Evidence for and a Theoretical Model of 10-km Wavelength Tropical Cyclone Boundary Layer Roll Vortices

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RESEARCH SUPPORTED BY: ONR PHYSICAL OCEANOGRAPHY & MARINE METEOROLOGY; NASA IOVWST. RADARSAT-1 IMAGERY ACQUIRED THROUGH HURRICANE WATCH AO

Multi-km-Scale Surface Wind Conv./Div. Patterns Synthetic Aperture Radar Surface Winds: 1-km pixels





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SLP-Filtered Tropical Cyclone (TC) SAR Wind Vector Fields

- SLP acts as a low-pass filter
 - Forces dynamical consistency
 - Winds at 1 km pixels
- Can calculate (hopefully) credible derivative fields
- Consistent signature in all SAR TC divergence fields:
 - $O(\sim 10 \text{ km})$ wavelength (λ) convergence bands
 - h (~ 1 km) is PBL depth
 - Look like PBL rolls, but aspect ratio (λ /h) is too large
- O(1-3 km) PBL rolls are very common feature of TC PBL



Wavelength: Larger-scale structures ~ 1500 to 2000 m Smaller-scale structures ~ 300 to 700 m

Velocity Perturbations: +/- 7 m s⁻¹ typical up to +/- "10s of" m s⁻¹ small-scale

Orientation: Typically along-mean TCBL wind, wide variability

Prevalence: Roll-scale structures common, (35% to 70%) Streak-scale structures: *Most likely usually present*

Hypothesis

• THE CONVERGENCE LINES ARE THE SIGNATURE OF LARGE ASPECT RATIO TC PBL ROLL VORTICES

- Theory and observations agree that the common TC PBL rolls have aspect ratio O(2-4)
- Large aspect ratio modes are slowly-growing; not expected to survive competition with much faster-growing dominant rolls.
- <u>Proposed Mechanism</u>: Upscale Energy Transfer from DOMINANT ($\lambda \sim 1-3$ km) modes into weak ($\lambda \sim 10$ km) modes THROUGH *RESONANT TRIAD WAVE-WAVE* INTERACTION
 - Based on 2-D Ekman layer model of Mourad and Brown (1990);
 - +6 contributions omitted in MB90
 - First step: Low-order truncation

<u>Single-Wave</u> Roll Theory Nonlinear Stability



- "Stretch" eigenvalue, λ_0 , in powers of nonlinear amplitude, A(t).
- Expand eigenfunction, q_{10} , in <u>harmonics</u> of fundamental wavenumber, α , and <u>forced modifications</u>
 - Forced fundamental modifications are orthogonal to linear mode
 - Determine Landau Coefficients (the λ_i)
- Estimate equilibrium Amplitude (dA/dt = 0) & structure, q = [u,v,w,T]^T

Standard Single-Wave PBL Roll Model

Table 5.1 Contributions to the nonlinear perturbation up to the fifth Landau Coefficient

Order	Landau	\mathbf{q}_0	\mathbf{q}_1	\mathbf{q}_2	\mathbf{q}_3	\mathbf{q}_4	q 5	\mathbf{q}_{6}	q 7	\mathbf{q}_{8}	q 9	\mathbf{q}_{10}	q ₁₁
1		MF											
А	λ ₀		\mathbf{q}_{10}				+						
A ²		q 01		q ₂₀									
A ³	λ_1		q ₁₁		\mathbf{q}_{30}								
A ⁴		q ₀₂		q ₂₁		\mathbf{q}_{40}							
A ⁵	λ_2		q ₁₂		q 31		q 50						
A ⁶		q ₀₃		q ₂₂		q ₄₁		q ₆₀			/		
A ⁷	λ_3		q ₁₃		q ₃₂		q 51		q 70				
A ⁸		q ₀₄		q ₂₃		q ₄₂		q ₆₁		980			
A ⁹	λ_4		q 14		q 33		q 52		q 71		q 90		
A ¹⁰		q 05		q ₂₄		q ₄₃		q ₆₂	/	\mathbf{q}_{81}		q ₁₀₀	
A ¹¹	λ_5		q ₁₅		q ₃₄		q 53		q 72		q 91		q 110

Truncated Contributions to Multi-Wave Roll Model

q= [u, v, w, T][⊤]

To 1st Nonlinear Landau Term:

 $\begin{array}{cccc} 0 & + A^2 q_{01} \\ A q_{10} + 0 & + A^3 q_{11} \\ 0 & + A^2 q_{20} \\ 0 & + 0 & + A^3 q_{30} \end{array}$

(mean flow modification) (fundamental wavelength) (1st harmonic) (2nd harmonic)

Low-Order Truncation Errors



Mean-flow Modification

Amplitude Estimation

Low-order truncation:

- Over-estimated amplitude
- "S-shaped" MF modification

Upscale Transfer Resonant Triad

- $\alpha = \beta + \gamma$ (mode A, mode B, mode C)
- My nonlinear solution method restricts me¹ to all unstable modes
- Require at least one wavenumber at fastest growing mode
- For upscale transfer (into small wavenumber), intermediate wavenumber is usually also fastgrowing

Truncated 3-Mode Roll Solutions

$$q_{\alpha} = Aq_{0,\alpha} + BCq_{1,\alpha}e^{i\phi} + A[A^{2}q_{2,\alpha} + B^{2}q_{3,\alpha} + C^{2}q_{4,\alpha}] + A^{2}q_{20,\alpha} + A^{3}q_{30,\alpha} + B[A^{2}q_{2,\beta} + B^{2}q_{3,\beta} + C^{2}q_{4,\beta}] + B^{2}q_{20,\beta} + B^{3}q_{30,\beta} + B[A^{2}q_{2,\beta} + B^{2}q_{3,\beta} + C^{2}q_{4,\beta}] + B^{2}q_{20,\beta} + B^{3}q_{30,\beta} + C[A^{2}q_{2,\gamma} + B^{2}q_{3,\gamma} + C^{2}q_{4,\gamma}] + C^{2}q_{20,\gamma} + C^{3}q_{30,\gamma} + C[A^{2}q_{2,\gamma} + B^{2}q_{3,\gamma} + C^{2}q_{4,\gamma}] + C^{2}q_{20,\gamma} + C^{3}q_{30,\gamma} + C^{3}q_$$

- YELLOW: contributions from single-wave theory; e.g. $q_{2,\alpha} = q_{11,\alpha}$.
- BLUE: new wave-wave & wave-mean flow interaction contributions.
- RED: Low-order phase-coupling terms.
- Also: mean-flow modifications due to each wave.

Truncated Model Amplitude (real) and Phase (imaginary)

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$$\frac{1}{A}\frac{dA}{dt} - i\frac{d\theta_A}{dt} = a_0 + a_1\frac{BC}{A}e^{i\phi} + [a_2A^2 + a_3B^2 + a_4C^2]$$

• $\frac{1}{B}\frac{dB}{dt} - i\frac{d\theta_B}{dt} = b_0 + b_1\frac{AC}{B}e^{-i\phi} + [b_2A^2 + b_3B^2 + b_4C^2]$
• $\frac{1}{C}\frac{dC}{dt} - i\frac{d\theta_C}{dt} = c_0 + c_1\frac{AB}{C}e^{-i\phi} + [c_2A^2 + c_3B^2 + c_4C^2]$

- $\phi = \theta_A \theta_B \theta_C$ (Wave phase imbalance)
- $\alpha = \beta + \gamma$ (resonant triad wavenumbers)
- The a_i, b_i, c_i are Landau coefficients, calculated via an orthogonalization assumption (nonlinear wave-wave & wave-mean flow interactions)
- <u>Highest-order</u> (bracketed) terms <u>force equilibrium</u>; dominated by single-wave contributions (a₂A², b₃B², c₄C²)
- Lower-order phase coupling allows inter-scale energy transfer, ENHANCES GROWTH RATE OF SLOWEST-GROWING MODE, ESPECIALLY DURING QUASI-LINEAR PHASE

Quasi-Linear Approximation Assume: A is fastest-growing C is slowest growing • $\frac{d\phi}{dt} = \Delta \omega - [|a_1|] \frac{BC}{A} \sin(\phi + \phi_{\alpha}) +$ $|b_1| \frac{A}{B} \sin(\phi - \phi_\beta) + A, B \text{ grow faster than C:}$ $\therefore \text{ This term dominates } \frac{d\phi}{dt}$ $|c_1| \frac{AB}{C} \sin(\phi - \phi_{\gamma})]$ • $\frac{dA}{dt} = a_0 A + |a_1| BC \cos(\phi + \phi_{\alpha})$ • $\frac{dB}{dt} = b_0 B + |b_1| AC \cos(\phi - \phi_{\beta})$ Enhanced growth of C if: $\phi - \phi_{c1} \sim 2n\pi$ • $\frac{dC}{dt} = c_0 C + |c_1| AB \cos(\phi - \phi_{\gamma})$ • $\Delta \omega = \omega_{\alpha} - \omega_{\beta} - \omega_{\gamma}$ • $\lambda_{\alpha} = a_0 + i \omega_{\alpha}$ (& etc., eigenvalues) • $c_1 = |c_1| e^{i\phi_{\gamma}}$, (& etc., QL Landau coef.)

Quasi-Linear Approximation (cntd.)

