Is Weather Chaotic? Coexistence of Chaos and Order Within a Generalized Lorenz Model

by

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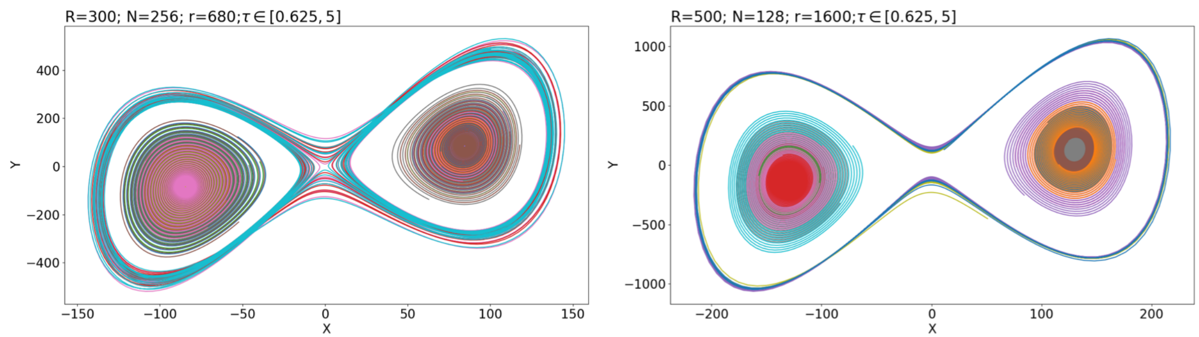

100th AMS Annual Meeting

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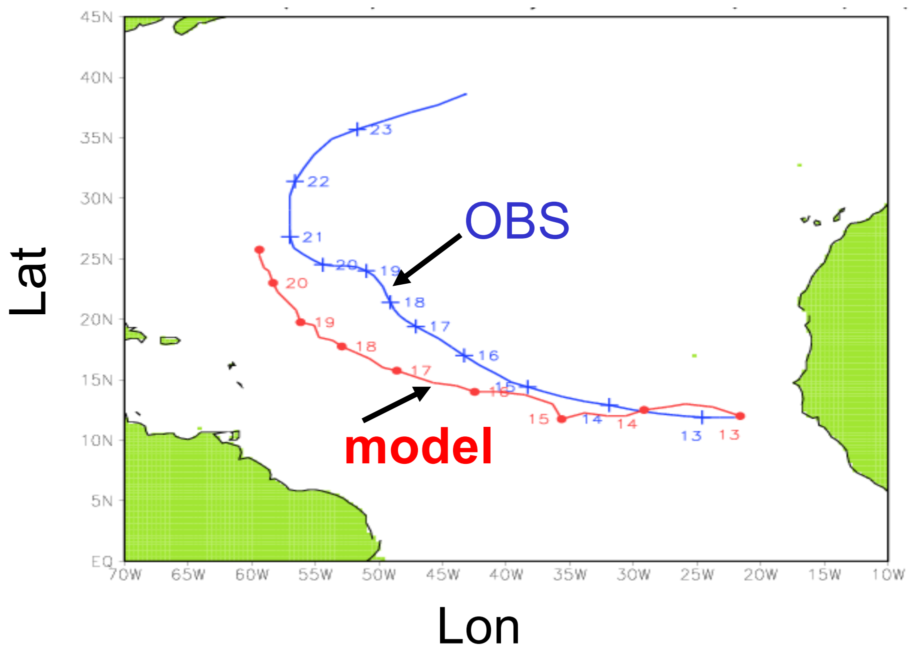
12-16 January 2020

Outline

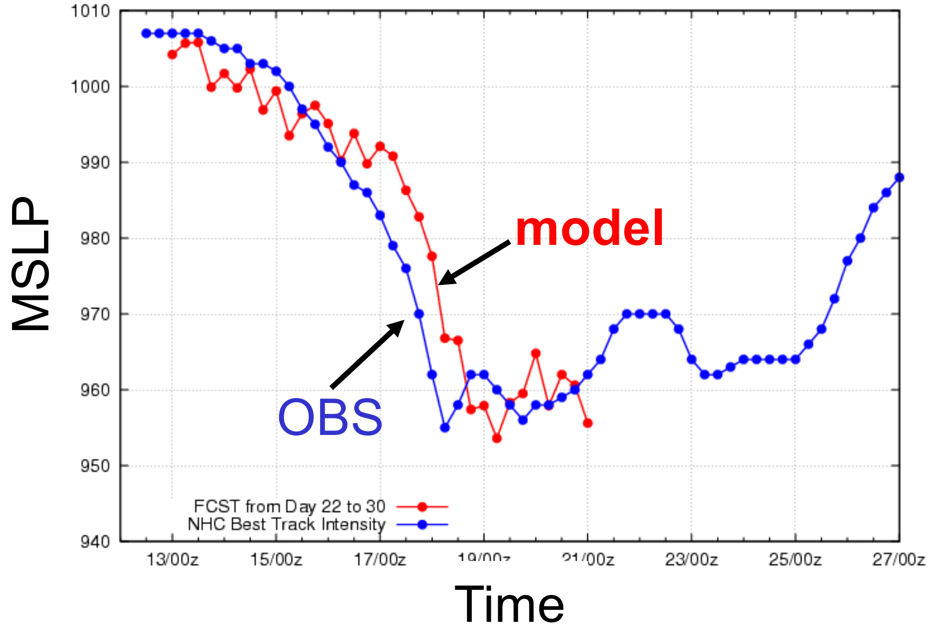
- Introduction
  - 30 day Predictions of African Easterly Waves (AEWs) and Hurricanes
  - Goals and Approaches
- The Lorenz (1963) Model and the Generalized Lorenz Model (GLM)
  - Three Types of Solutions (e.g., Steady-state, Chaotic, and Limit Cycle Orbits)
  - Major Features of the GLM
  - Two Kinds of Attractor Coexistence: Coexistence of Chaos and Order
- A Hypothetical Mechanism for Predictability of AEWs
- Summary and Outlook
- Appendix
  - Physical Processes in Various Lorenz Models (1969/1972, 1984, 1996/2005)
  - Homoclinic Orbits and Solitary Waves in the Non-dissipative Lorenz Model, Duffing, Nonlinear Schrodinger, Korteweg-de Vries (KdV) Equations
  - Instability, Turbulence, and Spatiotemporal Chaos in the Lorenz (1969), Leith(1971), and Kuramoto-Sivashinsky Equations
  - Coexistence of Diverged and Chaotic Hurricane Tracks

Goals: Understanding Simulations Between Day 22-30

Track Forecast



Intensity Forecast



Hurricane Helene: 12-24 September, 2006
(Shen et al, 2010; Shen, 2019b)

Our goals include addressing the following questions:

- Can global models have skill for extended-range (15-30 day) numerical weather prediction? Why?
- Is weather chaotic?

