A Modified Degree-Day Method for Volume and Timing Estimation of Snowmelt and Refreezing

Ana Žaknić-Ćatović, Ken W. F. Howard, William A. Gough & Zlatko Ćatović

University of Toronto Scarborough, Toronto, Ontario, Canada

UNIVERSITY OF TORONTO SCARBOROUGH

The critical information for the improvement of the accuracy of snowmelt water equivalent estimation comes from the knowledge of the temporal distribution of daily air temperature around the freezing threshold. The proposed Modified Degree Day method transfers the reliance of a classical degree-day approach from daily mean temperatures onto diurnal temperature-time extrema pairs further used as the connection points for the analytical air temperature approximation. Reproduced air temperature variability serves as a platform for the derivation of unique parameters used in estimation of volume and timing of diurnal melting and refreezing events.

MAIN OBJECTIVES

TO OVERCOME THE DEFICIENCIES OF A STANDARD DEGREE DAY (SDD) APPROACH AND OBTAIN REALISTIC ESTIMATES OF VOLUMES AND TIME DURATION OF DIURNAL MELTING AND REFREEZING EVENTS WHILE MAINTAINING THE SIMPLICITY OF A TEMPERATURE-INDEX METHOD.

STANDARD DEGREE-DAY APPROACH DEFICIENCIES

OBJECTIVES	METHOD FEATURES	PROBLEMS
SNOWMELT & REFREEZING ESTIMATES	*STANDARD DEGREE APPROACH*	REFREEZING PROCESS
DURATION OF MELTING AND REFREEZING EVENTS	SINGLE TEMPERATURE INDEX COARESNESS OF TIME STEP	TIMING OF WATER PHASE CHANGES
ACCURACY OF EXTREMA CONNECTION POINTS	*TEMPERATURE APPROXIMATION*	COW_{0-24} RELATED EXTREMA MISIDENTIFICATION
EXACT POSITION OF EXTREMA POINTS	REVERSAL OF DAILY EXTREMA INTERUPTION OF SEQUENCE	X,Y EXTREMA COORDINATES

STANDARD DEGREE-DAY METHOD DOES NOT RECOGNIZE THE PHYSICAL PROCESS OF WATER REFREEZING OR DURATION OF SNOWMELTING AND REFREEZING EVENTS

MODIFIED DEGREE-DAY APPROACH ADVANTAGES

OBJECTIVES	METHOD FEATURES	SOLUTIONS
SNOWMELT & REFREEZING ESTIMATES	*MODIFIED DEGREE APPROACH* (T, t) EXTREMA PAIRS	DAILY DEGREE DAY PARAMETERS
DURATION OF MELTING AND REFREEZING EVENTS	HOURLY TIME STEP	EFFECTIVE TEMPERATURE AND EFFECTIVE TIME
ACCURACY OF EXTREMA CONNECTION POINTS	*TEMPERATURE APPROXIMATION*	CLIMATOLOGICAL OBSERVING WINDOW COW_{N-D}
EXACT POSITION OF EXTREMA POINTS	CHRONOLOGICALLY ORDERED EXTREMA SEQUENCE	DIURNAL EXTREMA TIMING DET PARAMETER

EXACT POSITION OF DIURNAL EXTREMA POINTS PLAYS A CRITICAL ROLE IN AIR TEMPERATURE MODELLING

STANDARD DEGREE-DAY METHODOLOGY

The standard degree-day (SDD) method for the estimation of diurnal snowmelt water equivalent (SWE) seriously miscalculates volumes of water on days of symmetric air temperature fluctuation around the threshold point.

