J4.3 LARGE-EDDY SIMULATION ACROSS A GRID REFINEMENT INTERFACE USING EXPLICIT FILTERING AND RECONSTRUCTION

Lauren Goodfriend[1]*, Fotini K. Chow[1], Marcos Vanella[2], and Elias Balaras[2]
[1]Civil and Environmental Engineering, University of California, Berkeley, CA
[2]George Washington University, Washington, DC

1. INTRODUCTION

Large-eddy simulation (LES) is commonly used in atmospheric simulations when the domain size is small enough to allow grid spacing that resolves turbulence. At these resolutions LES is more accurate than the Reynolds-averaged Navier-Stokes (RANS) models used for mesoscale simulations, but far less computationally intensive than direct numerical simulation (DNS). Grid nesting, used to transfer the meso-scale boundary conditions to a micro-scale domain of interest, is another popular technique to increase the grid resolution in a cost-effective manner.

The use of LES on nested grids presents challenges not encountered in RANS or DNS simulations. On a two-way nested grid, the solution must be approximated across the grid refinement interface to calculate derivatives at the interface. This interpolation necessarily uses information at the grid scale, i.e. the solution points directly surrounding the interpolated point. Interpolation is not problematic in DNS or RANS codes, because the solutions resulting from these methods are smooth at all scales. Neither DNS nor RANS solutions contain substantial energy at the grid scale, so the grid scale is not reasonably represented. However, LES solutions are least accurate at the grid scale, because the grid scale is most heavily contaminated by the subgrid scale (SGS) stress approximation, as shown in Brandt (2006). Interpolation therefore introduces errors on both sides of the interface between grids of different refinements.

LES solutions can also reflect off of grid refinement interfaces, specifically on the outflow boundary from a fine to a coarse grid. A coherent structure that is resolvable on a fine grid but not a coarse grid may interact with a fine to coarse interface in several ways. The eddy may alias onto the coarse grid, represented as a motion with a larger wavelength, it may dissipate, or it may reflect. Reflection creates the more problematic errors, since the reflection appears as an accumulation of energy on the fine side of a grid refinement interface.

In grid nesting applications, modelers often want to develop smaller scales of turbulence as quickly as possible from the coarse to the fine grid, as discussed by Moeng et al. (2007) and Mirocha et al. (2011). The solution immediately inside the fine, interior grid has less resolved turbulence than it would at the same point of a fully developed uniform fine grid solution. The reduced resolved turbulent energy compared to a uniform fine grid solution is an error caused by the necessity of using nested grids.

Nested grids are an example of the wider class of block-structured non-uniform grids, in which grid spacing is refined by an integer factor. This paper focuses on block-structured non-uniform grids, in contrast to grid stretching or unstructured grids, in which grid spacing ratios may take on any value or be poorly defined.

In this paper, explicit filtering and reconstruction is used to mitigate the errors unique to using LES on a non-uniform block-structured grid. A series of experiments is performed using forced isotropic turbulence with a coarse-fine interface and periodic boundary conditions.

2. COMPUTATIONAL METHODS

2.1 Explicit filtering and reconstruction

Explicit filtering eliminates aliasing errors from the nonlinear advection term, as demonstrated by Lund (1997). In addition, as will be demonstrated here, explicit filtering can reduce reflection off grid refinement interfaces by forcing the filter-resolved scale on a fine grid to equal the grid-resolved scale on the coarse grid (Section 4). However, explicit filtering can reduce the accuracy of LES solutions when used without additional subfilter scale (SFS) modeling, as shown by Brandt (2008). The reconstruction model yields a more accurate SFS stress for explicitly filtered LES. Increased accuracy at the grid scale from more accurate SFS improves interpolation at the grid refinement interface, and therefore improves transfer of the solution between the grids.

