9.5 LATTICE BOLTZMANN METHOD FOR OCEAN OIL SPILL PROPAGATION MODEL AND SIMULATION

- A COMPARISON STUDY OF NAVIER STOKES MODEL AND ADVECTION DIFFUSION MODEL

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1. INTRODUCTION

Ocean oil spills have devastating impacts on marine ecosystems and human society in the surrounding coastal areas. The 2010 Deep Water Horizon spill in the Gulf of Mexico lasted 87 days and is estimated to have released over 3 million barrels of oil. It impacted over 1,600 miles of coastline, killed over 8000 marine animals/seabirds and caused direct economic loss from fishing and tour industries estimated at tens of billions of dollars [1]. In addition, the impacts to long-term public health and quality of life of millions of people are still unknown.

The transport of oil spilled into the ocean is a complex process that depends in a critical way on the current, wind, temperature and chemical composition of the oil and seawater [2]. Weathering further compounds the complexity of this process, a phenomenon which involves evaporation, emulsification, dissolution, oxidation and microbial processes. Spilled oil in oceans undergoes these physical, chemical and biological processes and will be transformed into substances with physical and chemical characteristics that differ from the original source material [3]. The authors in [3] describe the fate of oil spilled into oceans as going through three major phases: (i) after oil introduced into the oceans; (ii) transport the resulting degradation oil away from the source; and (iii) incorporate the residual substances into compartments of the earth's surface system. These compartments involve dissolution in the hydrosphere, deposition in the lithosphere, volatilization into the atmosphere, and ingestion by organisms in the biosphere. People have studied how each phase impacts the fate and behavior of spill oil, but it is not well understood how long each phase last and when and where one phase ends and another phase begins. Physical, chemical, and biological processes all interact to the spilled oil in oceans through all three phases. But one process may play a dominating role more than others in certain phase. Therefore, it is important to use suitable models in different phases that reflect the underlying behaviors of spilled oil.

Many models use the Advection-Diffusion Equation (ADE) with various ways of obtaining the ocean current, wind and tide data, to predict oil slick transport [4]. While solutions of the ADE provide changes in oil concentration over time and space, it does not compute the advection field, but uses an external ocean surface velocity field as input. ADE based models can work well for modeling the fate of spilled oil at certain phase, such as phase (ii). But it may not work well for other phases of oil spills, such as phase (i), where the oil concentration is very high in the vicinity of initial spill location. Therefore, it is important to consider the impact of velocity generated by rapid distribution of oil density. In such cases, we may need to take advantage of the Navier-Stokes Equation (NSE) that does conserve both mass and momentum while solving the velocity field.

We investigated the feasibility of using the Lattice Boltzmann Method (LBM) as a framework to model and simulate ocean oil spill propagation at the surface level. Previous research suggests that: (a) LBM has certain advantages compared to traditional analytical and numerical approaches for solving complex nonlinear ocean flow dynamic problems with reasonable accuracy and computational complexity [5]; (b) LBM can be a NSE or an ADE solver providing numerical solutions for ocean fluid problems that fit the governances of NSE or ADE [6]; (c) LBM provides a flexible structure to model multi-scale, multi-fluid ocean oil spills with a variety of boundary conditions [7]. Therefore, LBM can be a promising framework for modelling ocean oil spill fate and transport at different phases.

We develop a LBM model and simulation that is capable of providing numerical solutions for both NSE and ADE based models. To validate typical models, one of the most commonly used benchmarks for NSE solvers is Poiseuille Flow and for ADE solvers is Gaussian Hill with a simplified velocity field. However, the ocean surface current is a much more complex velocity field that is temporal-spatial dependent. The major contributions of our work are: (a) in addition to the most common benchmarks for LBM NSE and LBM ADE solvers, we tested the LBM ADE solver against a Finite Difference Method (FDM) ADE solver using a perturbation of the Taylor-Green velocity field. To the best of our knowledge, no such benchmark has been done in the past for an LBM ADE model using a velocity field as complex as the perturbed Taylor-Green field. Our test results show that the LBM ADE and the FDM ADE agree closely; (b) we developed a proof of concept prototype with a simple velocity projection schema that allows LBM NSE solvers to integrate several forces (ocean current, winds, tides, gravity) into a single ocean surface velocity field, which feeds into LBM ADE solvers to enhance its accuracy; (c) we experimented with coupling a LBM NSE solver with a LBM ADE solver using the NSE to solve the ocean surface velocity field, then feeding the velocity to the ADE solver for tracking oil concentration in slicks and their transport.

2. RELATED WORK

There are many publications regarding using LBM to model and solve ocean flow problems. Notably, Wolf-Gladrow's work [8] used the LBM to solve the linearized Munk Problem [9]. In another LBM application of ocean models, Nuraiman [10] used a 1D Shallow Water Equation representation of the Navier Stokes Equations coupled to the 2D Navier Stokes Equation to form a LBM using the Bhatnagar, Gross and Krook (BGK) kinetic theory [27]. But to our knowledge there have been only a few studies of oil spill tracking using the LBM. One of the most comprehensive studies was done by [11] and showed good agreement between simulated results and satellite observations from an oil spill in the Gulf of Beirut on July 15, 2006. The LBM model used a two relaxation parameter technique to facilitate numerical stability. In addition, a flux limiter computational technique was used to resolve sharp numerical boundaries, which led to negative densities. Further, an interpolation technique was used to permit a non-square lattice to resolve the flow along the elongated coastline studied. In addition, Ha and Ku used a LBM to simulate an adventive-diffusion formulation of the spread of an oil slick on the sea surface in [12] and confirmed the functionality of their model. Further, Li, Mei and Renwei solve the 2D convection-diffusion equation using the LBM in [13]. Other advection-diffusion equation solutions are presented in [14] by Vukadinovic et al. While not specifically studying oil transport, Li and Huang [15] used a coupled LBM formulation of the Shallow Water Equation and Contamination Concentration Transport. Excellent agreement was obtained between numerical predictions and analytical solutions in the pure diffusion problem and convection–diffusion problem. Banda and Seaid [16] also developed a LBM model to solve shallow water equations as the depth-averaged incompressible Navier-Stokes equations with conservation of momentum under the assumption that the vertical scale is much smaller than any typical horizontal scale and the pressure is hydrostatic. Then they apply their shallow water model to simulate pollutant transport in the Strait of Gibraltar.

Our literature review shows most of ocean oil spill and contamination transport models are based on the ADE. In addition to the above cited research work, GNOME (General NOAA Operational Modeling Environment), the modeling tool which the Office of Response and Restoration's (OR&R) Emergency Response Division has been using to predict the possible route, or trajectory, a pollutant might transport in a body of water [17]. GNOME is an ADE based tool, which relies on the accuracy of the ocean surface current velocity field to produce quality results.

ADIOS® (Automated Data Inquiry for Oil Spills) is NOAA's oil weathering model. It is an oil spill response tool that models how different types of oil weather (undergo physical and chemical changes) in the marine environment [18, 19]. As the authors in [3] have described, oil weathering is a process that spilled oil will be transformed into substances with mixed water and oil droplets which need to be modeled as multi species fluids.

