Snowfall frequency expressed by regression analysis with logarithms

AMS100

Poster #224

Introduction

- ✓ The frequency of phenomena that cause natural hazards such as heavy snowfall (**Fig. 1**) is an important factor in planning measures against those phenomena.
- ✓ In the frequency analysis, probability distribution functions of extreme values are commonly used to ascertain the return period, but complex procedures such as parameter setting are necessary in order to fit the data to the probability distribution.
- ✓ On the other hand, it has long been known that the relationship between the frequency and the magnitude of natural phenomena exceeding a certain threshold can be approximated by exponential and/or power-law distributions.

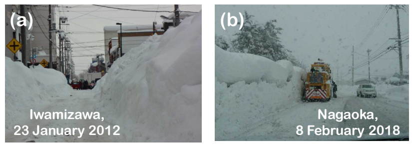

Fig. 1 Examples of heavy snowfall

Purpose

- ✓ To obtain a simple method (i.e, no using the probability distribution functions) for evaluating the frequency of snowfall events, suitability of exponential approximation for snowfall events was examined for snowy regions in Japan (**Fig. 2**).

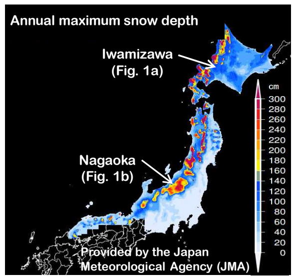

Fig. 2 Mean annual maximum depth of snow cover (1991-2010)

Methods

Sampling the snowfall events

- ✓ Using hourly snow depth SD observed during the past 16 - 35 winters at 270 meteorological observing stations of the Japan Meteorological Agency (JMA) (**Fig. 3**).
- ✓ Snowfall amount S (cm) was defined as the cumulative value of the positive difference in snow depth ΔSD each hour (**Fig. 5**).
- ✓ Each snowfall event was regarded as ending when the no snowfall (i.e., $\Delta SD \leq 0$ cm) continued to more than 6 hours.
- ✓ The events with snowfall amount S greater than 30 cm are regarded in this study. (**Fig. 6**)

Frequency of snowfall events

- ✓ Cumulative numbers of snowfall events N from classes of large snowfall amounts at intervals of 5 cm were counted (**Fig. 7**).
- ✓ Dividing the cumulative number of events N by the years of observation period provides the frequency of snowfall events $F(s \geq S)$ (number of events / year) (**Fig. 8**).
- ✓ The $F(s \geq S)$ means the occurrence number of snowfall events per year with snowfall amounts exceeding a certain S cm.

Regression analysis

- ✓ To obtain the estimation equation for the frequency of events exceeding a certain snowfall, the regression analysis between the common logarithms of frequencies $\log F(s \geq S)$ and the snowfall amounts S was carried out to each station.
- ✓ Stations which has less than 10 events were excluded from the analysis. Therefore, 237 stations (● in **Fig. 3**) were used.

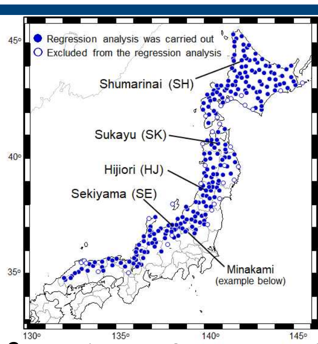

Fig. 3 Distribution of 270 meteorological observing stations of the JAM

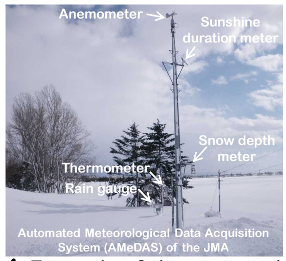

Fig. 4 Example of the meteorological observing station (AMeDAS) of JAM

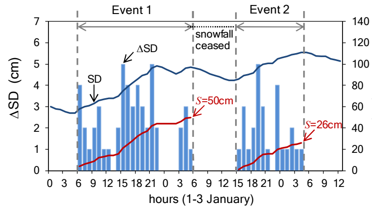

Fig. 5 An example of calculating snowfall amount S in each event

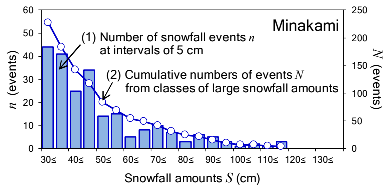

Fig. 7 Calculating the cumulative numbers of snow fall events N

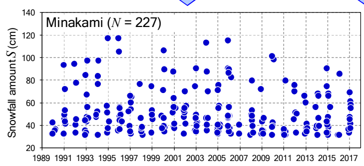

Fig. 6 Time series of sampled snowfall events exceeding 30 cm

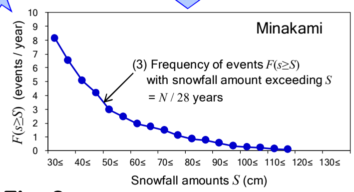

Fig. 8 Calculating frequency $F(s \geq S)$

Regression analysis ($\log F(s \geq S)$ vs S)