Originally developed for LES by Stolz and Adams (1999) and Stolz et al. (2001), the reconstruction model is essentially a scale similarity model like that of Bardina et al. (1983). This model maintains the benefits of explicit filtering while increasing the accuracy of the subfilter model. To formulate the reconstruction model, begin with the discretized ($\tilde{u}$) and filtered ($\bar{u}$)

*Corresponding author address: Civil and Environmental Engineering, 202 O'Brien Hall, University of California, Berkeley, CA 94720-1710, email: lgoodfriend@berkeley.edu

incompressible Navier-Stokes equations:

$$\frac{\partial \bar{\tilde{u}}_i}{\partial t} + \frac{\partial \overline{\bar{\tilde{u}}_i \bar{\tilde{u}}_j}}{\partial x_j} = -\frac{1}{\rho}\frac{\partial \bar{\tilde{p}}}{\partial x_i} + \nu \frac{\partial^2 \bar{\tilde{u}}_i}{\partial x_j \partial x_j} - \frac{\partial \tilde{\tau}_{ij}}{\partial x_j} \quad (1)$$

$$\frac{\partial \bar{\tilde{u}}_i}{\partial x_i} = 0 \quad (2)$$

$$\tau_{ij} = \overline{u_i u_j} - \overline{\bar{\tilde{u}}_i \bar{\tilde{u}}_j} \quad (3)$$

The SFS stress τ_{ij} is divided into a SGS stress and a resolvable subfilter scale (RSFS) stress

$$\begin{aligned} \tau_{ij} &= \overline{u_i u_j} - \overline{\bar{\tilde{u}}_i \bar{\tilde{u}}_j} & (4) \\ &= (\overline{u_i u_j} - \overline{\tilde{u}_i \tilde{u}_j}) + (\overline{\tilde{u}_i \tilde{u}_j} - \overline{\bar{\tilde{u}}_i \bar{\tilde{u}}_j}) & (5) \\ &= \tau_{ij}^{SGS} + \tau_{ij}^{RSFS} & (6) \end{aligned}$$

This formulation is interpreted as a mixed model: the SGS stress is represented by an eddy viscosity model

$$\tau_{ij}^{SGS} = \overline{u_i u_j} - \overline{\tilde{u}_i \tilde{u}_j} = -2\nu_T \bar{S}_{ij} \quad (7)$$

while the RSFS stress is solved by approximating $\tilde{u} \approx \tilde{u}^*$, where $\tilde{u}^*$ is the truncation of the infinite deconvolution series for filter kernel G:

$$\tau_{ij}^{RSFS} = \overline{\tilde{u}_i \tilde{u}_j} - \overline{\bar{\tilde{u}}_i \bar{\tilde{u}}_j} \approx \overline{\tilde{u}_i^* \tilde{u}_j^*} - \overline{\bar{\tilde{u}}_i^* \bar{\tilde{u}}_j^*} \quad (8)$$

$$\tilde{u}^* = \bar{\tilde{u}} + (I-G) * \bar{\tilde{u}} + (I-G) * [(I-G) * \bar{\tilde{u}}] + ... \quad (9)$$

$$\bar{\tilde{u}} = G * \tilde{u} \quad (10)$$

Here, I is the identity operator.

When explicit filtering is used, the three-dimensional numerical filter G is applied only to the advective term and to perform the approximate deconvolution. This model was found to be effective for simulating channel flow by Gullbrand and Chow (2003) and the atmospheric boundary layer by Chow et al. (2005) and Zhou and Chow (2011) when used with a dynamic eddy viscosity model.

In this paper, explicit filtering is used with both zero-level and one-level reconstruction. The "level" of reconstruction refers to the number of additional filtering operations required in the reconstruction series expansion. Zero-level reconstruction requires no additional filtering and estimates

$$\tilde{u}^* = \bar{\tilde{u}} \quad (11)$$

The RSFS stress in this case reduces exactly to the Bardina et al. (1983) scale similarity model. One-level reconstruction estimates

$$\begin{aligned} \tilde{u}^* &= \bar{\tilde{u}} + (I-G) * \bar{\tilde{u}} & (12) \\ &= 2\bar{\tilde{u}} - \bar{\bar{\tilde{u}}} & (13) \end{aligned}$$

The solution variable is $\bar{\tilde{u}}$, so the second bar on the second term of equation 13 must be applied explicitly.