An important aspect of the LBM is the ability to implement multicomponent fluids including immiscible fluids like oil and water [20, 21]. Simulating oil transport in seawater current has a number of very significant challenges including identifying the boundary layer between the seawater and oil. This challenge is complicated because the oil seawater mixture changes over time due to the effects of tides and wind as well as exposure to the atmosphere with solar radiation and temperature. Even without these complex physical and environmental effects, identifying the interface between the two dominant fluids, seawater and oil is a difficult free boundary problem. Assuming that the oil and water are immiscible, at least at the initial point of the release, Sébastien Leclaire et al. [22] in calculating the two fluid flows. The technical aspects of the LBM are based on a revised collision operator that uses a BGK single relaxation parameter technique for each fluid separately followed by a step which redistributes the two fluids to define an interface based on the gradient of the density of fluids. This step solves the free boundary problem and prepares the model for the streaming step.

LBM seems to be a promising tool, as a kinetic theory based modeling technique, that can support multimodal analysis for ocean oil spills including NSE and ADE based models, as well as multi-fluid models. It makes the LBM a uniform platform that can be applied to all three phases of ocean oil spill as described in [3].

3. LBM METHODOLOGY

We use the LBM to model ocean oil pollutants as a set of particles with certain density and mass located on a virtual grid (lattice) that maps over an area of ocean with boundary conditions representing coastal lines or islands. This model makes it possible to track particle spatial positions and microscopic momentum from a continuum to just a handful and similarly discrete in distinct steps. Particle positions are confined to the nodes of the lattice. Variations in momentum that could have been due to a continuum of velocity directions and magnitudes and varying particle mass are reduced (in a simple 2D model) to 9 directions and a single particle mass [23]. Figure 1 shows the Cartesian lattice and the velocities e_a (where a = 0, 1 … 8) is a direction index and e_0 = 0 denotes particles at rest. This model is known as D2Q9 as it is 2 dimensional and contains 9 velocities. It can be generalized to a 3 dimensional model as D3Q27 if we replace the lattice in D2Q9 with a cube with length, width and height are one lattice unit.

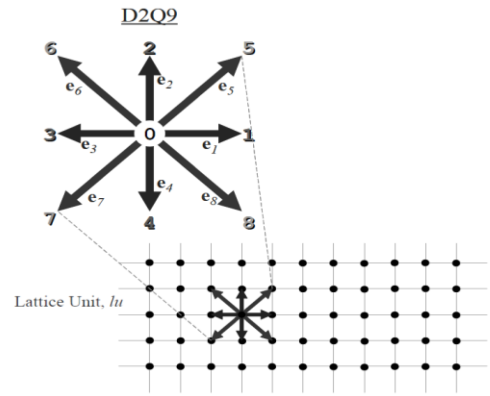


Fig.1 - A D2Q9 model with 9 velocities, including e_0 the particles at rest.

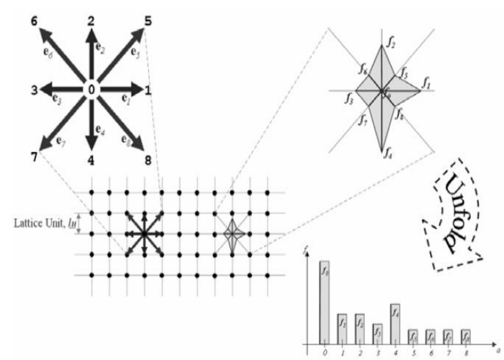


Fig.2 - Particle distribution function represents the percent of particles in the corresponding velocity bin.

The next step is to incorporate the single-species distribution function f, which has only nine discrete 'bins' instead of being a continuous function. The distribution function can conveniently be thought of as a histogram representing a frequency of occurrence. For example the shaded area in Figure 2 shows a likely oil pollutant propagation pattern after one time step. Accordingly, the macroscopic fluid density is defined as:

$$\rho = \sum_{a=0}^{8} f_a \qquad \text{(Eq. 1)}$$

The macroscopic velocity **u** is an average of the microscopic velocities $\mathbf{e}_a$ weighted by the directional densities f_a as defined as:

$$u = \frac{1}{\rho}\sum_{a=0}^{8} f_a e_a \qquad \text{(Eq.2)}$$

This simple equation allows us to pass from the discrete microscopic velocities that comprise the LBM back to a continuum of macroscopic velocities representing the fluid's motion.

When incorporating external forces, such as wind, gravity and others, that interact with the ocean water, Equation 2 can be modify as:

$$u = \frac{1}{\rho}\sum_0^8 e_a f_a + \frac{F\Delta t}{2\rho} \qquad \text{(Eq. 3)}$$

where the first term is the velocity due to mass density redistribution with conservation of momentum and the second term is due to external forces [6].

Equation 3 is a generalization of the LBM that is applicable to both NSE and ADE models. In the LBM NSE model, Equation 3 provides a mathematical base for developing a velocity projection schema that integrates ocean surface velocity into a LBM model as an external input, and then uses it to update local equilibrium distribution functions f^{eq}. On the other hand, in the LBM ADE model, we ignore the first term in Equation 3 and consider the second term as an advective velocity resulting from external forces.

The next steps are streaming and collision of the particles via the distribution function. The simplest approach to approximate the collision can be defined as:

$$f_a(x + e_a\Delta t, t + \Delta t) = f_a(x,t) - \frac{[f_a(x,t) - f_a^{eq}(x,t)]}{\tau} \qquad \text{(Eq. 4)}$$

where τ is a relaxation time used in the BGK operator. Although they can be combined into a single statement as above, collision and streaming steps must be separated if solid boundaries are present because the bounce back boundary condition is a separate collision. Collision of the fluid particles is considered as a relaxation towards a local equilibrium. The parameter τ is a relaxation time to reach equilibrium. A D2Q9 equilibrium distribution function f^{eq} is defined as:

$$f_a^{eq}(x) = w_a\rho(x)\left[1 + 3\frac{e_a u}{c^2} + \frac{2}{9}\frac{(e_a u)^2}{c^4} - \frac{3}{2}\frac{u^2}{c^2}\right] \qquad \text{(Eq. 5)}$$

where the weights w_a are 4/9 for a = 0, 1/9 for a = 1, 2, 3, 4, and 1/36 for a = 5, 6, 7, 8, and c is the basic speed on the lattice, one lattice unit per time step (1 *lu/ts)* in the simplest implementation. Note that if the macroscopic velocity **u** = 0, the equilibrium f_a are simply the weights times the fluid density.

To implementing the LBM model as a simulation program, Bao and Meskas [24] presented an algorithm outline that can be summarized as follows:

1. Initialize ρ, μ, f_a and f_a^{eq}
2. Streaming step: move $f_a \rightarrow f_a^*$ in the direction of e_a.
3. Compute macroscopic ρ and u from f_a^* using above equations (Eq 1. and Eq. 2).
4. Compute f_a^{eq} using (Eq. 5).
5. Collision step: calculate the updated distribution function using (Eq. 4): $f_a = f_a^* - (f_a^* - f_a^{eq})/\tau$.
6. Repeat steps 2 to 5.

where f_a^* holds intermediate values of density distribution after the streaming step.

