul Connolly (2018) that we modified for our use. The Shallow Water Model allows for varying different parameters. Our goal was to understand how the surface height changed with different parameter settings.
- We chose two parameters: the initial surface height and latitude. We varied the initial height field using a uniform westerly initial condition as the "off" and a sharp shear initial condition as the "on." We placed the domain at a latitude of 20 degrees as the "low" setting and moved it to a latitude of 45 degrees as the "high."
- The surface height at 30 hours into the simulation, when the sharp shear started to become unstable, was used as the response variable.
- Following a full factorial design, the four simulations were:

Factor	A	B
Off (low)	y_0: Uniform Westerly, Latitude=20 degrees	y_B: Sharp shear, Latitude=20 degrees
On (high)	y_A: Uniform Westerly, Latitude=45 degrees	y_{AB}: Sharp shear, Latitude=45 degrees

The Shallow Water Equation Simulations

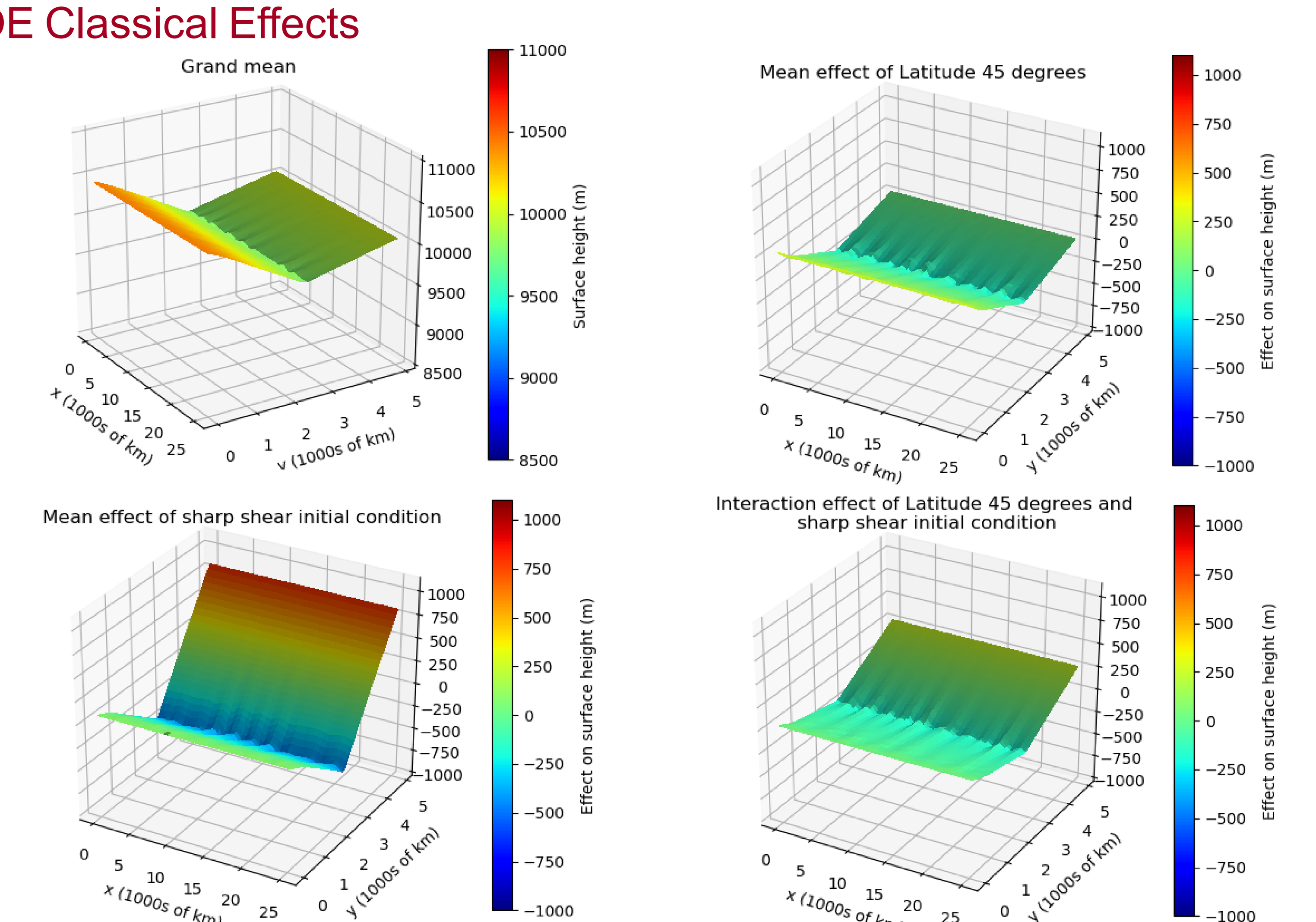


Effects versus Contributions for 2 Factors

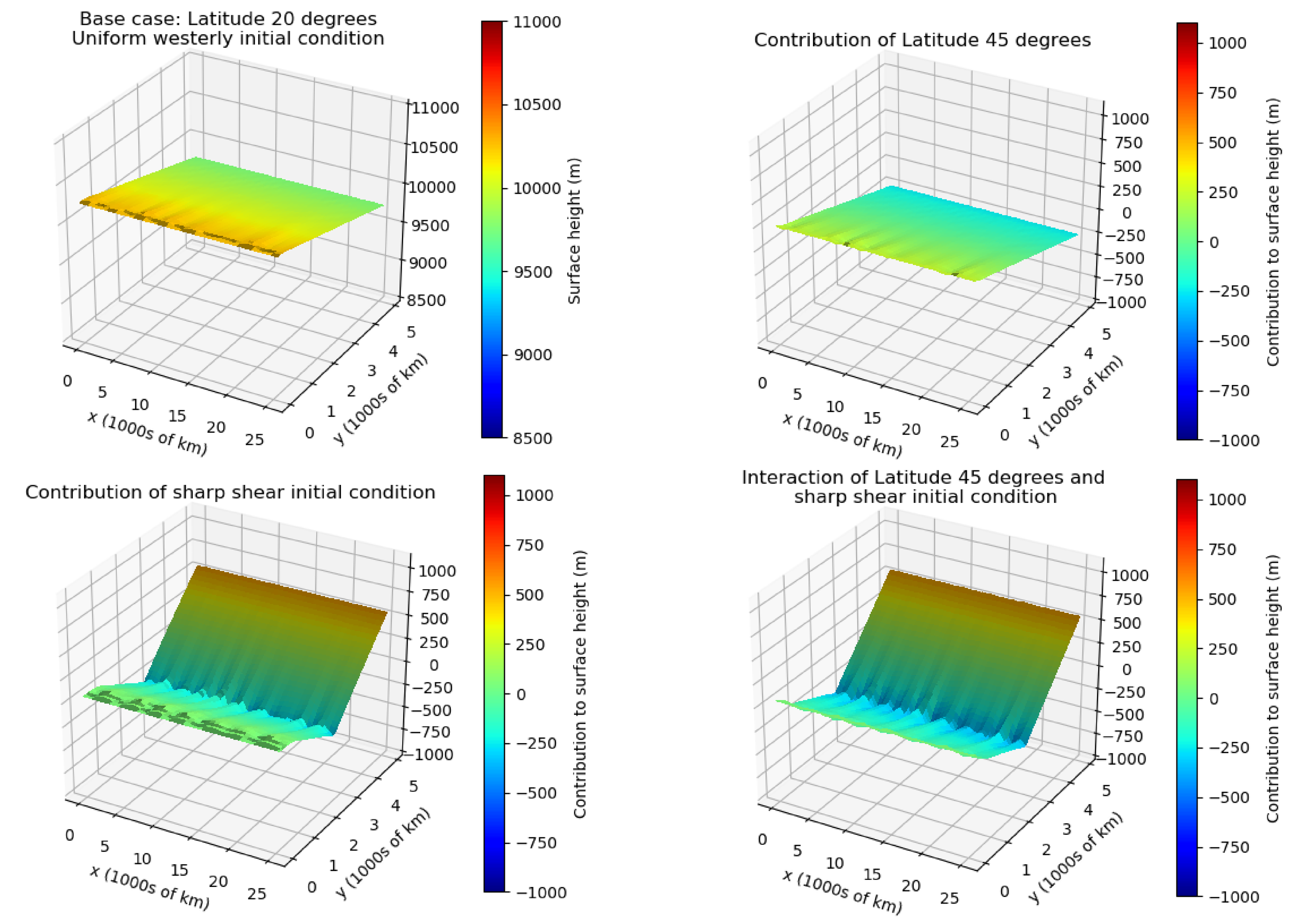
Factor	Design of Experiments: Effect	Design of Experiments: Interpretation	Stein and Alpert: Contribution	Stein and Alpert: Interpretation
A	$\frac{1}{2}(y_A+y_{AB}-y_B-y_0)$	Mean effect due to A	y_A-y_0	Pure contribution of A
B	$\frac{1}{2}(y_B+y_{AB}-y_A-y_0)$	Mean effect due to B	y_B-y_0	Pure contribution of B
AB	$\frac{1}{2}(y_{AB}-(y_A+y_B)+y_0)$	Interaction effect of A and B	$y_{AB}-(y_A+y_B)+y_0$	Contribution due to interaction of A and B
	Effects coding		Dummy coding	
	Effects are twice the linear regression coefficients		Contributions are the linear regression coefficients	
	Emphasizes mean		Emphasizes base (no factor) state	

Note: In the plots that follow, some scales differ to allow for differences in magnitude to be more obvious.