2.2 Variable filtering

Variable filtering was used by Vanella et al. (2008) to improve transitions of LES across a grid refinement interface. In this technique, the filter width is allowed to vary independently of the grid width (Figure 1). Variable filtering is used in this paper both with and without explicit filtering and reconstruction. To implement variable filtering for the non-explicitly filtered cases, the filter width used in calculating the eddy viscosity was varied. For the explicitly filtered cases, the explicit filter width was also adjusted in the variable filtering region with a linear combination of a three-point and five-point stencil discrete filter. The variable filtering region is 24 grid points on both sides of the fine grid, covering in total half of the fine grid. A wide variable filtering region was chosen to demonstrate this technique in its most effective form. The results presented use a linear filter width transition. A sinusoidal transition was also tested, but the results were very similar to the linear transition case.

3. SIMULATIONS

Simulations of homogeneous, isotropic turbulence were generated to test the efficacy of the reconstruction model in mitigating grid interface effects. This simple test case was chosen to isolate the effects of the grid refinement interface without the complicating effects of a near-wall region. The test domain was refined by a factor of two on the left side, with periodic boundary conditions on all boundaries (Figure 1). This grid mimics an infinitely repeating series of two-way nested grids.

The Navier-Stokes solver is an extension of the code written by Vanella et al. (2008). Second order Adams-Bashforth time stepping was used with centered second order finite differences in space on a staggered grid. The simulation was performed on a 96^3 grid adjoining a 48^3 grid (Figure 1). A linear forcing scheme developed by Lundgren (2003) and Rosales and Meneveau (2005) was used to maintain the turbulent kinetic energy (TKE). Conservation of mass was enforced at the grid refinement interface using flux matching from the fine to the coarse grid. Grid management is performed by Paramesh, written by MacNeice et al. (2000).

Simulations differed in the filter width transition, whether explicit filtering was used, and the level of reconstruction (Table 1). The only filter tested was a binomial approximation to a Gaussian filter. Previous tests on two other common LES filters, the trapezoidal and Simpson's rule approximations to the top-hat filter, found these filters to be unsuitable for the reconstruction method due to their spectral shapes. These results will be presented in an upcoming paper.

Filter type	Filter transition	Reconstruction
Gaussian	step	0-level
Gaussian	step	1-level
Gaussian	linear	0-level
Gaussian	linear	1-level
none	step	none
none	linear	none

Table 1: Tested filter types and transition types.

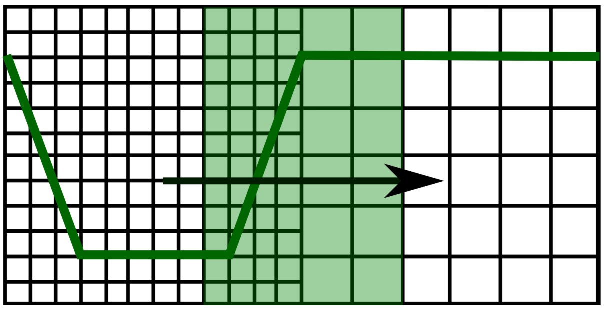


Figure 1: Schematic of the computational domain. The green line shows the filter width for the linear variable filter transition. The area highlighted in green is shown in the coherent structures figures. The arrow shows the direction of flow.

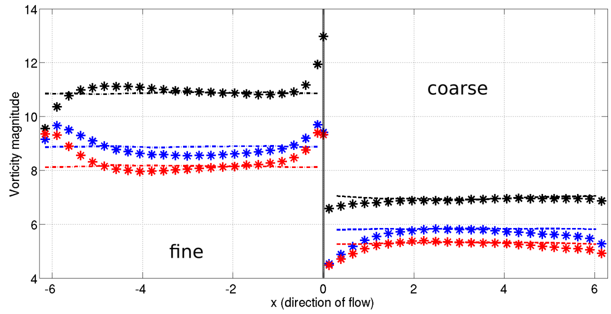


Figure 2: Vorticity magnitude results using a step filter width transition. ∗ no explicit filtering; ∗ explicit filtering with zero levels of reconstruction; ∗ explicit filtering with one level of reconstruction

4. RESULTS: VELOCITY STATISTICS

Vorticity magnitude is calculated to show resolved turbulence (Figure 2). The plots are averages over the yz-plane (perpendicular to the direction of flow) and over 96 snapshots in time collected every 300 timesteps after a spin-up period of 6000 timesteps. Results from uniform fine or coarse grid cases are plotted as dashed lines, and the non-uniform cases are plotted as symbols.