During the streaming and collision step, the boundary nodes require some special treatments for the distribution functions in order to satisfy the imposed macroscopic boundary conditions.

The LBM as described has been shown to be second order accurate in time and space to the 2D incompressible Navier Stokes Equations by Kruger [6] and separately by Wolf-Gladrow [8].

4. LBM MODELS AND BENCHMARKS

Inspirited by previous research works, we started the research with a goal of using LBM to model ocean oil spills in all phases. As the first step, we investigated the feasibility and efficacy of LBM by designing NSE and ADE models and conducting simulations under different configurations. To validate our models and simulations, we benchmarked our results to analytical solutions and/or other well-known published result.

4.1 LBM NSE Model with Poiseuille Flow

The first LBM model we designed and implemented was a NSE solver. We implement the model in both Python and Matlab programs. As one of the most commonly used benchmark for LBM NSE, we benchmarked our model and simulation to a Poiseuille flow in a channel with analytical solutions based on analysis and numerical work by Wolf-Gladrow [8 pp 189-192].

Our LBM NSE Poiseuille flow model was configured as a 64 by 64 square lattice with relaxation time τ = 5.0 and a steady force driven flow through the channel from left to right. The boundary conditions were set so that the top and bottom channel walls are solid nodes with non-slip conditions. The left and right of the channel are periodic boundary conditions. Figure 3 shows our LBM model agrees well with corresponding analytical profiles for both a “dry” node formulation using Zho He boundary conditions and half-way bounce back boundary conditions, as well as a “wet” node formulation for full-way bounce back boundary conditions.

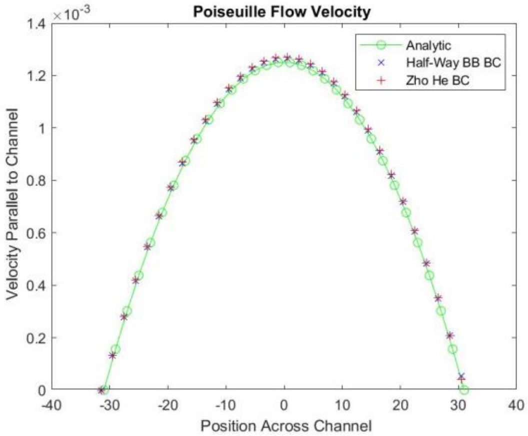


Fig. 3a LBM Numerical solution velocity profiles comparisons for “dry” node formulation

Fig.3a shows good agreement among the “dry” node boundary condition formulations, where the boundary node is located on land, not in the fluid and the corresponding analytic solution. A LBM NSE Model with Zho He boundary conditions [26] and LBM NSE Model with half-way bounce back boundary condition [8] and the analytic solution of the steady state solution of the NSE in the Poisueille flow example are shown.

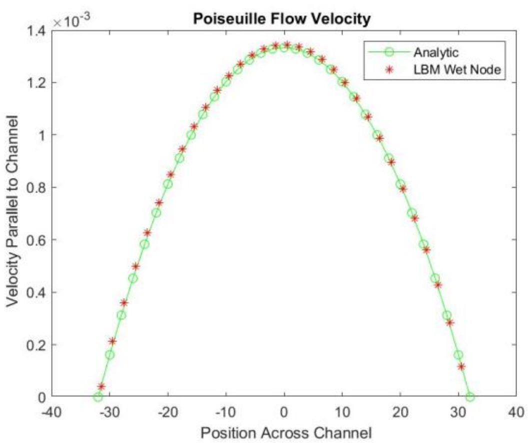


Fig. 3b LBM Numerical solution velocity profiles comparisons for “wet” node formulation

Fig. 3b shows excellent agreement between the “wet” node formulation [8] of the LBM NSE model of the Poiseuille channel, where the boundary node is placed on the channel edge in the fluid, and the corresponding analytic solution.

4.2 LBM ADE Model with Gaussian Hill

As mentioned in section 2, most ocean oil spill models are ADE based. Building on our LBM NSE model descripted in section 4.1, we analysed the similarity and difference between the LBM NSE and LBM ADE models in terms of their governing physical equations and the LBM equations.

When the NSE and the ADE are applied in near incompressible fluids, they can be expressed as equations below [6]:

(a) $$\frac{\partial u}{\partial t} + u\nabla u = -\frac{\nabla P}{\rho} + \nu\nabla^2 u + F \qquad \text{(NSE)}$$

where: u is fluid velocity; P is fluid pressure; ρ is fluid density; ν is fluid kinematic viscosity; F is an external force.

(b) $$\frac{\partial C}{\partial t} + u\nabla C = D\nabla^2 C + q \qquad \text{(ADE)}$$

where: C is mass concentration; D is diffusion coefficient; u is fluid velocity; q is a source term.

One of the strength of LBM method is that it can be used for both NSE and ADE models. The major difference between the two is that the NSE model conserves both mass and momentum while the ADE model only conserves mass.

The LBM stream and collision equation of the ADE is defined as:

$$g_a(x + e_a\Delta t, t + \Delta t) = g_a(x,t) - \frac{g_a(x,t) - g_a^{eq}(x,t)}{\tau_g}$$

(Eq.6)

where: $C = \sum g_a$ a = (0, …, 8)

Since the LBM equations for the NSE (Eq.4) and the ADE (Eq.6) are very similar, the algorithm outlined by Bao and Meskas [24] is also applicable to the ADE. For example, table 1 below shows the numerical quantities in both models side by side where the values of τ and τ_g are chosen independently.

Table 1. A list of numerical quantities in both models.

LBM NSE Model	LBM ADE Model
Calculate ρ(x,t) and u(x, t)	Calculate C(x, t)
Conserve mass and momentum	Conserve mass
Kinematic viscosity, $\nu = \frac{c^2}{9}(\tau - \frac{\Delta t}{2})$	Diffusion coefficient, $D = \frac{c^2}{9}(\tau_g - \frac{\Delta t}{2})$
Relaxation time, τ = 5.0	Relaxation time, τ_g=6.25
D2Q9 lattice (512 by 512)	D2Q9 lattice (512 by 512)
Lattice speed, c=Δx/Δt	Lattice speed, c=Δx/Δt
Calculated velocity field, u	Specified velocity field, u

We developed a 2D LBM ADE model with a 512 x 512 lattice and conducted a benchmark study of the LBM ADE model with a corresponding analytical solution, as well as, an ADE numerical solution based on a FDM (Finite Differential Method) [25]. Since most ADE model benchmarks were done with Gaussian Hills, we used a Gaussian Hill as an initial concentration distribution at location (200, 200) of the lattice. We used a hypothetical uniform ocean surface velocity field as an advection velocity,

$$U = (u_x, u_y) = (0.10, 0.10)$$

The Gaussian Hill analytic solution is defined as:

- Initial Gaussian Hill concentration:

$$C(X,t_0) = C_0 e^{-\frac{\left[(X-X_0-u_x t_0)^2+(Y-Y_0-u_y t_0)^2\right]}{2\sigma_0^2}} \quad \text{(Eq. 7)}$$

where: $C_0 = 1$, $\sigma_0 = 10$ and $t_0 = 0$.