The Lorenz Model and A Generalized Lorenz

The Lorenz 1963 Model
(3DLM, Lorenz, 1963)
Three Dimension

$$\frac{dX}{d\tau} = -\sigma X + \sigma Y,$$

$$\frac{dY}{d\tau} = -XZ + rX - Y,$$

$$\frac{dZ}{d\tau} = XY - bZ.$$

primary scale modes

A Generalized Lorenz Model
(GLM, Shen, 2019a)
Any Odd Dimension

$$\frac{dX}{d\tau} = -\sigma X + \sigma Y,$$

$$\frac{dY}{d\tau} = -XZ + rX - Y,$$

$$\frac{dZ}{d\tau} = XY - XY_1 - bZ,$$

an extension of the nonlinear feedback loop

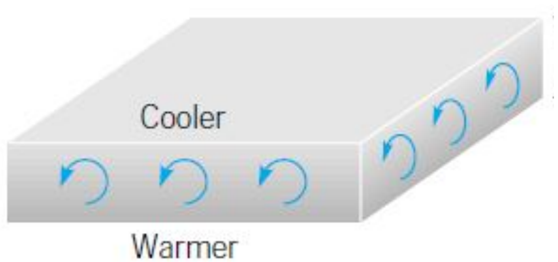
smaller scale modes

$$\frac{dY_j}{d\tau} = jXZ_{j-1} - (j+1)XZ_j - d_{j-1}Y_j, \quad j \in [1, N],$$

$$\frac{dZ_j}{d\tau} = (j+1)XY_j - (j+1)XY_{j+1} - \beta_j Z_j, \quad j \in [1, N],$$

$$N = \frac{M-3}{2}; \quad d_{j-1} = \frac{(2j+1)^2 + a^2}{1+a^2}; \quad \beta_j = (j+1)^2 b.$$

- σ – Prandtl number
- r – Rayleigh number
- b – Physical proportion



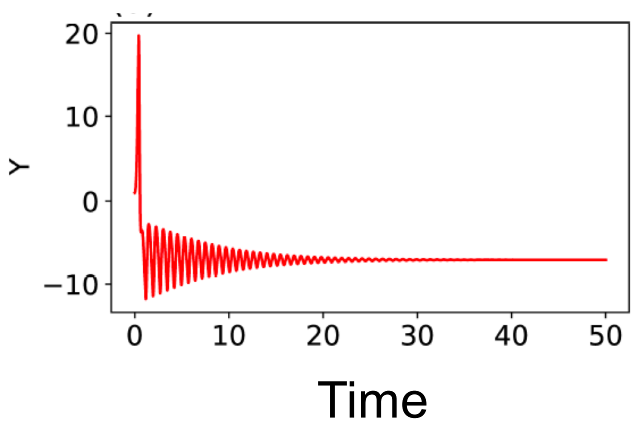
Three Kinds of Attractors and Butterfly Effect

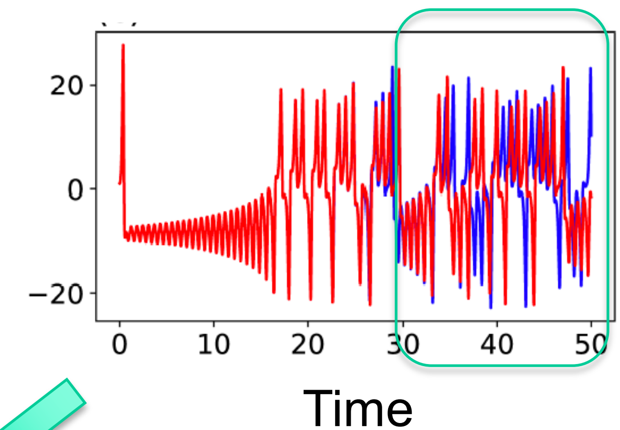
- Depending on the relative strength of heating, as compared to $r_c = 24.74$ and $R_c = 313$, three types of solutions within the 3DLM are:
  - ✓ control run (blue): $(X, Y, Z) = (0,1, \quad 0)$
  - ✓ parallel run (red): $(X, Y, Z) = (0,1 + \epsilon, 0)$, $\epsilon = 1e - 10$

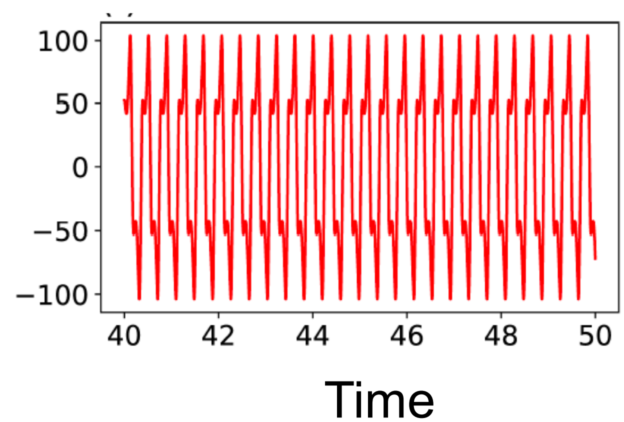
A steady-state solution $(r < r_c)$

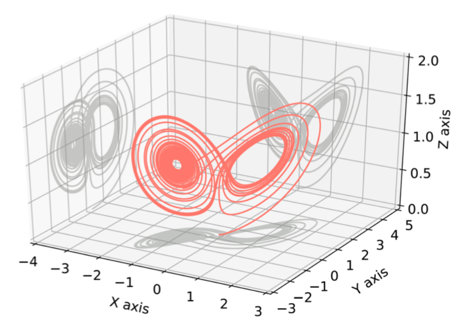
A chaotic solution $(r_c < \mathrm{r} < R_c)$

A limit cycle $(R_c < r)$









sensitive dependence

The LC, except for its phase, has no long term memory regarding ICs.

- Butterfly effect of the first kind (BE1)
- appearing within a finite range of parameters

Major Features of the GLM

As discussed in Shen (2019a) and Shen et al. (2019), the GLM with many M modes possesses the following features:

(1) any odd number of M greater than three; a conservative system in the dissipationless limit;

(2) three types of solutions (that also appear within the 3DLM);

(3) energy transfer across scales by the nonlinear feedback loop (NFL);

(4) slow and fast variables across various scales;

(5) aggregated negative feedback;

(6) increased temporal complexities of solutions associated with additional (incommensurate) frequencies that are introduced by the extension of the NFL (i.e., spatial mode-mode interactions);

(7) hierarchical scale dependence;

(8) two kinds of attractor coexistence;

- The 1st kind of Coexistence for Chaotic and Steady-state Solutions,
- The 2nd kind of Coexistence for Limit Cycle and Steady-state Solutions.