The ideal vorticity magnitude solution would follow the uniform grid solutions on their respective grids exactly, transitioning instantly between uniform grid solutions. All the fine grid structure would develop at the edges of the fine domain.

With these goals in mind, the results are examined to find how explicit filtering, filter transition, and level of reconstruction affect solution accuracy. In this paper, solution accuracy is defined in relation to the uniform grid solutions, which have one fewer source of error than the non-uniform solutions since they have no grid refinement interface effects.

4.1 No variable filtering

The step filter width transition, in which no variable filtering is used, is the standard choice. When no explicit filtering is used (denoted by black asterisks ∗), a substantial increase in vorticity magnitude is seen at the fine to coarse interface in Figure 2, due to reflection of eddies off the grid interface. After a small jump, the uniform fine grid solution is approached gradually, increasing from the smaller coarse side solution. This transition is parallel to the development of fine scale structure on an inner fine nest: as the flow advects inside the finer grid, the increased resolution allows more turbulence to be resolved.

As expected, when explicit filtering is used (denoted by blue asterisks ∗) the accumulation of energy at the grid refinement interface is reduced. However, the solution does not maintain good agreement with the fine or coarse uniform grid solutions compared to the non-explicitly filtered case. This result is an extension of the conclusions in Brandt (2008): explicit filtering compromises solution accuracy when the SFS stress model does not include active reconstruction. With the addition of another level of reconstruction (denoted by red asterisks ∗), agreement with the uniform grid solution is improved, particularly in the center of the fine grid. Note that the resolved vorticity is smaller with increased levels of reconstruction, because more of the energy is placed in the SFS stress.

The addition of explicit filtering causes an accumulation of energy on the fine side of both grid interfaces. This effect is not mitigated by reconstruction. Since no accumulation is present in the variably filtered case below (Figure 3), this effect appears to be an artifact of the step change in explicit filter width. These results suggest that a rapid change in explicit filter width, like a rapid change in grid width, can perturb the solution and generate reflections.

4.2 Variable filtering

When a variable filter width transition is used, an accumulation of energy at the fine to coarse interface remains for the non-explicitly filtered case (Figure 3). The use of explicit filtering eliminates the increase in vorticity magnitude at the grid interface, but reduces agreement with the uniform grid solutions, particularly on the coarse grid. An additional level of reconstruction improves agreement with uniform grid results, as

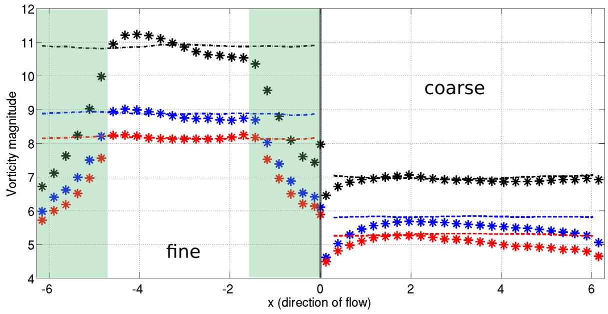


Figure 3: Vorticity magnitude results using a linear filter width transition. The green shaded region is where the filter width varies. $*$ no explicit filtering; $*$ explicit filtering with zero levels of reconstruction; $*$ explicit filtering with one level of reconstruction

for the non-variably filtered case.

Both with and without variable filtering, the non-uniform vorticity magnitude is too small on the edges of the coarse grid. At the fine to coarse grid interface (left side of the coarse grid), this result suggests that, in addition to reflecting, some eddies that are too small to pass through the interface are destroyed. The undershoot on the coarse side of the coarse to fine interface (right side of the coarse grid) appears to be an artifact of incorrect RSFS stress, since it is not present in the non-explicitly filtered case and is mitigated by additional reconstruction.

4.3 Transition type comparison

When no explicit filtering is used, the variable filter transition performs similarly to the step transition (Figure 4). The variably filtered case has a less substantial accumulation of energy at the fine side of the fine to coarse interface. This benefit is balanced against the observation that the step transition case maintains its agreement with the uniform fine grid solution closer to the coarse grid interfaces. When explicit filtering is not used, variable filtering has limited ability to improve the solution, since it takes effect only as a scaling on the eddy viscosity.

