A Climatological Analysis of Moist PV

Alex O. Gonzalez[1] **Christopher J. Slocum**[2]

[1]Department of Geological and Atmospheric Sciences, Iowa State University, Ames, Iowa
[2]NOAA/NESDIS Center for Satellite Applications and Research, Fort Collins, Colorado

Introduction

In the 1990–2000s, Wayne Schubert and colleagues published papers on generalizing potential vorticity (PV) to a moist, cloudy, precipitating atmosphere [1,2,3,4]. They suggested using virtual potential temperature as a thermodynamic variable in moist PV as it leads to the annihilation of the solenoidal term and in the limiting case of a dry atmosphere, moist PV reduces to Ertel's dry PV [3,4]. We highlight the differences between moist and standard PV on monthly timescales in the new ERA5 reanalysis, which is one of the few reanalyses that both predicts and outputs the mass water contents needed to calculate moist PV.

Standard vs. Virtual Potential Temperature

Standard and virtual potential temperature are defined as

$$\theta = \left(\frac{p_0}{p}\right)^{\kappa} T \quad \text{and} \quad \theta_\rho = \left(\frac{p_0}{p}\right)^{\kappa} T_\rho, \tag{1}$$

respectively, where virtual temperature is most generally given as

$$T_\rho = \frac{p}{\rho R_a} = \left(q_a + q_v \frac{R_v}{R_a}\right) T, \tag{2}$$

and q_a, q_v are the specific dry air mass content and specific humidity, respectively. To understand the differences between standard and moist PV, we take the difference between θ and θ_ρ, yielding

$$\theta_\rho - \theta = \left[\left(\frac{R_v}{R_a} - 1\right) q_v - q_l - q_r - q_i - q_s\right] \theta. \tag{3}$$

For consistency with ERA5 formulation, we assume $q_a = 1 - q_v - q_l - q_r - q_i - q_s$, where q_l, q_r, q_i, q_s are the specific water contents of liquid, rain, ice, and snow, respectively. From (3), water vapor acts to increase θ_ρ while all other forms of water act to decrease θ_ρ.