- Gaussian Hill mass concentration at time t:

$$C(X,t) = \frac{\sigma_0^2}{\sigma_0^2+\sigma_D^0} C_0 e^{-\frac{\left[(X-X_0-u_x t)^2+(Y-Y_0-u_y t)^2\right]}{2(\sigma_0^2+\sigma_D^0)}} \quad \text{(Eq. 8)}$$

where the specified diffusion coefficient D is 1.5 and $\sigma_D = \sqrt{2Dt}$.

The parameters of LBM ADE and FDM ADE models are listed in Table2.

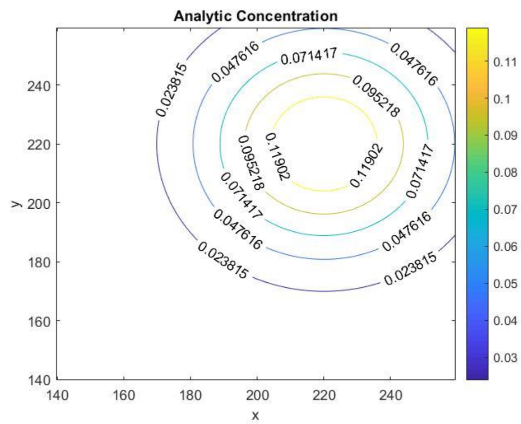
Table 2: LBM ADE and FDM ADE parameter list

LBM ADE	FDM ADE
Lattice grid: 512 by 512	N=512, X=[1:N], Y=[1:N]
Lattice velocity: $\Delta x/\Delta t=1$	$\Delta t = 1$
Relaxation time: $\tau_g=6.25$	Kappa=1.5
Initial Gaussian hill location: $(X_0,Y_0)=(200,200)$	Initial Gaussian hill location: $(X_0, Y_0)=(200, 200)$
Concentration $C_0= 1$,	Concentration $C_0= 1$,
$\sigma_0 = 10$	$\sigma_0 = 10$
External velocity: $U=(u_x, u_y)=(0.10, 0.10)$	External velocity: $U=(u_x, u_y)=(0.10, 0.10)$

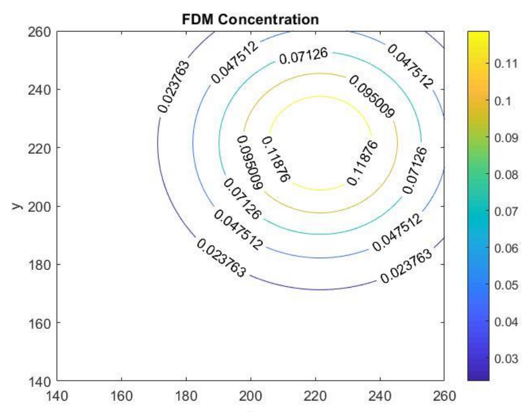
The LBM ADE numerical solution is implemented in Python. The FDM ADE numerical solution is implemented in MATLAB based on the Adams-Bashforth formulation with derivatives calculated using a discrete Fourier transform.

Our study results show very good agreement between LBM ADE solutions and both the analytic solution and the FDM ADE solution (Fig.4). It indicates that using the LBM ADE to model the transport of spilled oil on an ocean surface when the ocean velocity field is independent of the oil is valid.

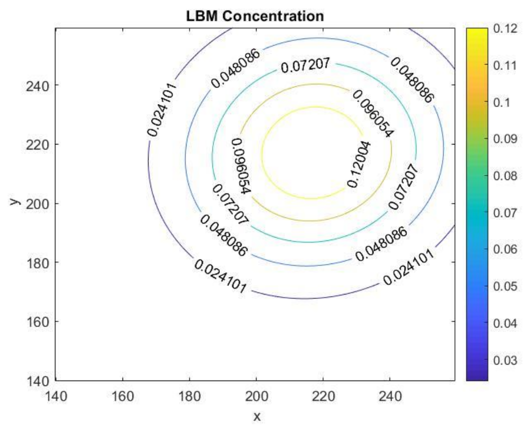
A challenge with the LBM is the coupling of the numerical relaxation parameter, τ_g and the diffusion, $D = c_s^2(\tau_g - \Delta t/2)$, where the speed of sound, c_s depends on the lattice size and time step [6]. In addition to having physical significant, τ_g also impacts the numerical stability of the model.



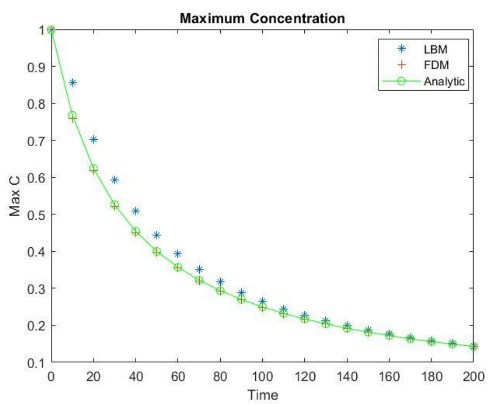
(a) GH Analytic solution with D=1.5 at t=200



(b) GH FDM solution with Kappa=1.5 at t=200



(c) GH LBM solution with τ_g=6.25 at t=200



(d) Maximum concentration over time
(e)

Fig. 4 Comparison of Gaussian Hills among the Analytic, the LBM ADE and the FDM ADE

Solving the LBM using the SRT (single relaxation time) gives results that depend on τ_g. This permits fine-tuning τ_g value to "dial-in" specific diffusion characteristics. To further investigate this factor, we tested the LBM ADE model with different values of τ_g. Table 3 shows the impacts of τ_g values on diffused concentrations measured by the maximum concentration and its location.

Table 3 Impacts of τ_g values to diffusions at t=200

Method	**C max**	**X max**	**Y max**
Analytic	0.142820	220	220
FDM	0.142506	221	221
LBM (τ_g=6.25)	0.138113	218	218
LBM (τ_g=6.00)	0.144020	217	217
LBM (τ_g=5.00)	0.174000	217	217

These results show an example where the maximum concentration at time equal 200 seconds can be dialled-in from 0.138 to 0.144 to approach the analytic solution of 0.143 by changing τ_g from 6.25 to 6.00.

4.3 LBM ADE Model with Perturbed Taylor-Green Velocity Field.

Our literature review shows most ocean oil spill and contamination transport models are ADE based. An example of such a model is GNOME which relies on the accuracy of the ocean surface current velocity field to produce quality results [17]. To ensure the model's feasibility, most models went through some degree of validation and benchmark tests. The most commonly used benchmark test for ADE based model is an initial Gaussian Hill concentration with a simplified adventive velocity field. However, in the real world, ocean surface current is a much more complex velocity field that is temporal-spatial dependent.

We conducted a benchmark study of LBM ADE against a FDM ADE solver using a temporal and spatial perturbation of the Taylor-Green velocity field. To the best of our knowledge, no such benchmarks have been done for an LBM ADE model using a velocity field as complex as the perturbed Taylor-Green field.