Aggregated Negative Feedback

model	r_c	heating terms	solutions	references
3DLM	24.74	rX	steady, chaotic, or LC	Lorenz (1963)
3D-NLM	n/a	rX	periodic	Shen (2018)
5DLM	42.9	rX	steady, chaotic, or LC/LT	Shen (2014a,2015a,b)
5D-NLM	n/a	rX	quasi-periodic	Faghih-Naini and Shen (2018)
6DLM	41.1	rX, rX_1	steady or chaotic	Shen (2015a,b)
7DLM	116.9	rX	steady, chaotic or LC/LT	Shen (2016, 2017)
7D-NLM	n/a	rX	quasi-periodic	Shen and Faghih-Naini (2017)
8DLM	103.4	rX, rX_1	steady or chaotic	Shen (2017)
9DLM	102.9	rX, rX_1, rX_2	steady or chaotic	Shen (2017)
9DLMr	679.8	rX	steady, chaotic, or LC/LT	Shen (2019a)

rc: a critical value of the Raleigh parameter for the onset of chaos; LC: limit cycle; LT: limit torus

- Higher-dimensional LMs require larger heating parameters for the onset of chaos, indicating aggregated negative feedback (Shen, 2019a).
- The aggregated negative feedback may change the stability of non-trivial critical points, leading to attractor coexistence.

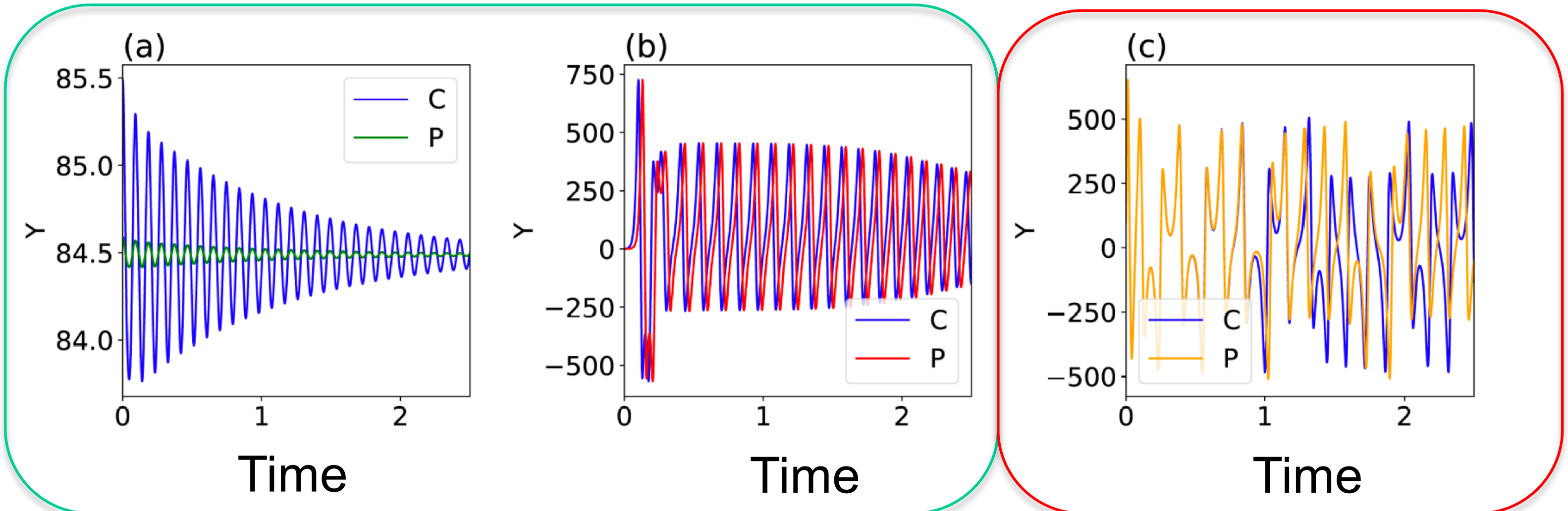
The First Kind of Attractor Coexistence

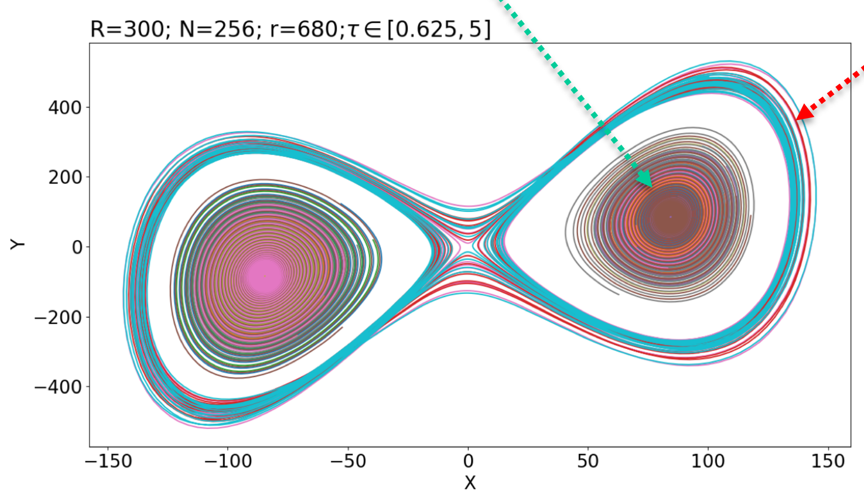
- The 1st kind of attractor coexistence within the 9DLM indicates final state sensitivity to ICs.

non-chaotic orbits

chaotic orbit

r=680



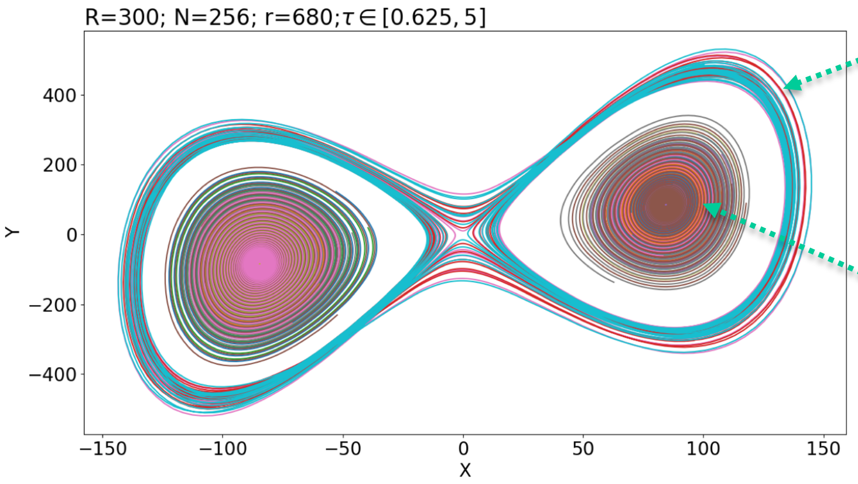


The coexistence of chaotic and non-chaotic orbits displays distinct regions of attraction.

(Note that the figure displays 256 orbits with different ICs)

Two Kinds of Dependence on ICs

- The 9DLM with attractor coexistence displays two kinds of data dependence.



a chaotic attractor displays:
- sensitive dependence on ICs
- i.e., BE1

a point attractor displays:
- insensitivity to ICs
- i.e., no BE1

- The role of initial tiny perturbations is different within the 3D- and 9D-LM.