ERA5 Monthly Virtual Potential Temperature

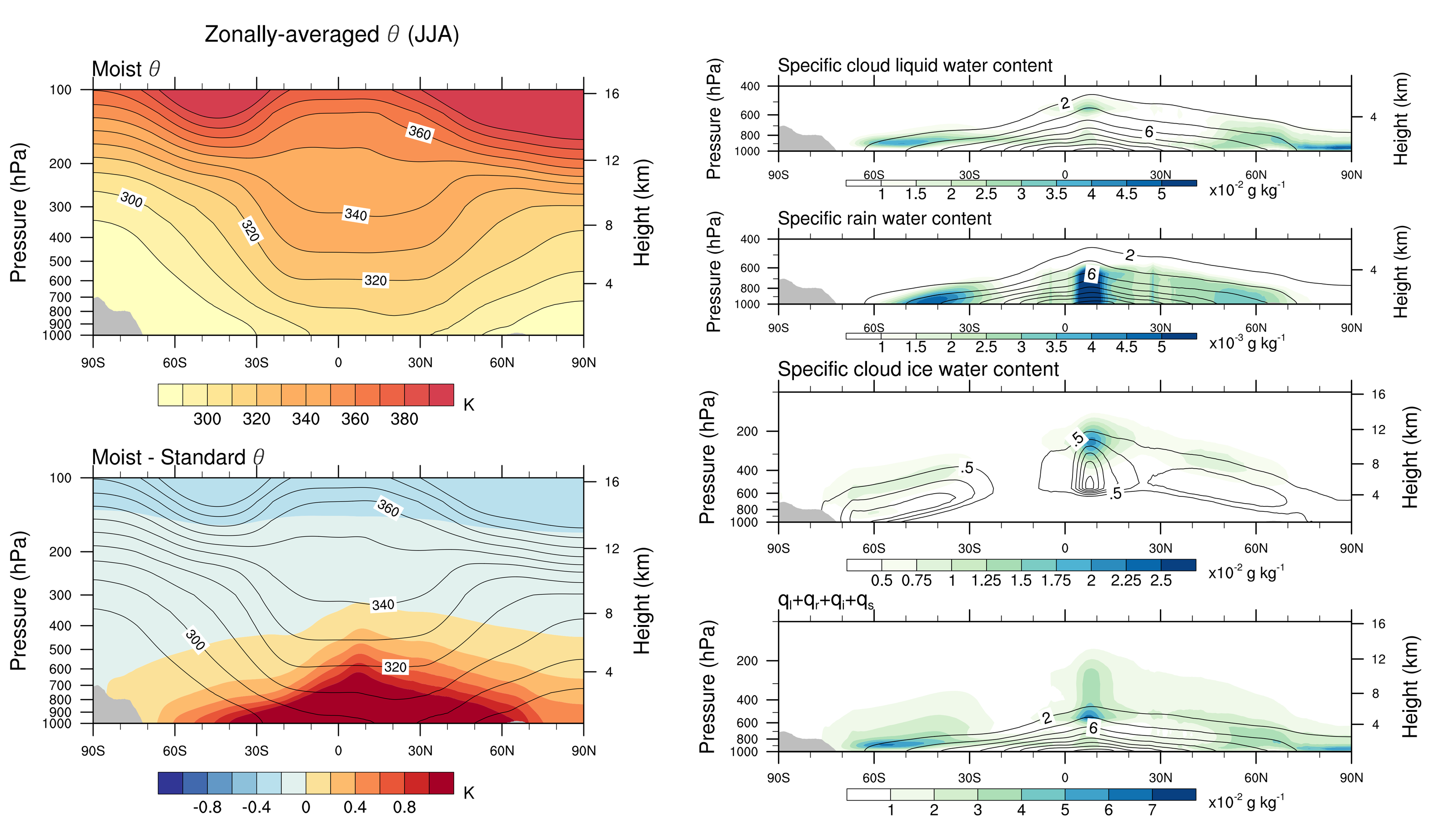

Figure 1: Left) June–August zonal mean shaded θ_ρ and $\theta_\rho - \theta$ (θ overlaid). Right) Shaded q_l, q_r, q_i, and $q_l + q_r + q_i + q_s$, with overlay of q_s in third panel from the top and q_v for all other panels.

- $\theta_\rho > \theta$ at low levels and $\theta_\rho < \theta$ at mid-upper levels, with the largest effects in the tropics through the subtropics.
- Specific humidity is the largest contributor while cloud liquid and ice water are secondary contributors, $O(10^2)$ smaller.
- Vertical θ_ρ gradients are smaller and meridional θ_ρ gradients are larger at low levels, which typically lead to smaller PV.

ERA5 Monthly Moist PV

The difference between standard PV (P) and moist PV (P_ρ) in pressure coordinates is given by

$$P_\rho - P = -g\zeta_a \cdot \nabla\left(\theta_\rho - \theta\right), \tag{4}$$

respectively, where

$$\zeta_a = \left(-\frac{\partial v}{\partial p}, \frac{\partial u}{\partial p}, \zeta_r + 2\Omega \sin\phi\right), \quad \nabla = \left(\frac{\partial}{a\cos\phi\,\partial\lambda}, \frac{\partial}{a\partial\phi}, \frac{\partial}{\partial p}\right). \tag{5}$$

Horizontal and vertical derivatives are computed using spherical harmonics and center finite differences, respectively. ζ_a is unchanged between P and P_ρ, thus potential temperature gradients are what distinguish P and P_ρ.

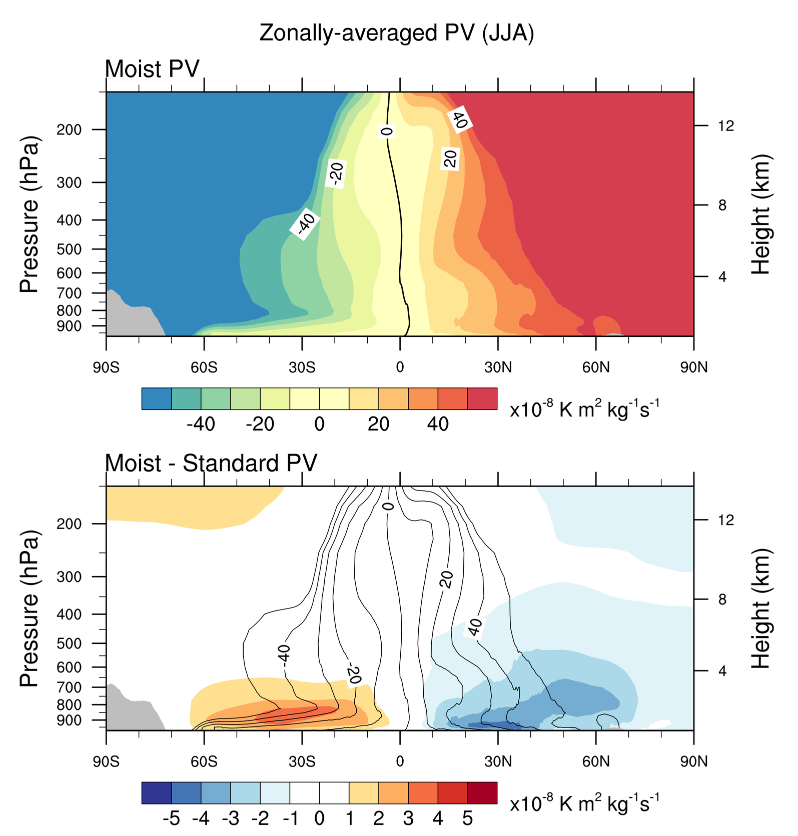

Figure 2: June–August zonal mean shaded P_ρ and $P_\rho - P$ (P_ρ overlaid).