In this, perturbed Taylor-Green, study we consider a 2D ADE model with a concentration field, $C = C(x, y, t)$ and a fluid velocity field, $u = (u_x, u_y)$ where both concentration field and fluid velocity field are temporal-spatial dependent. The initial concentration is given as:

$C_0 = C(x, y, t_0) = sin(x)$, for x in [-π, π], independent of y for a 2D square mesh $(x, y,)$.

We choose a perturbed Taylor-Green flow as the fluid velocity field:

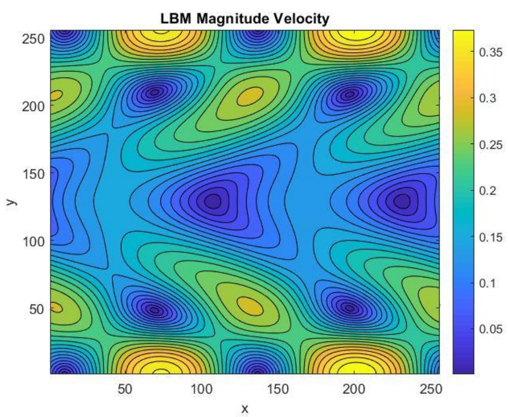
$$u = \begin{bmatrix} \sin(x)\cos(y) + \varepsilon(2\sin(x-\omega t)\cos(2y) - k) \\ -\cos(x)\sin(y) - \varepsilon\cos(x-\omega t)\sin(2y) \end{bmatrix}$$

One challenge in our study is to choose parameters in a scaled relationship between LBM ADE model and FDM ADE model while keeping the numerical solutions stable and physically similar. Based on the dimensionless and unit analysis in [6], Table 4 below describes both FDM ADE and LBM ADE model configurations.

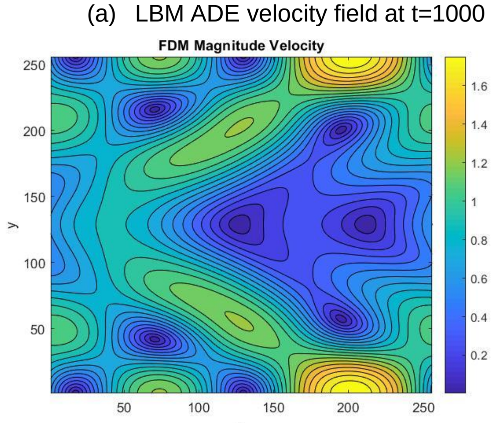
Table 4 FDM and LBM ADE model configurations

FDM Parameters	**LBM Parameters**
NX=NY=256	NX=NY=256
Xmax = π, Xmin = -π	Xmax=256, Xmin=1
TG velocity field: v-scale=1.0 $\omega = 1.0$ $\varepsilon = 0.3$ $k = 0.1$	TG velocity field: v-scale=0.25 $\omega = 0.01$ $\varepsilon = 0.3$ $k = 0.01$
$kappa = 0.00183$	$kappa = 0.0183,$ $\tau_g = 0.5549$
$\Delta x = 0.02454, \Delta t = 0.001$	$\Delta x = 1.0, \Delta t = 1.0$
Nsteps = 10,000	Nsteps = 1000
Total time = 10	Total time = 10

The LBM model is implement in both Python and MATLAB while the FDM model is implement in MATLAB. Both models use a 256 by 256 grid and double periodic boundary conditions. Even though the LBM and FEM methods approach solving the problem in different ways, our benchmark results show that the LBM ADE and FDM ADE agree closely. This study validates the LBM ADE capability to model oil transport in a temporal-spatial depended ocean fluid. Figure 5 shows comparison of perturbed velocity fields between LBM ADE (a) and FDM ADE (b).



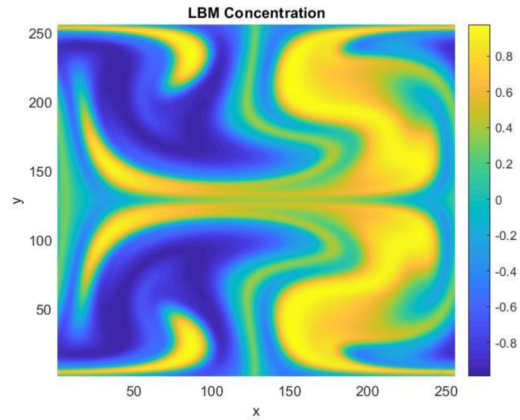
(a) LBM ADE velocity field at t=1000



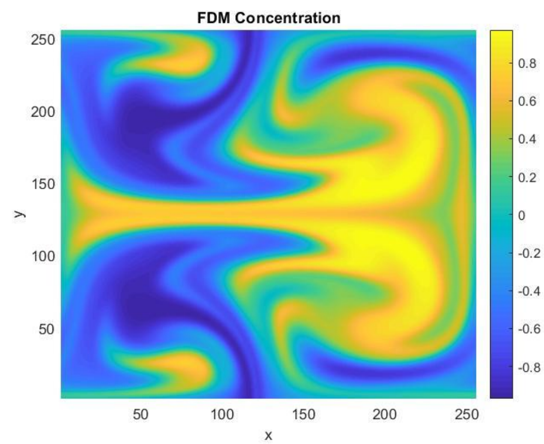
(b) FDM ADE velocity field at t=10000

Fig. 5 Velocity fields of LBM and FDM

Figure 6 shows a comparison of concentrations between LBM ADE (a) and FDM ADE (b).



(a) LBM ADE C with Taylor-Green Perturbation



(b) FDM ADE C with Taylor-Green Perturbation

Fig. 6 Concentrations, C, of LBM results (a) and FDM results(b)

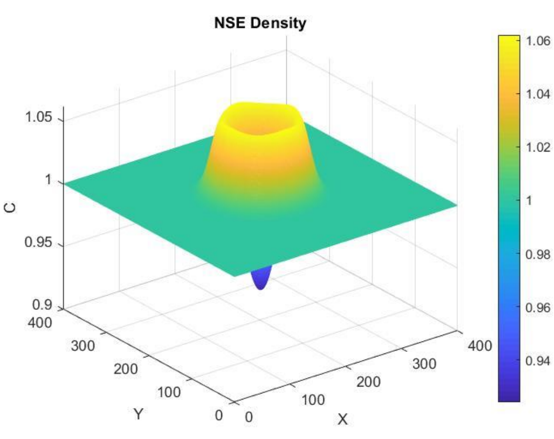
5. LBM NSE and ADE Models Comparison

LBM, as a kinematic modelling technique, is effective for providing numerical solutions as both NSE and ADE solvers. The major difference between the two models is that NSE conserves both mass and momentum while ADE only conserves mass. While the ADE is the most commonly used model for ocean oil spill and pollutant transports, there are little or no reported studies using NSE for such applications. Most of the ADE models use ocean surface current velocity fields as external inputs and ignore interactions from the velocity generated due to the fluid mass properties of the oil transport. Some of the models, i.e. GNOME, acknowledge such omissions and justify it based on the fact that spilled oil forms a slick which is carried by ocean currents. The ocean current moves the oil rather than the oil induced current moving the ocean surface.