Additionally, the SDD method fails to observe the physical process of water refreezing and does not have the capability of estimating the daily duration of snow-melting and refreezing events.

$$T_{max} = 8°C$$
$$T_{min} = -6°C$$
$$T_{ave} = 1°C$$

$$M_R = C^+ \cdot T_{ave}$$

$$M_R = 4\left(\frac{mm}{C° d}\right) \cdot 1°C$$

$$M_R = 4\left(\frac{mm}{d}\right)$$

M_R = MELT RATE $(mm \cdot day^{-1})$

C^+ = MELTING CONSTANT $(mm \cdot °C^{-1} \cdot day^{-1})$

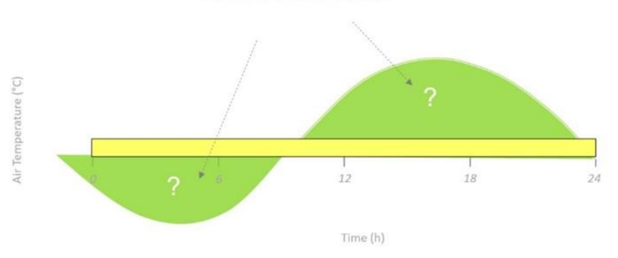

Fig 1. *Example of a miscalculation of a diurnal snowmelt water equivalent by the SDD method on a day with a symmetrical air temperature variation.*

METHOD DIFFERENCES IN DAILY VOLUME ESTIMATION

In simple analytical terms, the difference between SDD and MDD volume estimation can be presented as follows:

SDD: $$M_V = C^+ \cdot T_{ave} \cdot 1\,day$$

$$MM_V = C^+ \cdot T^+_{eff} \cdot t^+_{eff}$$

MDD: $$MR_V = C^- \cdot T^-_{eff} \cdot t^-_{eff}$$

C^+ and C^- are melting and refreezing constants $(mm\,℃^{-1} day^{-1})$.

EXTREMA PAIRS FOR TEMPERATURE REPRODUCTION

The choice of a Climatological Observing Window (COW) greatly affects the accuracy of the diurnal extrema search.

The COW currently in use or (COW_{0-24}) is associated with a systematic "cold bias" in air temperature records due to frequent mischaracterization of minima.

Correct information on the 'turning points' of diurnal air temperature function and timing of diurnal extrema occurrence are of critical importance for the accuracy of a degree-day method.

A Modified Degree-Day (MDD) approach removes dependence on daily air temperature mean by transferring reliance onto daily "true" or mathematical temperature extrema pairs at known times of occurrence.

Mathematical extrema, or points on a temperature-time curve in which daily temperature changes its sign, are identified using (COW_{N-D}).

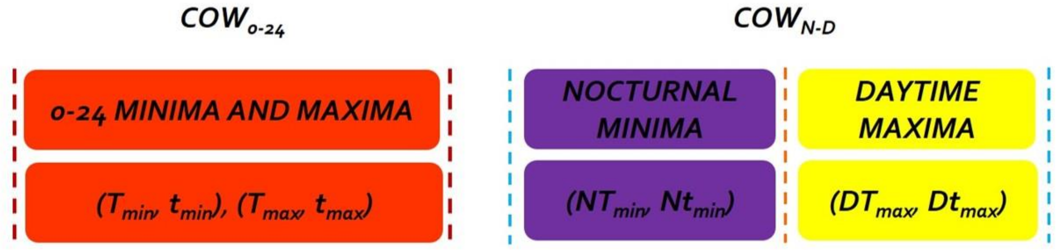

Fig 2. *Climatological Observing Windows for identification of extrema pairs*

DIURNAL EXTREMA TIMING (DET)

OCCURRENCE TIMES OF DAILY AIR TEMPERATURE MINIMUM (t_n) AND MAXIMUM (t_x)

AIR TEMPERATURE EXTREMUM IS A SPECIFIC POINT WITHIN AN OBSERVING WINDOW CHARACTERIZED WITH (T,t) COORDINATES

MODIFIED DEGREE-DAY METHODOLOGY

The COW_{N-D} discretization of time enables more accurate identification of radiatively driven air temperature extrema and as such provides a platform for the derivation of two newly introduced modified degree-day parameters.