The variable filter transition is preferred when using explicit filtering and one level of reconstruction (Figure 5). The step filter transition is too large on both edges of the fine grid and does not match the uniform fine grid case well. The linear filter width transition yields better agreement with the uniform fine grid. The variable and step filter transition solutions reach the uniform fine grid solution at the same distances down- and upstream from the coarse grid. Thus, the variable filter width does not effectively slow the transition between fine and coarse grid resolved turbulence states. However, the variable filter transition produces worse agreement with the uniform coarse grid results, as shown by the larger undershoot on the right side of

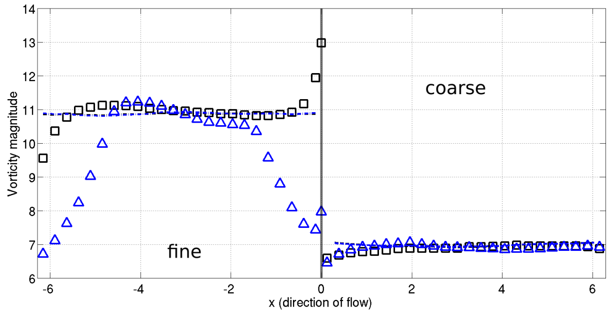


Figure 4: Vorticity magnitude results using no explicit filtering. $\square$ no variable filtering; $\triangle$ variable filtering

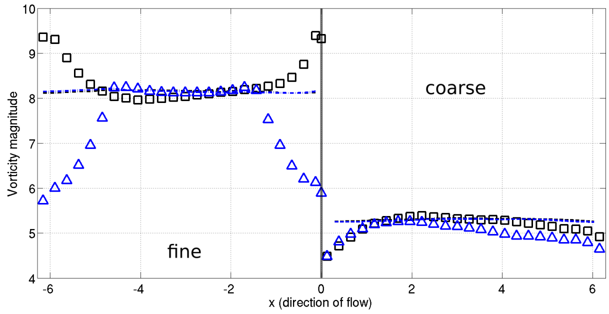


Figure 5: Vorticity magnitude results using explicit filtering and one level of reconstruction. $\square$ no variable filtering; $\triangle$ variable filtering

the coarse grid when using variable filtering.

When explicit filtering is used, variable filtering does not increase the distance required to develop fine scale structure downstream of a coarse to fine interface, such as when nesting into a smaller-scale grid. This result is perhaps counterintuitive, since the variable filter forces length scales to remain larger. As discussed in section 4.1, a step change in filter width appears to induce perturbations in the flow, which also slows transition to the uniform fine grid solution. Variable filtering can be an effective technique on nested grids using explicit filtering, since it does not inhibit generation of fine scales compared to non-variable filtering, and offers the advantage of a smoother transition across grid sizes.

5. RESULTS: COHERENT STRUCTURES

Single time snapshots of coherent structures are plotted for the step filter width transition (Figure 6) and the variably filtered case (Figure 7). Coherent structures are calculated as contours of the second invariant of the velocity gradient tensor Q

$$Q = -\frac{1}{2}\left(\frac{\partial \bar{\tilde{u}}_i}{\partial x_j}\frac{\partial \bar{\tilde{u}}_j}{\partial x_i}\right) \tag{14}$$

The plotted contours are isosurfaces where $Q = 8$. This contour was chosen because it highlights discontinuities in coherent structures (i.e. velocity derivatives) at the grid refinement interface. The lack of a coherent structure does not imply lack of turbulence, it means only that the turbulence in that region is less energetic.

Without variable filtering, explicit filtering and reconstruction generate a small improvement in coherent structure continuity (Figure 6). When no explicit filtering is used, very few coherent structures pass through the grid interface (Figure 6a). When one level of reconstruction is used, about half of the eddies travel across the grid interface (Figure 6c). Explicit filtering with zero levels of reconstruction produces intermediate results, but resembles the non-explicitly filtered case more (Figure 6b). This result suggests the importance of higher levels of reconstruction when using explicit filtering. Explicit filtering and reconstruction improves coherent structure continuity by increasing accuracy of the solution at the grid scales needed for interpolation at the interface.