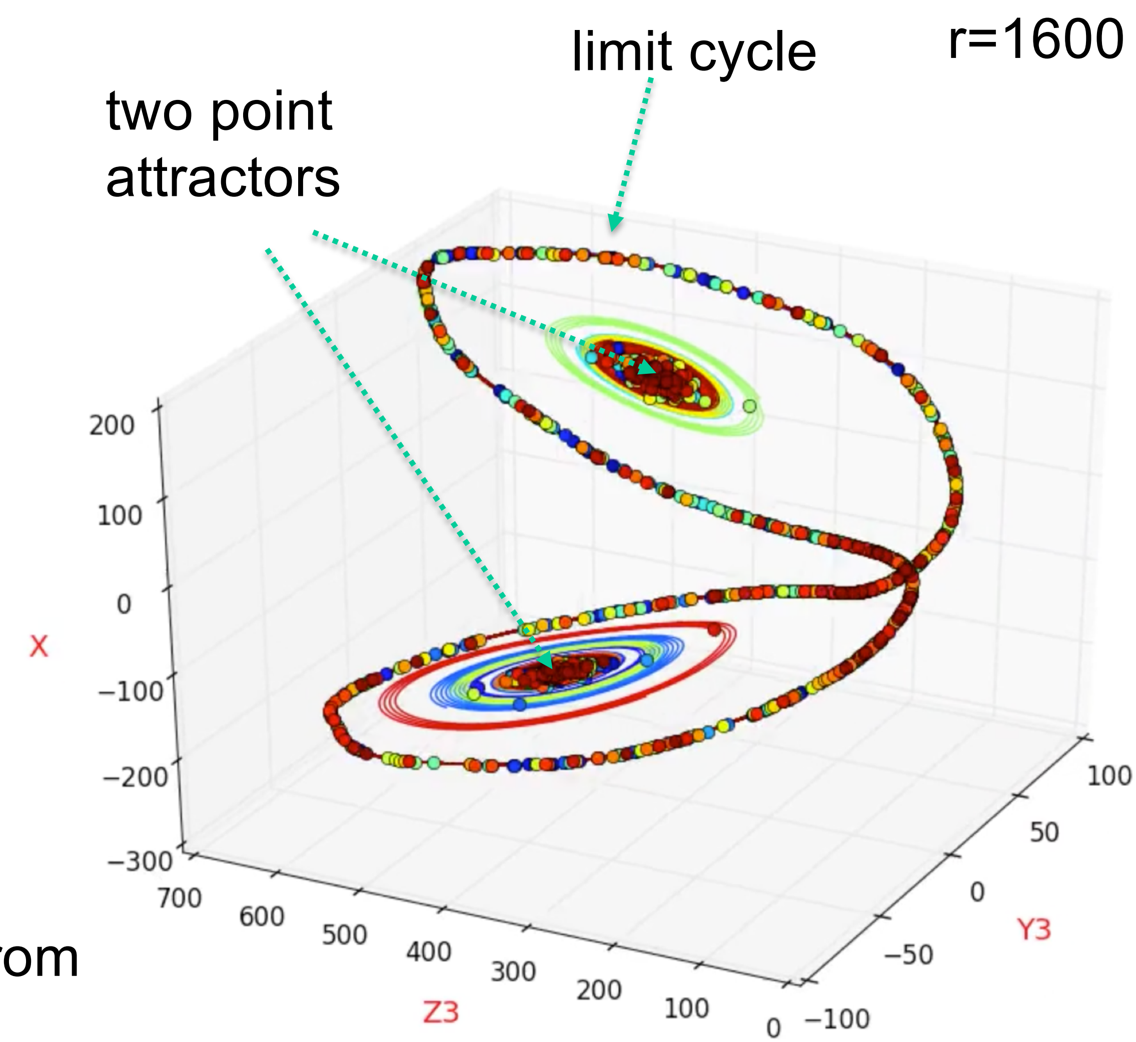
3DLM	9DLM
always important	important or unimportant
BE1 always appears	BE1 may or may not appear

- As a result, the 9DLM is more realistic, as compared to the 3DLM.

The Second Kind of Attractor Coexistence

- For the 2nd kind of attractor coexistence, a limit cycle (LC) that is an isolated closed orbit coexists with point attractors.

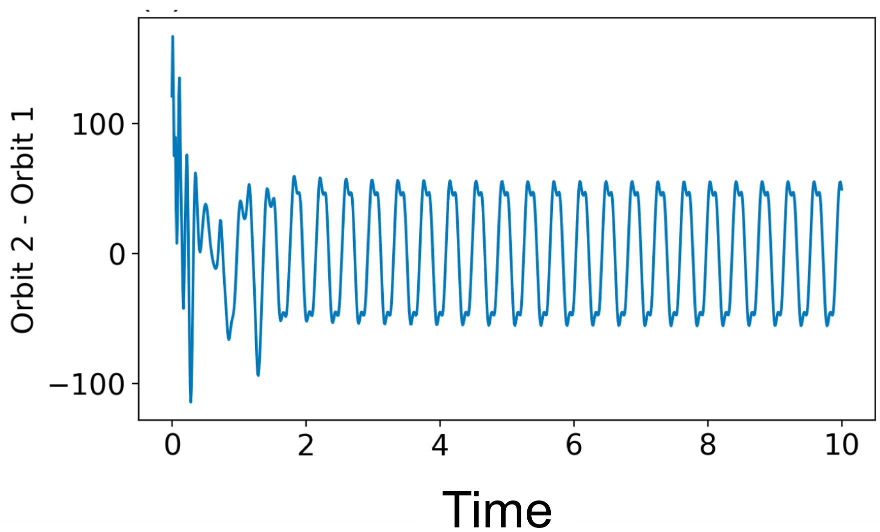
- Time evolution of 2,048 orbits in the X-Y_3-Z_3 space using the 9DLM.
- The total simulation time is $\tau = 3.5$.
- Transient orbits are only kept for the last 0.25 time units, i.e. for the time interval of [max (0, T-0.25), T] at a given time T.

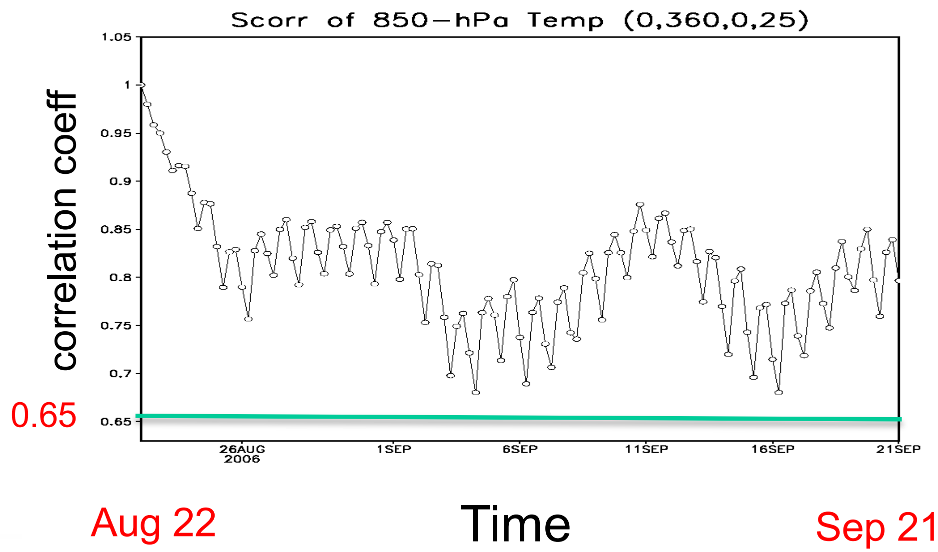


- The animation is available from https://goo.gl/sMhoUb.