Effective temperature (T_{EFF}) and effective time (t_{EFF}) parameters target the exact segments of the diurnal temperature-time curve directly related to diurnal periods of melting and refreezing.

As illustrated in Fig.3, the MDD methodology correlates diurnal snowmelt and meltwater refreezing volumes to the corresponding areas under or above the temperature-time T(t) curve.

ARC AREAS (A AND B) ENCLOSED BY THE POSITIVE OR NEGATIVE TEMPERATURE CURVE SEGMENTS ARE BOUND BY THE CORRESPONDING INTERVALS OF POSITIVE OR NEGATIVE TEMPERATURE

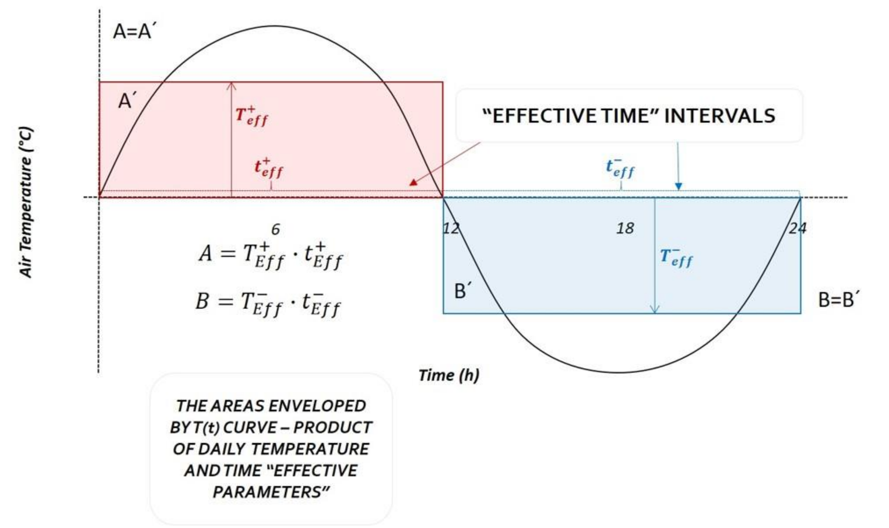

Fig 3. *Graphical illustration of snow-melting and refreezing parameters.*

The physical meaning of MDD parameters and the difference between SDD and MDD approaches are further emphasized in a theoretical case of a symmetric temperature oscillation (black curve) around temperature threshold, thus setting the SDD snowmelt water equivalent estimate to zero, on a day when both melting and refreezing conditions occur.

ANALYTICAL AIR TEMPERATURE APPROXIMATIONS

The accuracy of air temperature approximation depends to a minor degree on a selection of the analytical expression. However, correct information on the timing of daily air temperature extrema is critically important for the success of the approximating function and accurate calculation of the MDD parameters.