Variable filtering improves the beneficial effects of reconstruction (Figure 7). Even without explicit filtering, variable filtering improves the transition of eddies across the grid interface compared to the step transition (Figure 7a). Since the filter width is continuous across the grid interface, eddies on the fine grid are larger when variable filtering is used and therefore pass onto the coarse grid more easily. When one level of reconstruction is used, most of the eddies pass through the grid interface (Figure 7c). As for the step filtering case, explicit filtering with zero levels of reconstruction resembles the non-explicitly filtered case (Figure 7b).

Animations of these figures may be found at efmh.berkeley.edu/lgoodfriend/AMSBLT2012.html.

6. DISCUSSION AND FUTURE WORK

This paper examines the use of explicit filtering and reconstruction to improve LES results on block-structured non-uniform grids, such as nested grids. The use of variable filtering is also investigated.

Explicit filtering reduces energy accumulation at the grid refinement interface because it ensures that all eddies are resolvable on both the fine and the coarse grid. The grid refinement interface becomes more "transparent" to the solution, so coherent structures reflect less. Agreement with the uniform grid solutions is reduced when explicit filtering is used without higher levels of reconstruction. Level one reconstruction improves explicit filtering solutions by more accurately estimating the SFS stress and visibly improves transition of coherent structures across a fine to coarse grid refinement interface because the solution is more accurate at the grid scale.

When explicit filtering and reconstruction are used, variable filtering improves agreement with the uniform fine grid solution. Explicit variable filtering does not delay development of resolved turbulence downstream of a coarse to fine interface, such as an outer to inner grid nest transition, making it a viable alternative to step filter width transition. Without explicit filtering, variable filtering offers no clear benefit, because it has no direct control over turbulence scales in this case.

The explicitly filtered solutions do not converge to their uniform coarse grid solution. It is possible that explicit filtering slows the convergence on the coarse grid, and the observed results are an effect of an insufficiently long domain. Tests are currently being run on a doubly long domain to address this concern.

These preliminary results suggest that variable filtering and explicit filtering with reconstruction can improve performance of LES on nested grids. It will be interesting to compare results using explicit filtering and reconstruction with more popular methods of controlling grid interface effects, such as the relaxation condition of Davies (1983) and Lehmann (1993). For the next step, tests will be run with more levels of reconstruction to find if more reconstruction continues to improve explicitly filtered results. Tradeoffs from using more reconstruction will be examined: increased accuracy, increased computational time, and accumulation of numerical errors from repeated filtering. These results will be applied to a similar test of boundary-layer flow to isolate the effects of the near-wall region versus the grid refinement interface.

REFERENCES

Bardina, J., Ferziger, J., and Reynolds, W. (1983). Improved subgrid scale models based on large eddy simulation of homogeneous, incompressible, turbulent flows. Report TF-19, Thermosciences Division, Dept. Mechanical Engineering, Stanford University.