A Hypothetical Mechanism for the Predictability of the 30-day Runs

Limit Cycle	African Easterly Waves (AEWs)
oscillatory errors	oscillatory correlation coefficients





strong heating + nonlinearity	during summer (JAS)

- The realistic simulation of Hurricane Helene (2006) from Day 22 to 30 became possible as a result of the realistic simulation of the
  1. periodicity (or recurrence) of AEWs and
  2. downscaling process of the 4th AEW.

Concluding Remarks

1. Within the 3DLM, chaotic solutions, displaying the sensitive dependence of solutions on initial conditions (ICs), appear only within a finite range of the Rayleigh parameters. The feature of sensitivity to ICs is referred to as the butterfly effect of the first kind (BE1). (By comparison, the the butterfly effect of the second kind is a metaphor for indicating the enabling role of a tiny perturbation in producing an organized large-scale system.)
2. Major features of the generalized Lorenz model (GLM) include: (a) Aggregated negative feedback; (b) Two kinds of attractor coexistence, leading to two kinds of data dependence. The BE1 does not always appear.
3. "*As with Poincare and Birkhoff, everything centers around periodic solutions*," Lorenz and chaos advocates focused on the existence of non-periodic solutions and their complexities.
4. We propose that the entirety of weather possesses a dual nature of chaos and order. The duality should be taken into consideration to revisit the predictability problem.

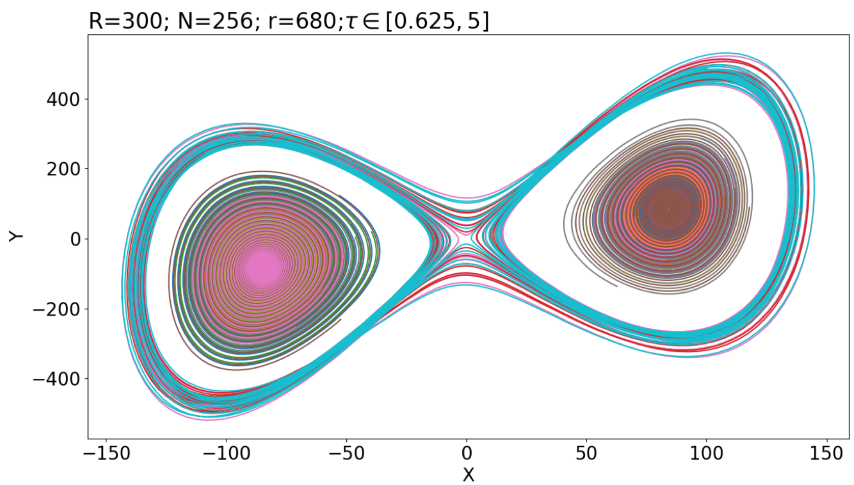
Future Tasks

- Detect (Nonlinear) Oscillatory Signals by
  - applying the Parallel Ensemble Empirical Mode Decomposition (PEEMD) to decompose data into oscillatory modes and non-oscillatory trend mode (Shen et al. 2017; Wu and Shen, 2016);
  - performing the Recurrence Analysis (Reyes and Shen, 2019a,b);
  - performing the Kernel Principle Component Analysis for classification of solutions (Cui and Shen, 2019, under revision).
  - [Apply the above to analyze MJO signals and compare results with those using the analysis of Real-time Multivariate MJO (RMM) Index]
- Improve the Simulations of Nonlinear Oscillatory Signals (e.g., limit cycle or quasi-periodic orbits) by
  - reducing numerical dissipations to avoid computational chaos (e.g., the Logistic equation vs. the Logistic map; Lorenz, 1989)
  - examining the potential impact of increased resolutions and newly added components on the generation of new incommensurate or commensurate frequencies, leading to quasi-periodic solutions.

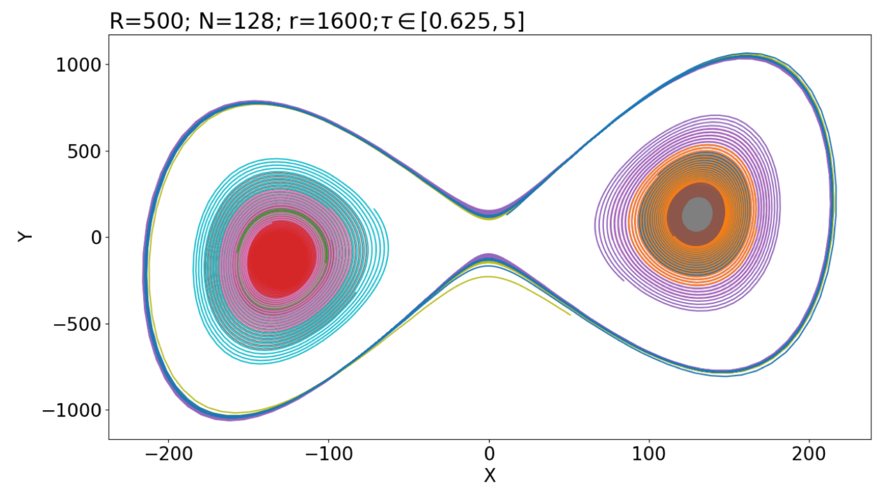
Takeaway Messages

- The entirety of weather possesses a dual nature of chaos and order.
  - The above refined view is neither too optimistic nor pessimistic as compared to the Laplacian view of deterministic predictability and the Lorenz view of deterministic chaos.

- ``*there is no reason that the limit of predictability is a fixed number*'' as suggested by Prof. Arakawa (Lewis, 2005, MWR).
  - In some cases, we obtained realistic predictions with a predictability of over two weeks.



The first kind of attractor coexistence



The second kind of attractor coexistence

Acknowledgments and References

- We thank Drs. R. Anthes, B. Bailey, J. Buchmann, D. Durran, M. Ghil, B. Mapes, Z. Musielak, T. Krishnamurti (Deceased), G C Layek, C.-D. Lin, J. Rosenfeld, R. Rotunno, I. A. Santos, C.-L. Shie, S. Vannitsem, and F. Zhang (Deceased) for valuable comments and discussions.

- Selected references are listed below. Additional information can be found in Appendix and the following link: https://bwshen.sdsu.edu.