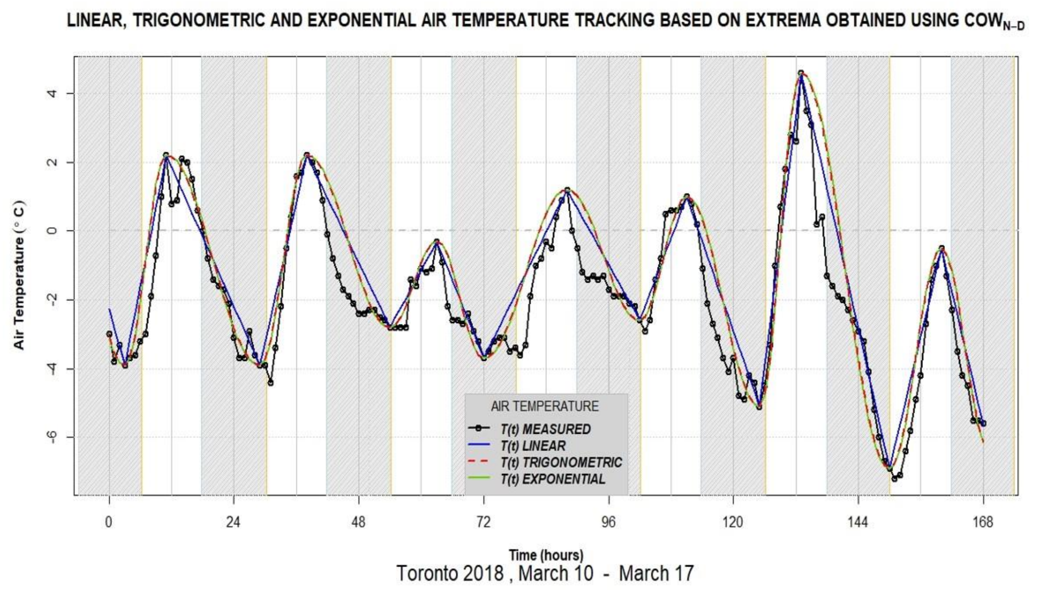

Fig 4. *Example of different analytical approximation fits connected in consecutive extrema points obtained using COW_{N-D}.*

MDD PARAMETERS ESTIMATION

Obtaining MDD parameters using linear air temperature approximating function requires the calculation of triangular areas bounded by the approximating curve and the temperature threshold line. The length of the base corresponds to the time interval associated with the positive or negative temperature, while the height of the triangle is determined by the corresponding temperature maximum or minimum:

$$T^+_{eff} = \frac{T_{max}}{2}$$

$$T^-_{eff} = \frac{T_{min}}{2}$$

The effective time parameters are determined from the time-intercepts of the T(t) segments and the threshold temperature:

$$t = \frac{T_{max}t_{min} - T_{min}t_{max}}{t_{max} - t_{min}}$$

Time intervals between neighbouring time-intercepts define the effective melting and refreezing times.

HYBRID DEGREE-DAY ALGORITHM

Finally, the MDD approach is not seen as a replacement for the regular SDD method, so much as a tool that can be applied when the SDD methodology is likely to become unreliable. This is best achieved by using the Hybrid SDD-MDD algorithm that invokes the MDD approach only when the necessary conditions arise (Fig 5).

SDDM corresponds to the Standard Degree-day Melting, SDDR to the Standard Degree Day Refreezing, MDDR to the Modified Degree-Day Melting and MDDR to the Modified Degree-Day Refreezing.

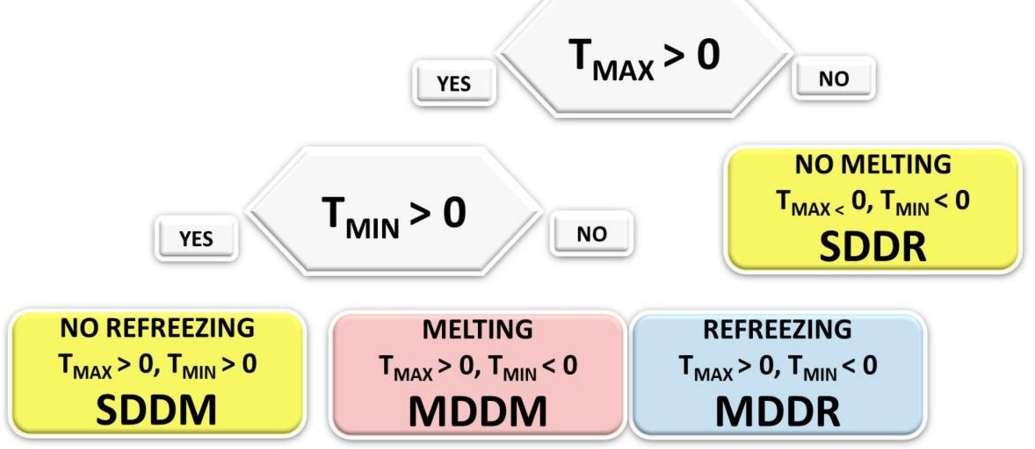

Fig 5. *Decision tree for estimation of melting and refreezing volumes.*

A practical example of the delineation of triangular melting and refreezing areas based on estimated MDD parameters is demonstrated in the figure below. Linear air temperature approximation line connects mathematical extrema pairs from the application of COW_{N-D}.