DOE Classical Effects

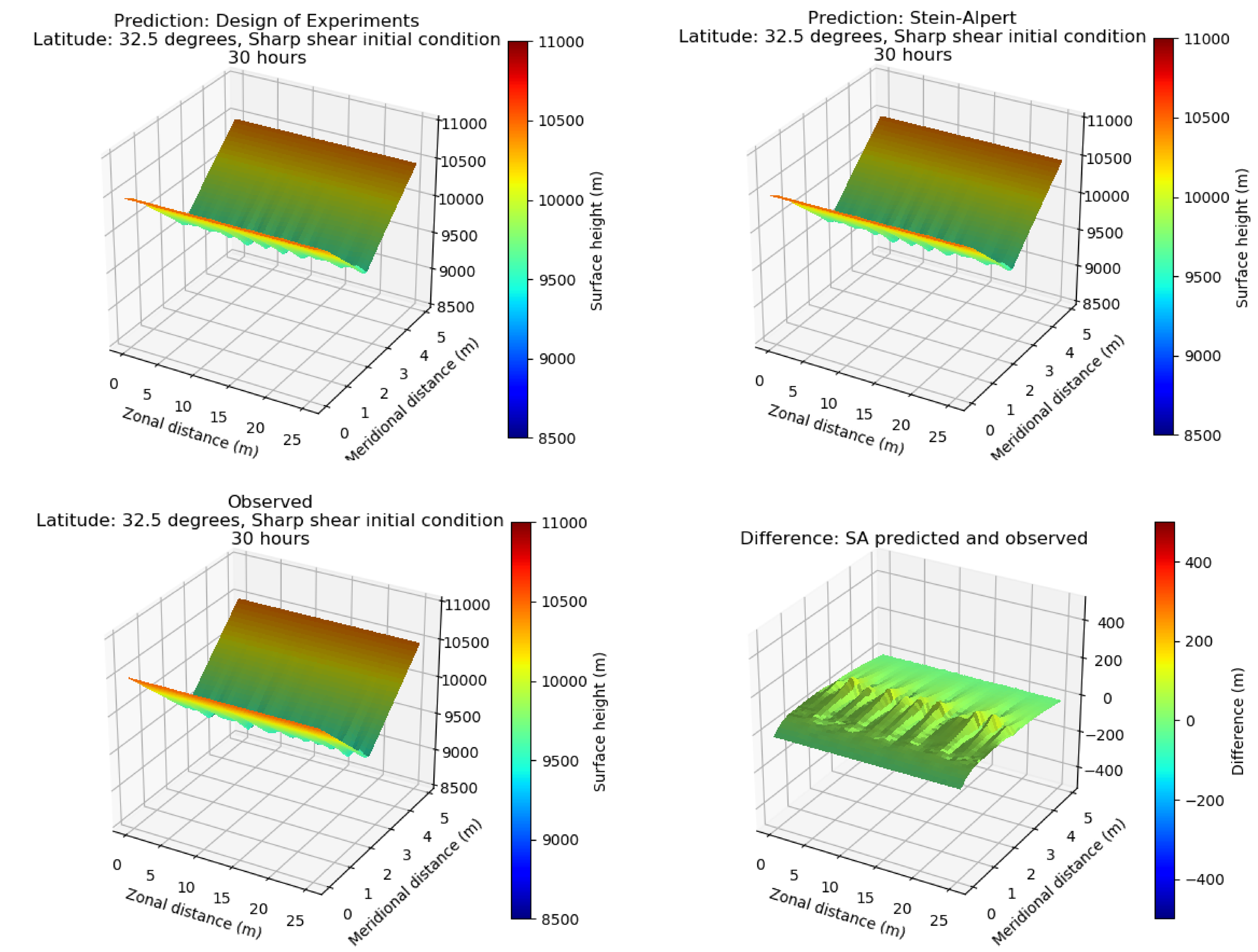


Stein-Alpert Contributions



Predictions Using Both Methods at 32.5 Degrees Latitude

- After performing factor separation on the four runs, both methods can be used to create multiple regression equations to predict results at different values for the chosen factors. Here we predict the surface height at a latitude halfway between the two original latitudes, 32.5 degrees latitude, with a sharp shear initial condition.



- The predictions from DOE and SA are identical. The observed value at 30 hours is close to the predicted at 30 hours.
- We ran the simulations to 96 hours and the results became unpredictable at that point so the data is not shown here. DOE and SA did continue to predict identical values.

Conclusions

- DOE and SA are both based on multiple linear regression. The models from each method predict the same values.
- The coefficients for the regression equations differ because the coding is different. In the Shallow Water example, the corresponding effects and contributions were fairly similar in some cases (e.g. the Sharp Shear contribution and effect) and quite different in others (e.g. the Latitude contribution and effect).
- Which method should be used depends on the goals of the researcher and should be clearly stated and understood so the results can be properly interpreted. Neither method is inherently better.
- The predictions diverge from the observations as the surface becomes more unstable so the results from either method at that point should not be relied upon.

References

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