Selected References:

1. Shen, B.-W.*, R. A. Pielke Sr., X. Zeng, J.-J. Baik, T.A.L. Reyes#, S. Faghih-Naini#, R. Atlas, and J. Cui#, 2019: Is Weather Chaotic? Coexistence of Chaos and Order within a Generalized Lorenz Mode. Institut Henry Poincare, Paris, France, 12-15 November 2019.I (available from ResearchGate: http://doi.org/10.13140/RG.2.2.21811.07204) (recorded talk: http://bit.ly/2Z7ytRB)
2. Shen, B.-W.*, 2019a: Aggregated Negative Feedback in a Generalized Lorenz Model. International Journal of Bifurcation and Chaos, Vol. 29, No. 3 (2019) 1950037 (20 pages). https://doi.org/10.1142/S0218127419500378
3. Shen, B.-W.*, 2019b: On the Predictability of 30-day Global Mesoscale Simulations of Multiple African Easterly Waves during Summer 2006: A View with a Generalized Lorenz Model. Geosciences 2019, 9(7), 281; https://doi.org/10.3390/geosciences9070281
4. Shen, B.-W.*, T. Reyes#, and S. Faghih-Naini#, 2018: Coexistence of Chaotic and Non-Chaotic Orbits in a New Nine-Dimensional Lorenz Model. In: Skiadas C., Lubashevsky I. (eds) 11th Chaotic Modeling and Simulation International Conference. CHAOS 2018. Springer Proceedings in Complexity. Springer, Cham. https://doi.org/10.1007/978-3-030-15297-0_22
5. Shen, B.-W.*, 2018: On periodic solutions in the non-dissipative Lorenz model: the role of the nonlinear feedback loop. Tellus A: 2018, 70, 1471912, https://doi.org/10.1080/16000870.2018.1471912.

Appendix

#17

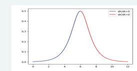
Physical Processes in Various Lorenz Models

Lorenz 1963 Model (3DLM):	**Lorenz 1972/1969 Model:**
• nonlinear and chaotic • limited scale interactions (with 3 modes) • Lyapunov exponent (LE) analysis • KE and PE, PDE based (Rayleigh-Benard Convection)	• multiscale but linear (21 modes) • growth rate analysis using a realistic basic state • KE, PDE based (a conservative system with no forcing or dissipation)
Lorenz 1984 Model (with 3 ODEs):	**Lorenz 1996/2005 Model:**
• missing detailed derivations • limited scale interactions • non-zero volume of the solution	• nonlinear and chaotic with multiple spatial scales • equal weighting in dissipations • KE, not PDE based

Lorenz (1989) discussed computational chaos using the following models with "limit cycle (LC) solutions": (i) a logistic equation; (ii) a simplified 3DLM (with LC solutions); and (iii) a simple 2D LC model.

#18

Homoclinic Orbits and Solitary Waves

A 3D non-dissipative LM (Shen, 2018)	Solutions	Other Equations
$X'' - \sigma r X + \frac{X^3}{2} = 0$ (Eq. 15 of Shen, 2018)	$X = \sqrt{2\sigma} cn(\sqrt{\sigma}\tau + 3K, k^2 = 1/2)$ cn: elliptic function (Eq. C9 of Shen, 2018)	Duffing Eq. $X'' + \delta X' + \alpha X + \beta X^3 = \gamma \cos(\omega\tau)$ $\delta = 0 \; and \; \gamma = 0$
$X' = \pm\sqrt{\sigma r X^2 - \frac{1}{4}X^4}$ (Eq. D4 of Shen, 2018)	 $X(\tau) = 2\sqrt{\sigma r} sech(\sqrt{\sigma r}\,\tau)$ (Eq. 34a of Shen, 2018)	Nonlinear Schrodinger (NLS) Eq. $(r')^2 + \delta r^2 + \frac{\gamma}{2} r^4 = E = 0$ $\delta < 0 \; and \; \gamma > 0$ (solitary wave of the amplitude)
$Z'' + 3\sigma Z^2 - 4\sigma r Z = 0$	$Z(\tau) = \frac{X^2(\tau)}{2\sigma}$ (Eq. 34c of Shen, 2018)	Korteweg-de Vries (KdV) Eq. $u_t + 6uu_x + u_{xxx} = 0$ $u = f(x - ct)$ $f'' + 3f^2 - cf = A = 0$

Homoclinic (and periodic) orbits	Solitary (and conidial) waves

#19

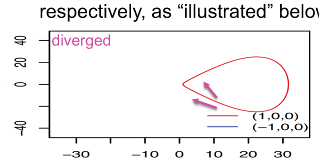
Instability, Turbulence, and Spatiotemporal Chaos

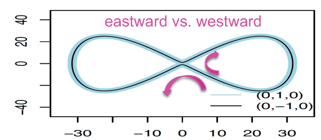
Equations	Physical Processes
Lorenz (1969) $\frac{\partial \nabla^2 \psi}{\partial t} = -J(\psi, \nabla^2\psi)$	• The Eq. contains nonlinearity but no forcing or dissipation. • The basic state that possesses a realistic spectrum was applied.
Leith (1971) $\left[\frac{d}{dt} + \nu k^2 - \alpha(k)\right] u_i(k,t) = -\frac{1}{2} i P_{ijk}(k) \sum_{p+q=k} u_j(p,t) u_k(q,t)$	• In addition to nonlinearity, the Eq contains ✓ instability function $(-\alpha(k)u_i)$ and ✓ dissipation $(\nu k^2 u_i)$ terms. • $\alpha(k)$ is added to obtain a stationary solution. • The basic state that possesses a realistic spectrum was applied.
Leith and Kraichnan (1972)	• similar to the above
Kuramoto Sivashinsky Equation $\frac{\partial u}{\partial t} = -uu_x - u_{xx} - u_{xxxx}$	• The Eq contains ✓ nonlinearity $(-uu_x)$, ✓ forcing $(-u_{xx})$, and ✓ dissipation $(-u_{xxxx})$ terms. • A simplified Eq with no $-u_{xxxx}$ is not well posted.

#20

(Coexisting) Diverged and Chaotic Tracks

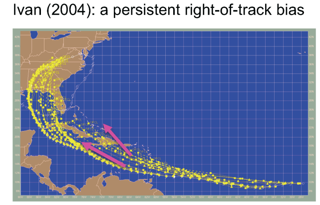
➢ Within a global model, diverged and chaotic tracks may coexist and are associated with instability and an unstable (chaotic) saddle point, respectively, as "illustrated" below.

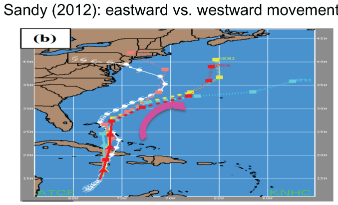




Ivan (2004): a persistent right-of-track bias

Sandy (2012): eastward vs. westward movement





• Shen et al. (2006): Geophys. Res. Lett., 33, L05801. • Shen et al. (2013): *Geophys. Res. Lett.* 40. 2013, DOI: 10.1002/grl.50934.