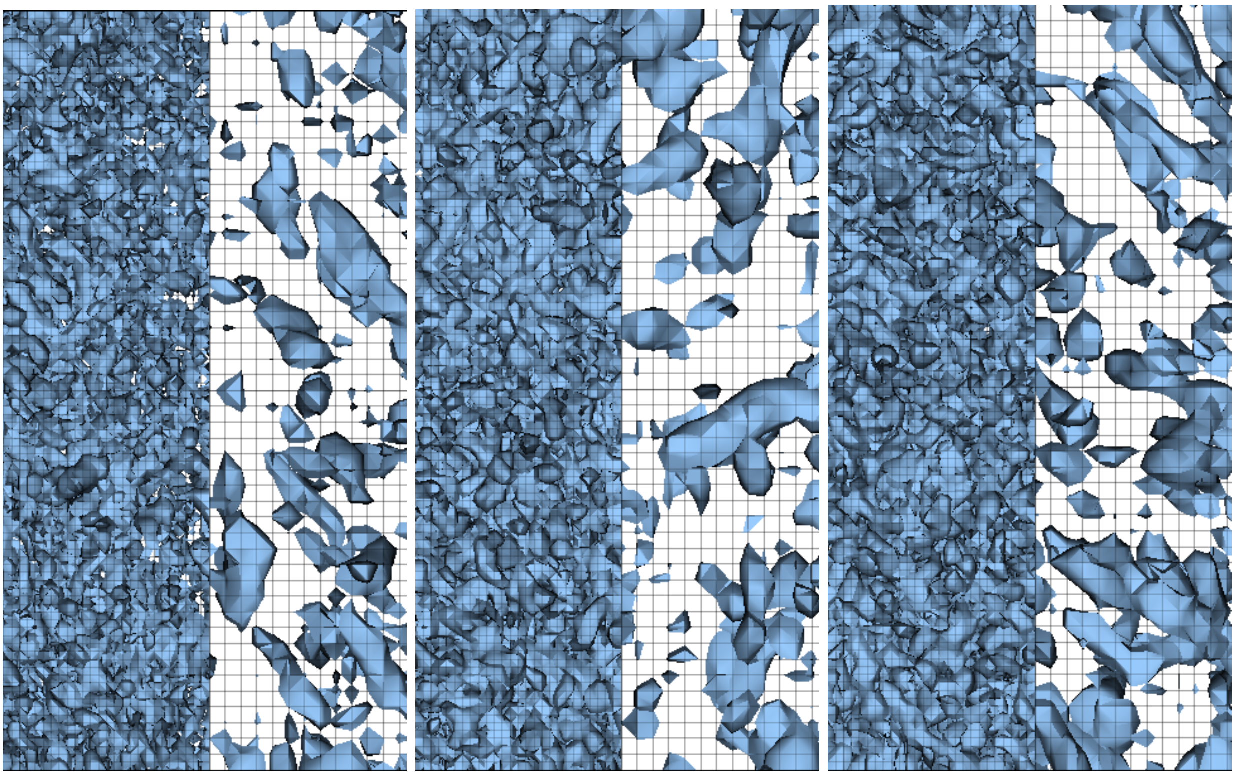
Brandt, T. (2006). A priori tests on numerical errors in large eddy simulation using finite differences and explicit filtering. *International Journal for Numerical Methods in Fluids*, 51(6):635–657.

Brandt, T. (2008). Usability of explicit filtering in large eddy simulation with a low-order numerical scheme and different subgrid-scale models. *International Journal for Numerical Methods in Fluids*, 57(7):905–928.

Chow, F., Street, R., Xue, M., and Ferziger, J. (2005). Explicit filtering and reconstruction turbulence modeling for large-eddy simulation of neutral boundary layer flow. *J. Atmos. Sci.*, 62(7):2058–2077.

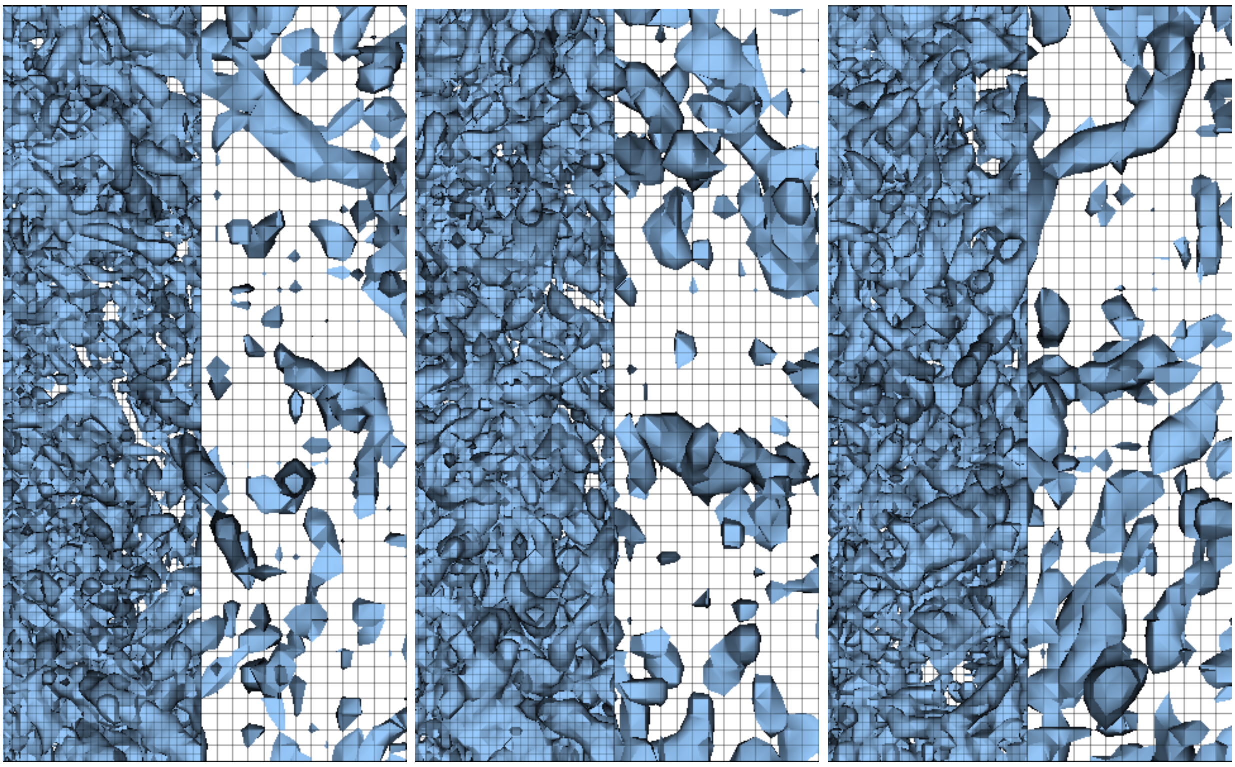
Davies, H. C. (1983). Limitations of some common lateral boundary schemes used in regional NWP models. *Monthly Weather Review*, 111:1002.