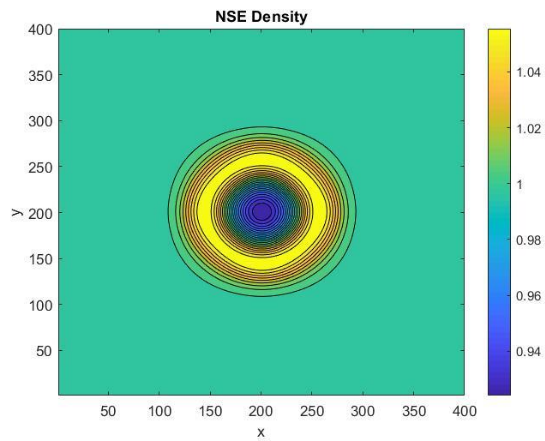
The National Research Council published a report [3], based on the work of teams of experts, which characterizes the fate of oil spilled into oceans in three major phases: (i) after oil introduced into the oceans; (ii) transport the resulting degradation oil away from the source; and (iii) incorporate the residual substances into compartments of the earth's surface system. Can the NSE play a role in modelling ocean oil spills? Where is the NSE applicable in the life cycle of spilled oil in the ocean?

In order to better understand the feasibility of using LBM NSE to model ocean oil spills and its transport at the surface level, we developed a LBM NSE model and compared it with a LBM ADE model. We conducted two simulation experiments one using an initial Gaussian Hill without considering ocean surface velocity and a second with an ocean surface velocity obtained from an ocean current model [Reference]. In both, we used a Gaussian Hill for representing the initial concentration of oil pollutant particles and then compare the pollutant spread between LBM NSE and ADE models.

In the first experiment, the LBM NSE model reveals that the Gaussian Hill collapses at the center after initial distribution and the mass propagate outwards in a ring form.



(a) LBM NSE Gaussian Hill at t=100



(b) LBM NSE mass propagations at t=100

Fig.7 LBM NSE mass distribution and propagation.

While we do believe what Fig.7 shows is the correct numerical solution of the NSE solver, it is not the right model or solution for the ocean oil spill in which the center should retain the maximum of mass and propagate outward. In this case, the ADE is a better model for ocean oil spill and transport on ocean surface as indicated by the results in section 4.2. This scenario suggest a coupling between the LBM NSE solver for calculating a velocity field followed by a LBM ADE solver using the results from the LBM NSE as the initial concentration for modelling advection diffusion effects. Figure 8 show the concentration at t=100.2 resulting from the ADE model when starting with the NSE density and velocity at t=100. Here the concentration is quickly redistributed mitigating the collapsed Gaussian Hill.

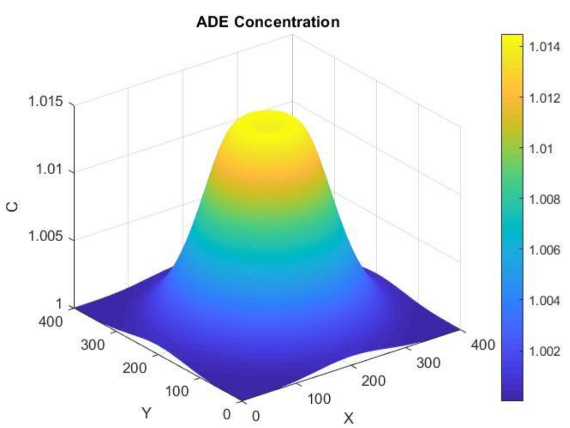


Figure 8 (a) ADE Gaussian Hill at t=100.2 starting from LBM NSE density and velocity at t=100.

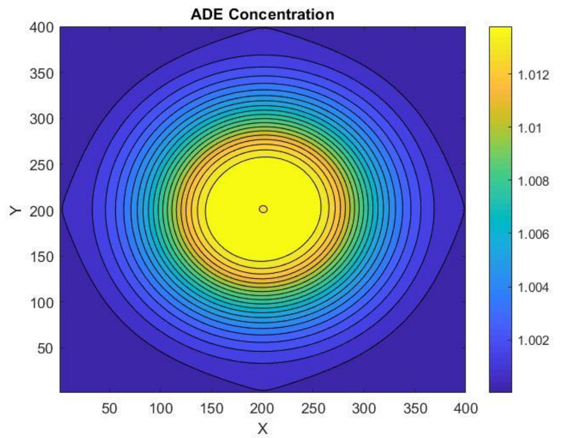


Figure 8 (b) ADE concentration at t=100.2 starting from LBM NSE density and velocity at t=100.

It is interesting to point out that there is a velocity field generated due to mass redistribution under NSE, in this case, even without the present of ocean surface current velocity. This velocity field, which we refer to as the NSE induced velocity, can interact with ocean current velocity at local level and alter the velocity field for a short period time. Eventually the ocean current will prevail as Gaussian Hill mass dissipates over time. This velocity interaction is potentially significant near "ground zero" of the oil spill for the initial phase transport.

To further study the velocity field interaction, we conducted a second simulation experiment to compare LBM NSE and ADE with Gaussian Hill and a realistic ocean surface current velocity field from the Unified Wave Interface-Coupled Model (UWIN-CM), a fully coupled atmosphere–wave–ocean system [26]. We used bilinear interpolation spatially to generate a velocity field for the LBM computational domain at each time step. The ocean surface velocity field is integrated into the LBM NSE model using the velocity project schema we descripted in (Eq. 3) and in LBM ADE model as an advective velocity descripted in (Eq. 8). We used a subset of data from the ocean model to cover a 1 degree square area of the Gulf of Mexico centered at -88.4 longitude and 28.8 latitude over three days from Feb. 07, 2016 at 16:00:00 until Feb. 10, 2016 at 18:00:00. Figure 9 shows a map of the area in the Gulf of Mexico being modelled.

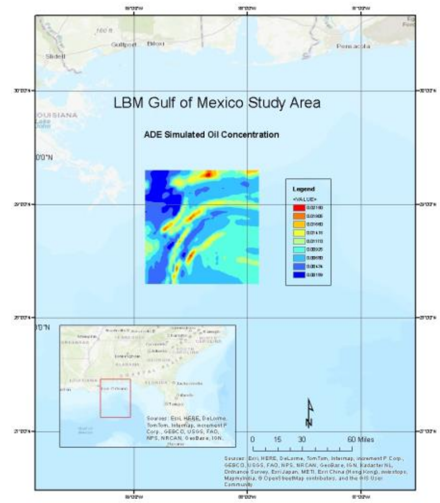


Figure 9. The map of the Gulf of Mexico shows the study area of a simulated hypothetical oil spill concentration centered at 28.9 latitude and -88.5 longitude in the vicinity of the Deep Water Horizon accident. The LBM ADE simulation used UWIN-CM velocity fields.

Figure 10 shows the initial ocean surface velocity of the LBM domain at t=0 step.