- Slides for this presentation are available from http://bit.ly/2tt0yqM.
- Shen, Poster #450 at AMS 2020, 4-6 PM, January 13, 2020, Hall B.
- Zeng, a town hall, 12:15-1:15 PM, January 15, 2020, Room 512.

Thank you!

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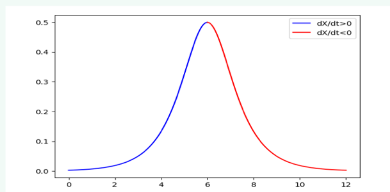
$X' = \pm\sqrt{\sigma r X^2 - \frac{1}{4} X^4}$ (Eq. D4 of Shen, 2018)	 $X(\tau) = 2\sqrt{\sigma r} sech(\sqrt{\sigma r}\,\tau)$ (Eq. 34a of Shen, 2018)	Nonlinear Schrodinger (NLS) Eq. $(r')^2 + \delta r^2 + \frac{\gamma}{2} r^4 = E = 0$ $\delta < 0 \;\; and \;\; \gamma > 0$ (solitary wave of the amplitude)
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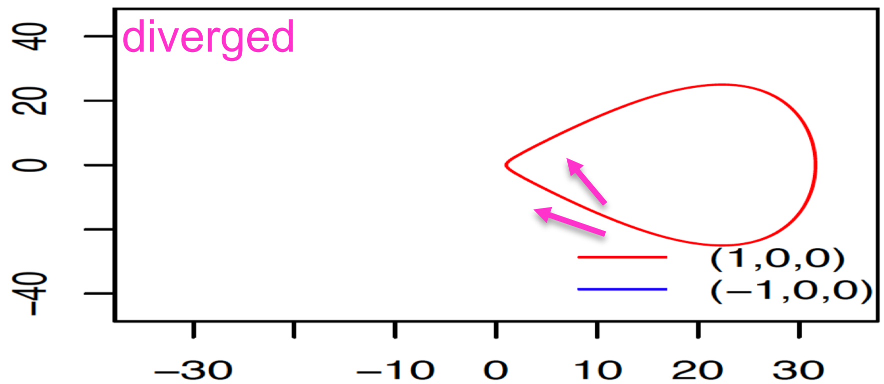
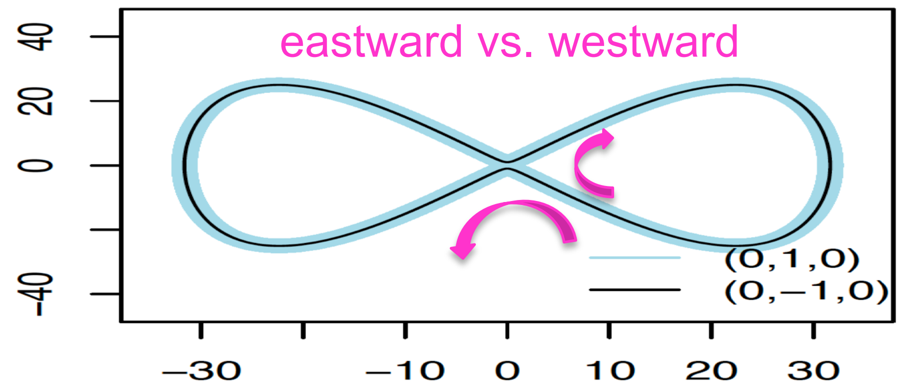
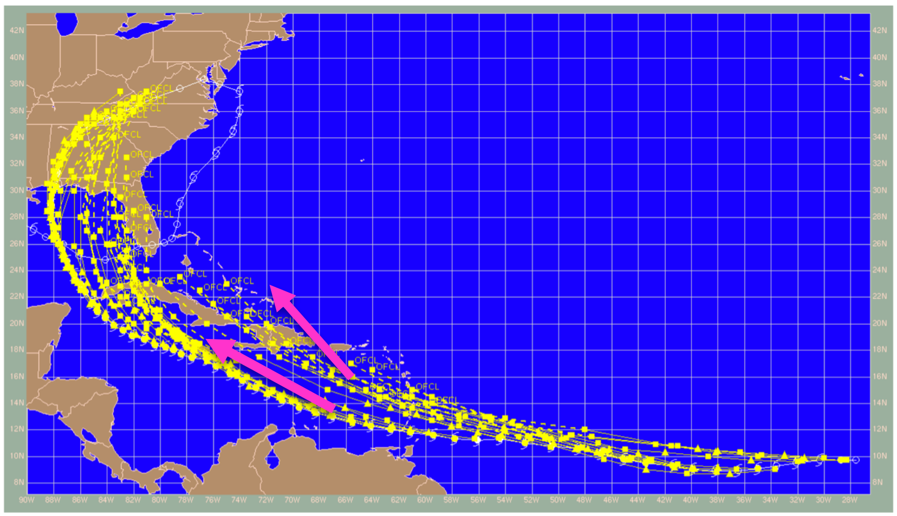
(Coexisting) Diverged and Chaotic Tracks

- Within a global model, diverged and chaotic tracks may coexist and are associated with instability and an unstable (chaotic) saddle point, respectively, as "illustrated" below.

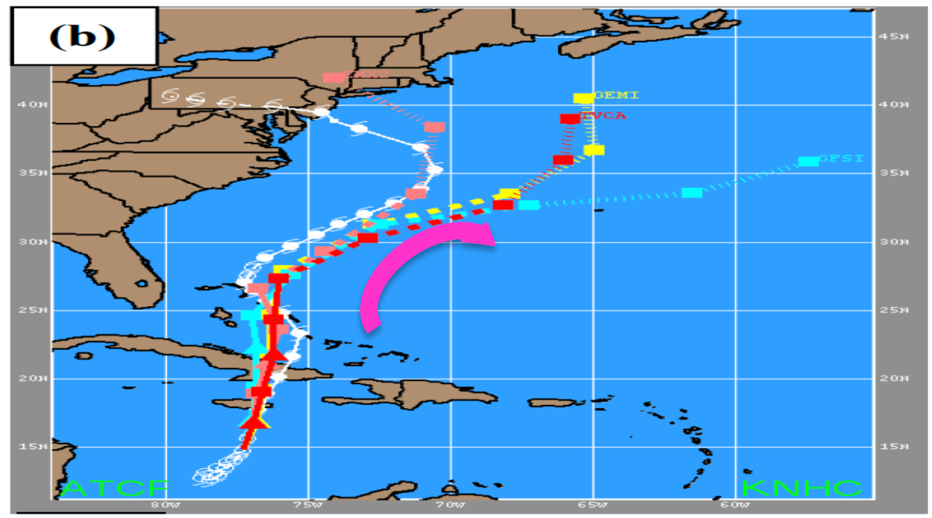




Ivan (2004): a persistent right-of-track bias



Sandy (2012): eastward vs. westward movement



- Shen et al. (2006): Geophys. Res. Lett., 33, L05801.
- Shen et al. (2013): *Geophys. Res. Lett.* 40. 2013, DOI: 10.1002/grl.50934.