- As anticipated, $P_\rho < P$, particularly at low to mid levels, with the largest effects in the tropics into the subtropics.

ERA5 Monthly Moist PV over Africa/Atlantic

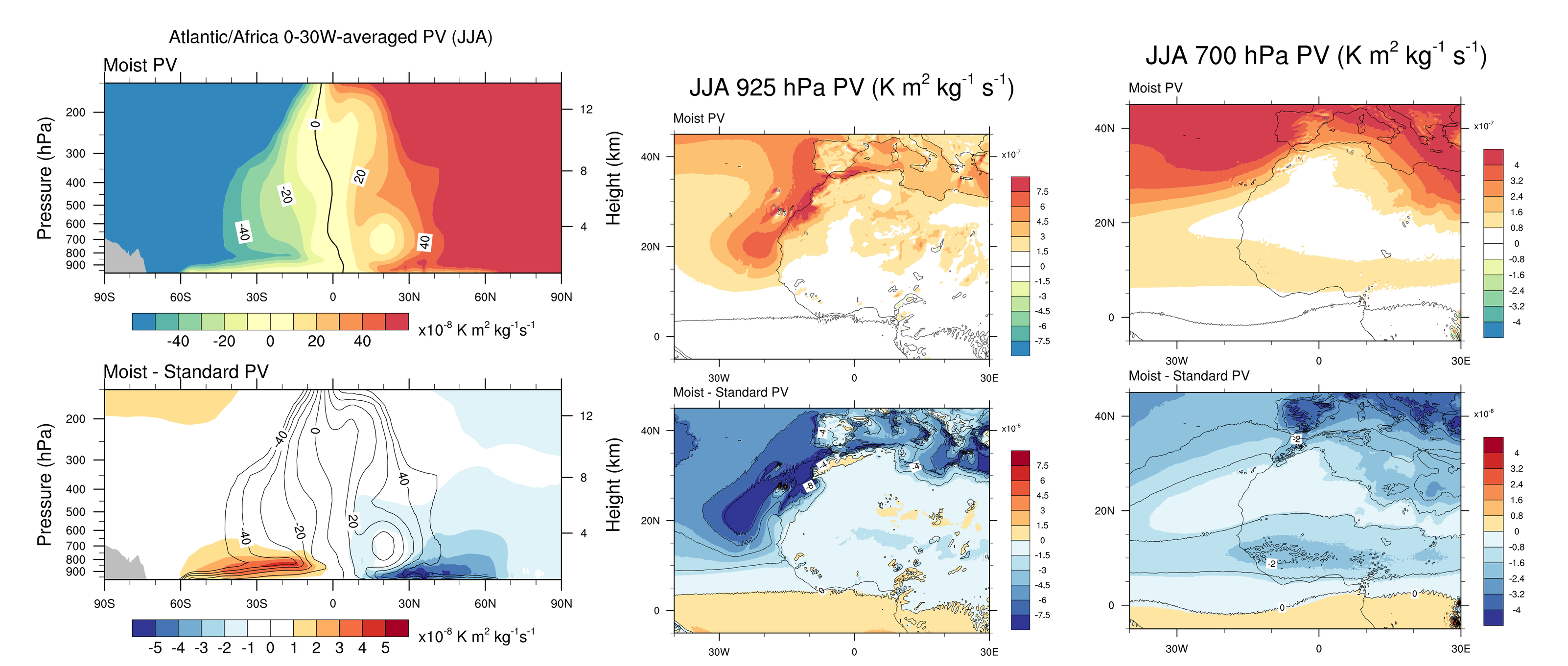

Figure 3: Left) June–August $0 - 30°$W mean shaded P_ρ and $P_\rho - P$ (P overlaid). Middle, Right) June–August shaded P_ρ and $P_\rho - P$ at 925 hPa and 700 hPa. The vertical component of $P_\rho - P$ is overlaid in the bottom panels of the middle and right subplots.

- Two phenomena of interest are stratocumulus clouds west of Africa and African meridional PV gradient reversal near 700 hPa.
- In stratocumulus regions, $P_\rho < P$, where high specific humidity below cloud liquid water leads to $(\partial\theta_\rho/\partial p) < (\partial\theta/\partial p)$.
- The meridional PV gradient reversal decreases over Africa because of moisture and clouds in the ITCZ to the south, where again $(\partial\theta_\rho/\partial p) < (\partial\theta/\partial p)$ dominates.

Conclusions and Future Work

Moist PV is typically smaller than standard PV on monthly time scales as the vertical component of PV dominates over the meridional component on large space and time scales. We suggest that the differences between moist and standard PV are larger on smaller space and time anomalies and deserve further study in remotely sensed observations and reanalyses. Future work will also involve writing and sharing scripts with the community that diagnose moist PV using either pressure or model level data.

Acknowledgments

We acknowledge helpful comments by Wayne Schubert, Paul Ciesielski, and Rick Taft. This work is supported by startup funding from Iowa State University.

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