Results – examples of some stations

Regression analysis for some stations

- ✓ **Figure 9** shows the examples of the regression analysis between $\log F(s \geq S)$ and S for four stations located heavy snow areas.
- ✓ The regression analysis for each station revealed a strong linear correlation between the logarithm of frequency $\log F(s \geq S)$ and the snowfall amount S at a statistically significant level of 1%.
- ✓ The regression equation to estimate the frequency of snowfall is expressed in Eq.(1),

$$\log F = a\,S + b \qquad (1)$$

where a and b are the regression coefficients.

- Natural logarithm of the base e (i.e., $\ln F(s \geq S)$) also can be used in this analysis. It mean that the exponential function can be used for evaluating the frequency of snowfall events.
- In statistical terms, the frequency of snowfall events follows the generalized Pareto distribution (GPD), including the exponential distribution with a shape parameter of zero.

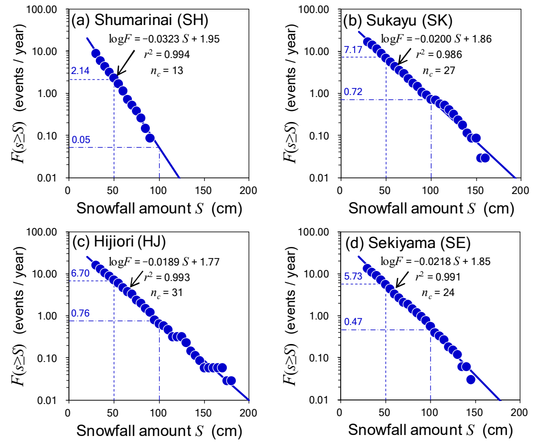

Fig. 9 Snowfall amounts S versus common logarithms of frequencies of snowfall events $\log F(s \geq S)$

Hiroki Matsushita*, Wataru Takahashi, and Joji Takahashi

Civil Engineering Research Institute for Cold Region, Public Works Research Institute, Japan

*e-mail: matsushita-h@ceri.go.jp

Results – all stations

Comparison of estimated and observed frequencies

- ✓ **Figure 10** shows comparison of frequencies observed and estimated from regression equations for 237 stations.
- ✓ In case of the frequency of snowfall exceeding 30 cm (**Fig. 10a**), data distribute around the equal line but some data slightly scatter.
- ✓ In case of the frequency of snowfall exceeding **50 cm and more** (**Fig. 10b - d**), estimated values corresponds **closely** to observed values.

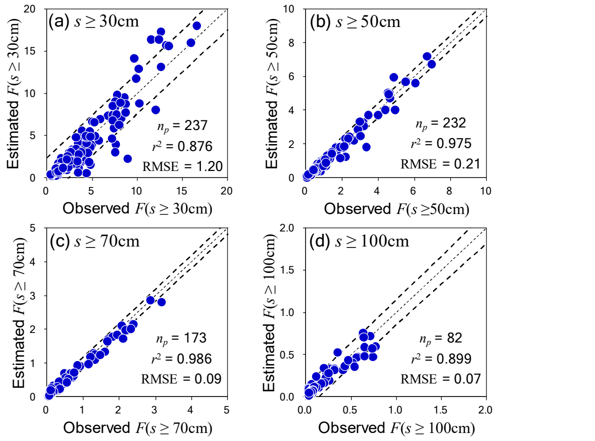

Fig. 10 Comparison of estimated and observed frequencies $F(s \geq S)$. n_p: number of stations, thick broken lines: 95 % confidence interval for estimated values.

Root mean squared error (RMSE) and coefficients of determination r^2

- ✓ The frequencies estimated from the regression equations at all 237 stations were compared with observed values at 5-cm snowfall intervals (**Fig. 11**).
- ✓ In the range of snowfall amounts **between 40 cm and 100 cm**, the coefficients of determination r^2 **exceed 0.9**.
- ✓ Moreover, the root mean squared errors (**RMSEs**) are **less than 0.5**. In cases of snowfall amounts exceeding 65 cm, the RMSEs are less than 0.1.
- ✓ Consequently, using the **simple regression analysis** with the **logarithm of frequency** enables the easy evaluation of the frequency of snowfall events exceeding a certain snowfall amount.

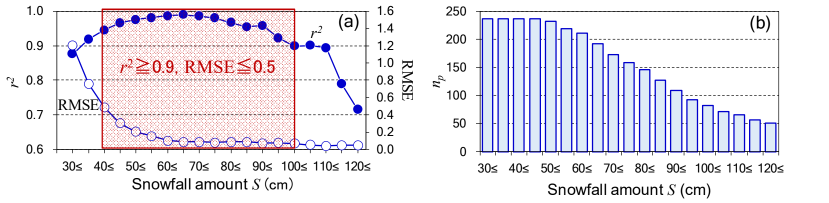

Fig. 11 (a) The coefficients of determination r^2 and the root mean squared errors (RMSEs) between estimated and observed frequencies, (b) the number of stations n_p at 5-cm snowfall intervals.