Spatially Oscillatory Tracks of Hurricane Karl (2004)

J. Beven (2004)

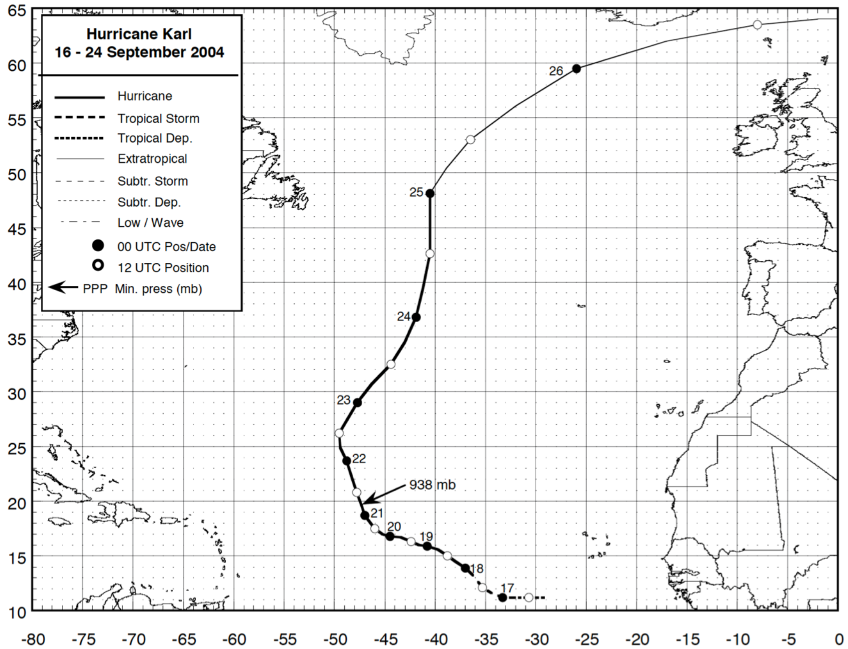
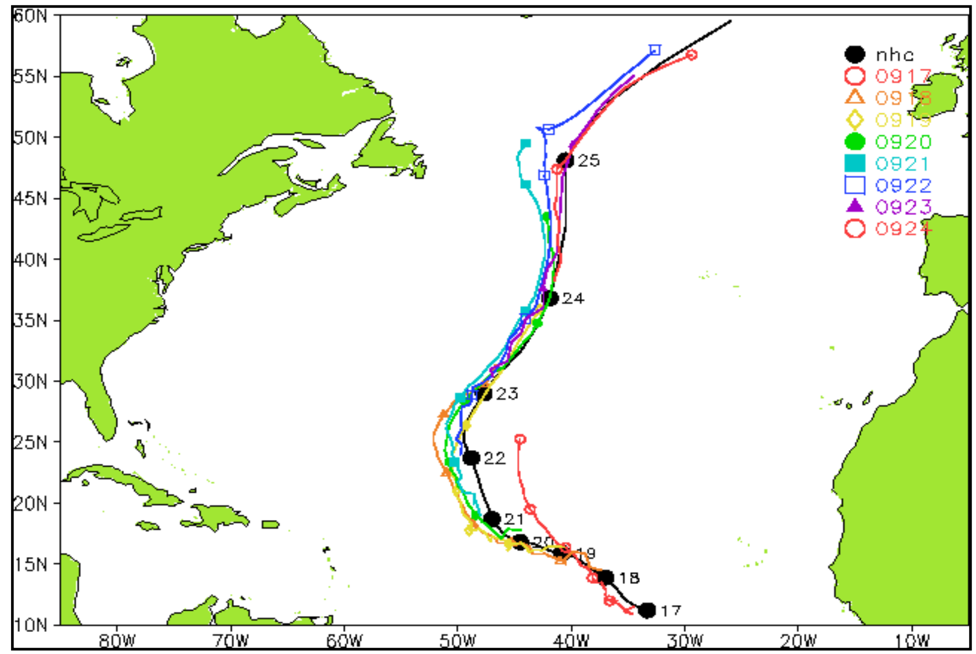


Figure 1. Best track positions for Hurricane Karl, 16 – 24 September 2004. Track during the extratropical stage is based on analyses from the NOAA Ocean Prediction Center and the United Kingdom Meteorological Office.

KARL (CAT-4), 8 runs

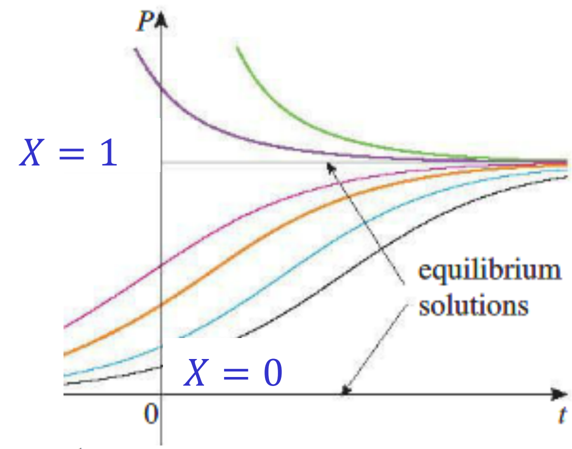


- *“Average official track errors for Karl are lower than the average official track errors for the 10-yr period 1994-2003, about 20% lower at 12-36 h increasing to 50-60% lower at 96 and 120 h.” (Beven, 2004).*
- *”Some of the track forecast models had average errors lower than the official. These include the GUNA consensus model, which was better at all times except 12 h, and the GFS global model, which was better at all times except 12 and 120 h” (Beven, 2004).*

Computational Chaos: An Illustration

- Logistic Equation (Flow)

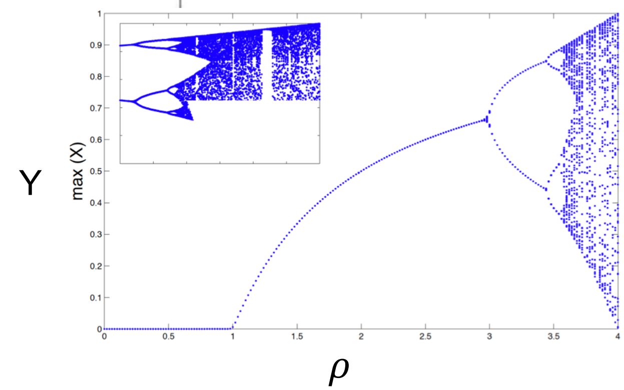
$$\frac{dX}{dt} = rX(1-X).$$



- Logistic Map

$$Y_{n+1} = \rho Y_n(1-Y_n)$$

$$\rho = 1 + r\Delta t \text{ and } X_n = \frac{1+r\Delta t}{r\Delta t} Y_n,$$



- Terminal Velocity

$$\frac{dW}{dt} = -g + \alpha \mid W \mid (-W), \qquad \alpha = \frac{3}{4}\frac{\rho}{\rho_p}\frac{0.46}{D}$$

The magnitude of dW/dt is proportional to W^2
(Shen and Lin, 1995)