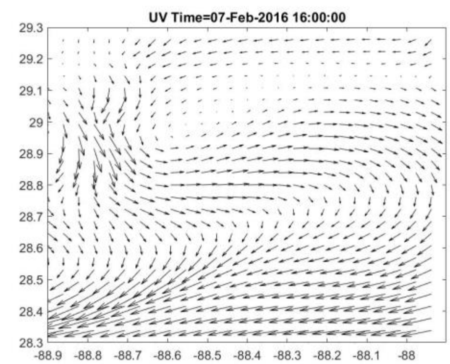


Fig.10 NSE ocean velocity field at t=0

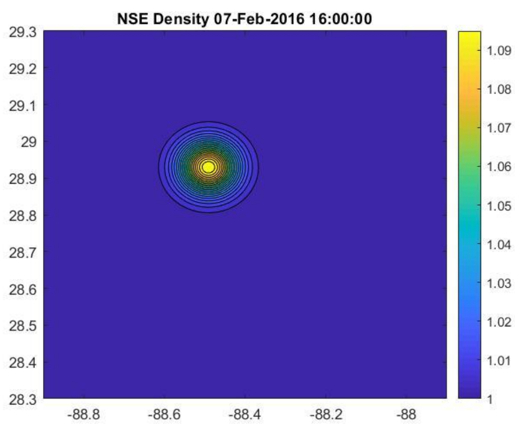
The details of physical and LBM spatial-temporal scales are given in Table 5.

Table 5. Physical and LBM Spatial-Temporal Scales

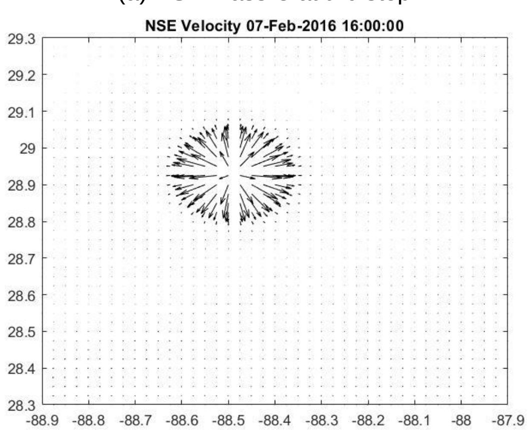
Physical Scale	LBM Scale
A grid with 200x200 cells. Cell size 500m by 500m. Centered at (-88.4, 28.8)	A lattice with 200 by 200 cells. Cell size. Lattice cell size: $\Delta x = \Delta y = 1$ lsu (lattice space unit)
Duration 3 days start 07-Feb 2016 16:00 Δt=15 m Total steps: t=0 to 296	$\Delta t = 1$ ltu (lattice time unit); total steps: t=0 to 296
Physical velocity: $U_p=V_p$=500m/900s=0.56m/s	Lattice velocity: $U_L=V_L= \Delta x/\Delta t$=1 lsu/ltu;

In the LBM model, a major challenge of using Gaussian Hill as spilled oil mass concentration is to estimate the quantity and volume of spilled oil at the surface in a relative comparison with the body of water in the study area, which will determine the parameters of a Gaussian hill as σ and C_0. As a proof of concept prototype, we set the Gaussian Hill with σ=10 and C_0=0.1 with a backdrop of ocean water with C=1 uniformly. We realize that this may not be applicable in a real oil spill application where these parameters need to be carefully calibrated according to the specify oil spill and the type of oil. For a stable numerical solution, we scaled the ocean velocity by 10^{-2} when applying the velocity project schema.

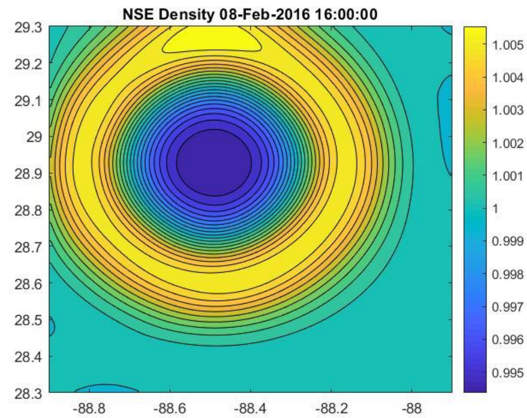
The NSE simulation results show the movements of mass and the interactions between ocean current velocity and NSE induced velocity at different time steps in Figure 11.



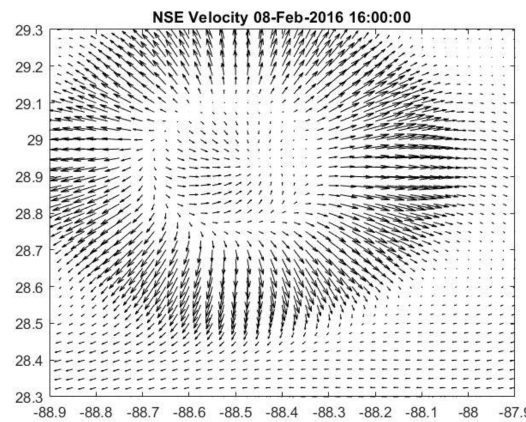
(a) NSE mass C at t=0 step



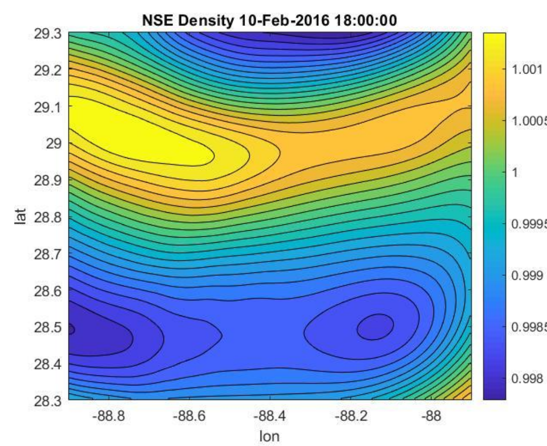
(b) NSE ocean velocity field at t=0 step