Discussions

Return period

- ✓ The reciprocal number of frequency ($1/F$) is mean occurrence interval (i.e., the **return period** RP).
- ✓ In practical use, expression using the return period makes it easy to image the extreme snowfall amount corresponding a certain period (**Fig. 12**).
- ✓ The regression equation to estimate the return period RP of snowfall exceeding a certain value S is expressed in Eq.(2),

$$S = c \log RP + d \qquad (2)$$

where c and d can be obtained using a and b in Eq.(1).

$$c = -1/a, \quad d = -b/a \qquad (3)$$

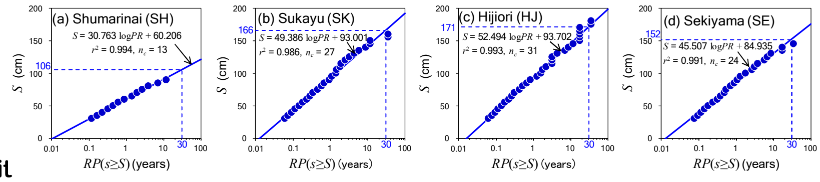

Fig. 12 Snowfall amount S versus return period $RP(s \geq S)$

Regional characteristics

- ✓ **Figure 13** shows regional distributions of the frequency of snowfall events exceeding 50 cm $\log F(s \geq 50\text{cm})$ and the **coefficient a** of regression equation.
- ✓ In regions where the frequencies are higher, coefficients a (slope of regression line) tend to be low (**Fig. 13**).
- ✓ The coefficients **b/a correlate to the number of events per year** (**Fig. 14c**).
- ✓ The b/a will correspond to the **location parameter μ** of the probability distribution function for the **exponential distribution** (**Eq. (4)**), but its detailed reason is a future task.

$$F(x) = 1 - \exp\left(-\frac{x-\mu}{\sigma}\right) \qquad (4a)$$

$$\ln\left(1 - F(x)\right) = -\underset{\substack{\uparrow \\ a}}{\frac{1}{\sigma}}x + \underset{\substack{\uparrow \\ b}}{\frac{\mu}{\sigma}} \qquad (4b)$$

in Eq.(1) $\quad \therefore b/a \approx \mu$

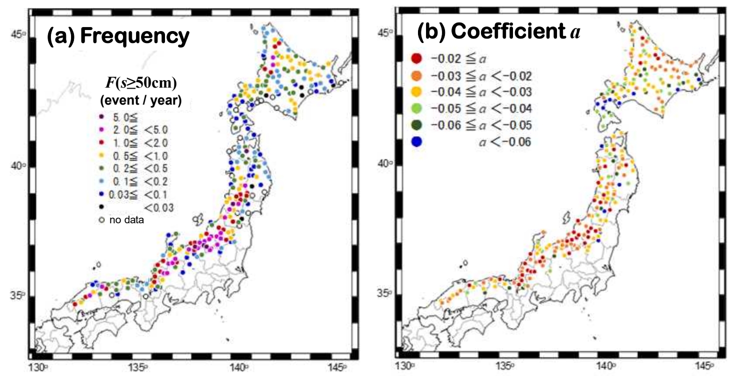

Fig. 13 Regional distributions of (a) the frequency of snowfall amount exceeding 50 cm $F(s \geq 50\text{cm})$ and (b) the coefficient a of regression equation

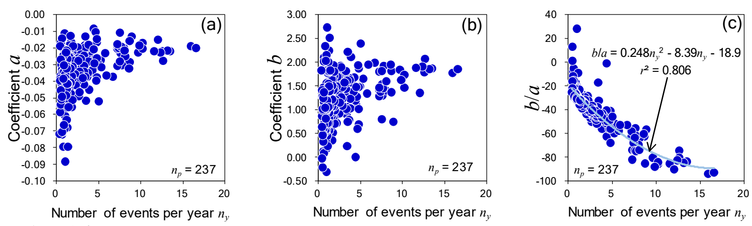

Fig. 14 Relationships of the number of snowfall events per year n_y with (a) the coefficient a, (b) the coefficient b, and (c) b/a.

Conclusions

- ✓ Using the **simple regression analysis** with the **logarithm of frequency** (i.e., the exponential function) **enables the easy evaluation** of the frequency of snowfall events exceeding a certain snowfall amount.
- ✓ The method proposed in this study will make it **easy to assess the frequency** of phenomena related to snow disasters, such as heavy snowfall, avalanches during snowfall (**Fig. 15**), and atmospheric icing.

Fig. 15 Example of avalanche