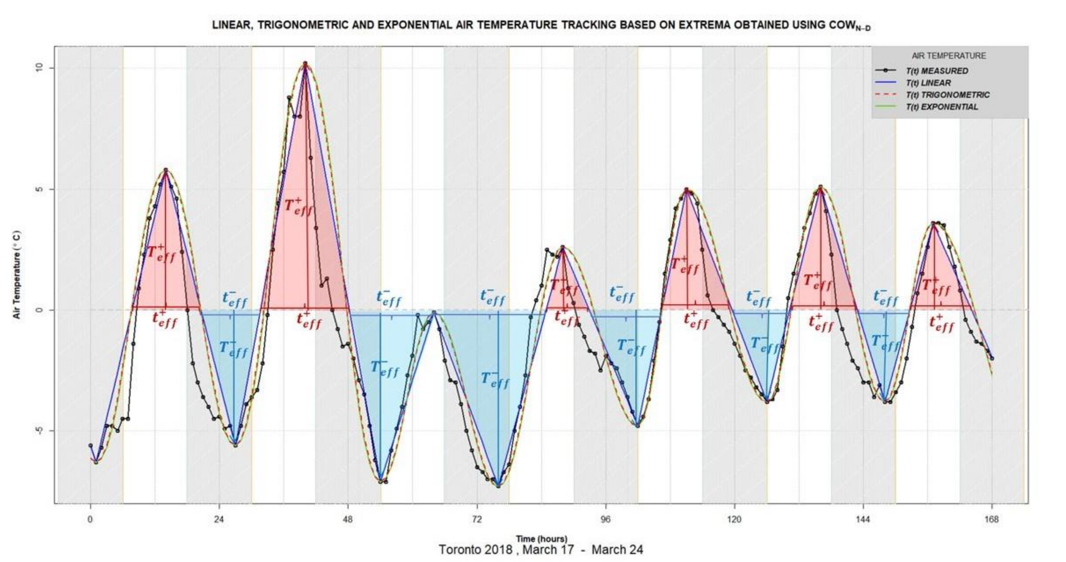

Fig 6. *A weeklong example of air temperature oscillation around the threshold point and melting and refreezing areas for volume estimation.*

BENCHMARKING RESULTS AND CONCLUSION

The MDD method was evaluated for multiple sites in the snow-belt region of Canada where the availability of hourly records of daily temperatures allowed the required MDD input parameters to be calculated reliably and thus be used for comparative purposes. A good agreement was obtained for all sites against reference benchmarks confirming the validity of the MDD approach for diurnal estimation of snow melting and water refreezing quantities.

Day	Σ^+	BENCHM	T^+_{Eff}	t^+_{Eff}	$MDDM_{Lin}$	$T24_{ave}$	SDD^+	Σ^-	BENCHM	T^-_{Eff}	t^-_{Eff}	$MDDR_{Lin}$	$T24_{ave}$	SDD^-
1	11.10	44.40	1.10	9.38	41.26	-0.85	-3.40	-29.40	-58.80	-1.95	17.26	-67.32	-0.85	-1.70
2	10.29	41.14	1.10	10.29	45.26	-0.85	-3.40	-42.90	-85.80	-1.95	19.04	-74.26	-0.85	-1.70
3	N/A	N/A	Tx<0	>24	N/A	-1.55	-6.20	N/A	N/A	-1.40	>24	N/A	-1.55	-3.10
4	8.34	33.36	0.60	8.34	20.01	-1.25	-5.00	N/A	N/A	-1.85	>24	N/A	-1.25	-2.50
5	4.80	19.18	0.50	4.80	9.59	-0.80	-3.20	-29.10	-58.20	-1.30	16.08	-41.81	-0.80	-1.60
6	10.59	42.38	2.30	10.59	97.46	-0.25	-1.00	-52.70	-105.40	-2.55	15.91	-81.15	-0.25	-0.50
7	N/A	N/A	Tx<0	>24	N/A	-3.70	-14.80	N/A	N/A	-3.45	>24	N/A	-3.70	-7.40