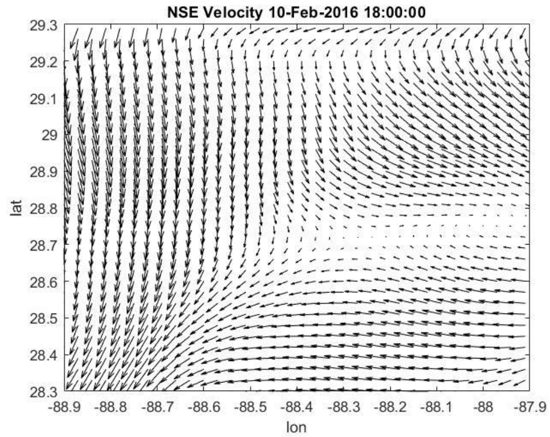
(c) NSE mass C after t=96 steps



(d) NSE Ocean velocity field after t=96 steps



(e) Final NSE mass C after t=296 steps

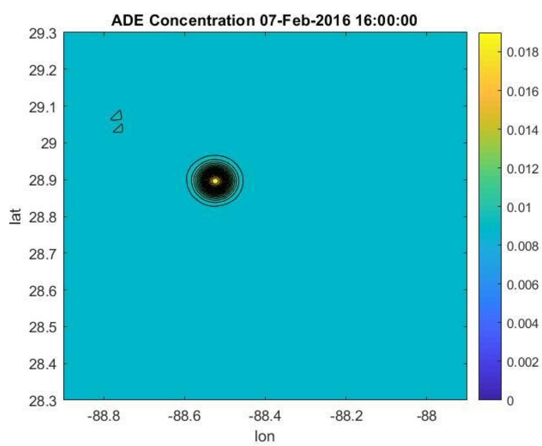


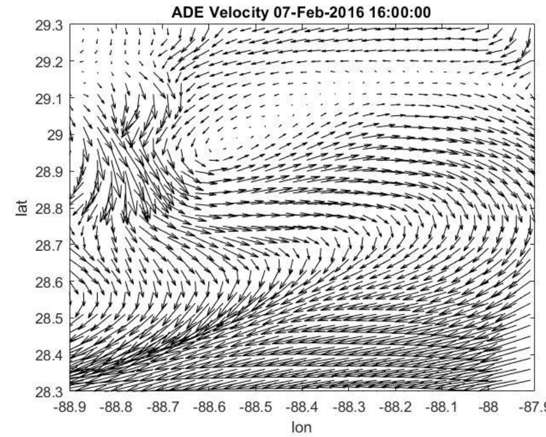
(f) Final NSE velocity field after t=296 steps

Fig.11 NSE Gaussian hill mass distribution and velocity field snap shots at different time steps.

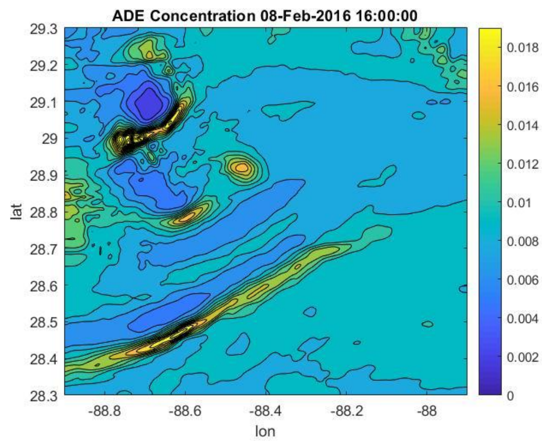
Our study shows the NSE could be a useful model for velocity interactions in the local area of high concentration near the center of the oil spill. These interactions can alter the velocity field significantly in a local area with sub-meso scale (10m to 100m) providing a more accurate estimation of the ocean surface current.

Since the ADE model only computes the mass concentration over space and time taking an ocean current velocity as inputs, there are no resulting velocity field interactions. The ADE simulation results show the movements of mass concentration and ocean current velocity at different time steps in Figure 12.

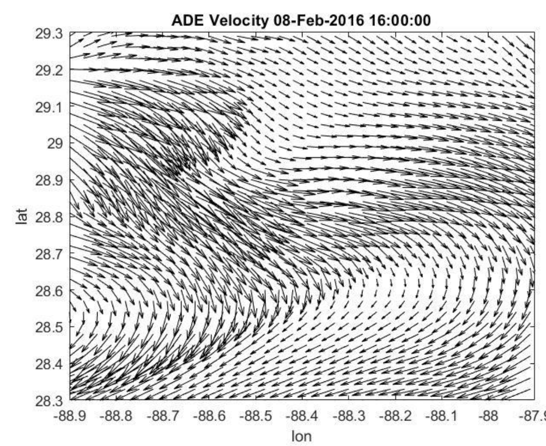


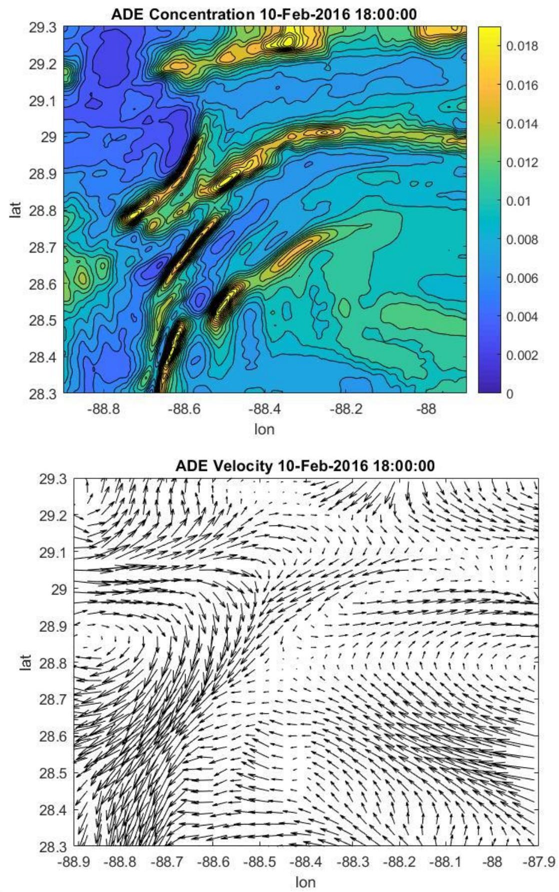


(a) ADE mass C and velocity at t=0 step



(b) ADE mass C and velocity after t=96 steps





(c) ADE mass C and velocity after t=296 steps

Fig.12 ADE Gaussian hill mass distribution and velocity field snap shots at different time steps.

Our findings opened another area of study, to couple NSE and ADE models. While ADE plays a major role for modelling the spilled oil transport with an external adventive velocity field, NSE can model the velocity interaction and produce a more accuracy velocity field, which will be feed into the ADE model, especially at the initial period of an oil spill in a local area near the source of the spill. There may be several other areas where NSE can be applied, such as, multi fluid model where oil and seawater are mixed and oil droplets are formed during the weathering process.

6. CONCLUSION

Our study and simulation results show how LBM can be used to model ocean oil spills over the life cycle of spiled oils in ocean, given that the LBM can support NSE and ADE based models. We validated both LBM NSE and ADE models via benchmarks with Poiseuille flow in a channel and Gaussian Hill mass concentrations with analytic solutions. In particular, we made a novel contribution to benchmark LBM ADE using a spatial-temporal depended perturbed Taylor-Green velocity field. Our work brings ADE benchmarks a step closer to testing ADE model in real ocean current dynamics.

We conducted a comparison study between LBM NSE and ADE models and their possible applications in modelling ocean oil spill. Our simulation experiments with both NSE and ADE models with simulated ocean surface velocity fields from the UWIN-CM model enabled us to test how NSE and ADE models behave differently given similar initial Gaussian Hill mass concentration and realistic ocean surface currents in the Gulf of Mexico near the location of the Deep Water Horizon oil spill.

As to future work, we will extend our work to use LBM model for multi-fluids in oil weathering process and to use LBM NSE to model the oil droplet formations in the mixture of oil and seawater.

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