Gullbrand, J. and Chow, F. (2003). The effect of numerical errors and turbulence models in large-eddy simulations of channel flow, with and without explicit filtering. *J. Fluid Mech.*, 495:323–341.



(a) No explicit filtering (b) Zero levels of reconstruction (c) One level of reconstruction

Figure 6: Coherent structures using a step filter width transition.



(a) No explicit filtering (b) Zero levels of reconstruction (c) One level of reconstruction

Figure 7: Coherent structures using a linear filter width transition.

Lehmann, R. (1993). On the choice of relaxation coefficients for davies' lateral boundary scheme for regional weather prediction models. *Meteorology and Atmospheric Physics*, 52(1-2):1–14.

Lund, T. (1997). On the use of discrete filters for large eddy simulation. *Annual Research Briefs*, pages 83–95.

Lundgren, T. (2003). Linearly forced isotropic turbulence, annual research briefs. *Center for Turbulence Research, Stanford University*, pages 461–473.

MacNeice, P., Olson, K., Mobarry, C., de Fainchtein, R., and Packer, C. (2000). PARAMESH: a parallel adaptive mesh refinement community toolkit. *Comput. Phys. Commun.*, 126(3):330–354.

Mirocha, J., Kirkil, G., Bou-Zeid, E., Chow, F., and Kosovic, B. (2011). Transition and equilibration of neutral atmospheric boundary layer flow in one-way nested large eddy simulations using the weather research and forecasting model. *Mon. Wea. Rev.* submitted.

Moeng, C.-H., Dudhia, J., Klemp, J., and Sullivan, P. (2007). Examining two-way nesting for large eddy simulation of the PBL using the WRF model. *Mon. Wea. Rev.*, 135:2295–2311.

Rosales, C. and Meneveau, C. (2005). Linear forcing in numerical simulations of isotropic turbulence: Physical space implementations and convergence properties. *Phys. Fluids*, 17(9).

Stolz, S. and Adams, N. (1999). An approximate deconvolution procedure for large-eddy simulation. *Phys. Fluids*, 11(7):1699–1701.

Stolz, S., Adams, N., and Kleiser, L. (2001). An approximate deconvolution model for large-eddy simulation with application to incompressible wall-bounded flows. *Phys. Fluids*, 13(4):997–1015.

Vanella, M., Piomelli, U., and Balaras, E. (2008). Effect of grid discontinuities on large-eddy simulation statistics and flow fields. *J. Turbulence*, 9(32):1–23.

Zhou, B. and Chow, F. (2011). Large-eddy simulation of the stable boundary layer with explicit filtering and reconstruction turbulence modeling. *J. Atmos. Sci.*, 68(9):2142–2155.