$C^+ = 4\,mm/(°C\,d)$ $C^- = 2mm/(°C\,d)$

Fig 7. *Comparison of SDD and MDD volume estimates from Figure 4.*

Broader implementation of the MDD approach would benefit from an event-based discretization of time as an alternative to a fixed, 24-hour presentation of results that fragments periods of melting and refreezing.

RECOMMENDATIONS FOR PRACTICAL APPLICATION OF MDD METHOD

1. **MDD modelling based on inferences of Typical Seasonal Extrema (TSE) pairs from available historical high-frequency air temperature records.**
2. **MDD forecasting of potential snowmelt and refreezing events combining daily TSE estimates with short-term air temperature forecasts.**
3. **Theoretical MDD modelling of potential melting and refreezing conditions using Milanković's adjusted formula for temporal oscillations of vertical temperature profiles enables theoretical estimation of TSE pairs based on insolation and physical properties of the snow-air boundary layer.**

MILANKOVIC'S SURFACE TEMPERATURE FORMULA

The formula for temporal oscillations of vertical temperature profiles, derived by Milankovic in his studies of long-term climate change due to secular perturbations of Earth's orbit and rotation, can be adjusted for use on shorter time scales.

This formula represents a solution of 1D heat diffusion equation within a plane-parallel layer with a boundary condition at the top surface derived from energy balance between periodic insolation, radiative cooling of the top surface and vertical heat conduction (Reference [2]):

$$T(z,t) = T_{ave} + T_z{'}z + T_{amp}e^{-\frac{z}{a}\sqrt{\frac{\pi}{P}}}\cos\left(\frac{2\pi}{P}t - \Delta - \frac{z}{a}\sqrt{\frac{\pi}{P}}\right),$$

where a is thermal diffusivity ($m\,s^{-1/2}$), P is a period while Δ and T_{amp} are the temperature - insolation time delay and amplitude of temperature oscillations, respectively. These two parameters can be further expressed as:

$$\Delta = \tan^{-1}\left(\frac{G}{G-1}\right), T_{amp} = \frac{I\cos\varphi\cos\delta}{\sqrt{1-2\,G\,(1-G)}},$$

with

$$G = \frac{-K/a\sqrt{\pi/P}}{4\,E\,\sigma\,(273+T_{ave})^3}.$$

where E represents emissivity of a top layer, K its thermal conductivity while I is the solar constant, and φ and δ are geographic latitude and solar declination, respectively. Both I and δ are considered constant within a period P (one day), while they vary annually.

Using Milankovic's general approach, (Reference [3]), daily temperature averages, T_{ave} for any day of the year and any location on Earth can be estimated from his energy balance equation at the top surface.

Note 1: Parameters T_{ave}, T_{amp} and ε depend on location (φ), season (I, δ), and the top layer thermal characteristics A, E, K, a!

Note 2: At the top surface (snow-air boundary), Milankovic's formula becomes:

$$T(0,t) = T_{ave} + T_{amp}\cos\left(\frac{2\pi}{P}t - \Delta\right).$$

The approach outlined above achieves the objective of obtaining daily theoretical estimates of TSE pairs, **(t_{max},T_{max}), (t_{min},T_{min}),** based on insolation and physical properties of the top layer.

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