- $\frac{d\phi}{dt} \sim |c_1| \frac{A_0 B_0}{C_0} e^{(a_0 + b_0 c_0)t} \sin(\phi \phi_{\gamma})$ - Can solve by separation of variables - Net effect: $\phi - \phi_{\gamma} \rightarrow 2n\pi$ • $\frac{dC}{dt} \rightarrow c_0 C + |c_1| AB$
 - Accelerates growth of mode C

Quasi-Linear Phase-Coupling: Energizing Slowest-Growing Mode



Accelerated growth of slow mode (C)



Starts when $(\phi - \phi_{c1}) \rightarrow 2\pi n$

Phase Imbalance: QL Landau Coefficient: $\phi = \theta_A - \theta_B + \theta_C$ $c_1 = |c_1| e^{i\phi_Y}$

TC PBL Mean Flow from Foster (2009)





GROWTH RATE CONTOURS VS. WAVENUMBER AND ORIENTATION ANGLE



0 contour is shown as dash-dot

ions in Fig. 4 for $a_0 > 0$. The $a_0 = 0$ contour is Contour interval is 0.0005 (nondimensional).

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Large aspect ratio modes are too slowly-growing To compete with lower aspect ratio modes



Contours are Overturning Flow Streamfunction (ψ) **Colors** are:

- top, VERTICAL VELOCITY (W) (top)
- bottom, ALONG-ROLL (U⊥)

NOTE: LARGE ASPECT RATIO MODES EXTEND ABOVE THE PBL TOP (~1.2 KM)

Any Other Evidence?

Small-Scale Spiral Bands Observed in Hurricanes Andrew, Hugo, and Erin

ROBERT GALL, JOHN TUTTLE, AND PETER HILDEBRAND

National Center for Atmospheric Research, Boulder, Colorado (Manuscript received 7 July 1997, in final form 10 October 1997)



GALL ET AL.

FIG. 5. Correlation field from the correlation analysis of the Andrew data from Fig. 1, using $\lambda = 10$, white contours, superimposed on the Fig. 1 radar reflectivity field. Contours start at 0.3 and increment by 0.1. Reflectivity scale same as Fig. 1.

Note: Their definition of aspect ratio is different

5. Conclusions

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The above analysis provides clear evidence for the existence of relatively small-scale ($\sim 10 \text{ km}$) spiral features of rather deep vertical extent near the eyes of three hurricanes. We suspect that these small-scale spiral features are present in most intense hurricanes. Using the radar and in situ data, we have been able to describe many features of these small-scale spiral bands, but the question remains, what are they? At this point we can only offer suggestions, since a definitive description of their structure is not available from the data at hand.

We know that the small-scale bands are on the order of 5 km deep and have spatial scales on the order of 10 km. The higher-reflectivity regions have stronger updrafts and greater equivalent potential temperature than do regions of lower reflectivity. These small-scale bands appear to line up approximately with the low-level wind, where the radial component of the flow is part of the total wind. They move approximately with the mean tangential wind throughout their depth. They exist near the center of the hurricane where the environment is saturated nearly everywhere and the lapse rates are neutral to moist ascent. It is our suggestion that these smallscale rainbands are similar to the rolls that exist in the planetary boundary layer and other flows, that are driven by the boundary layer shear in the presence of convection. Such rolls often have 1:2 aspect ratios, such as described here. We are confused by the depth of the these hurricane bands, however, since they are much deeper than we would have expected for classic boundary layer rolls. Perhaps the environment in the inner 100 km or so of the hurricane is unstable enough, wet enough, and contains enough liquid water (so that weak downdrafts associated with the band structure undergo moist adiabatic processes) and these properties hold through deep enough layers that deep structures typical of boundary layer rolls could develop. Definitive an-

MWR, **126**, 1998

JULY 1998

Motivation for Study

- Can this signal be used to improve SAR (or UHR scatterometer) surface wind retrievals?
 - Orientation relative to the surface wind, mean shear, mean PBL wind?
- Do the large rolls affect PBL fluxes?
- Do they affect air-sea interaction?
 Wind stress curl?

Signature may be clearer in (p=const) WSC than in DIV

Divergence



Wind Stress Curl



Summary

- ALL SAR TC SCENES SHOW SURFACE WIND ORGANIZATION AT O(10 KM) WAVELENGTH; CONSISTENT WITH LARGE ASPECT RATIO PBL ROLLS
 - 1 km SAR wind pixels (from 25 m σ_0 pixels); SLP-filtered winds
 - Consistent with Gall et al. (1998) radar data and brand-new Gruskin & Tripoli numerical modeling research
- LOW-ORDER PHASE COUPLING IN WAVE-WAVE INTERACTION MODEL FEEDS ENERGY FROM THE DOMINANT ROLLS INTO THE SLOWLY-GROWING LONG-WAVELENGTH ROLLS
 - TC PBLs nearly always form O(1-2 km) wavelength rolls
 - Simple and reasonable mechanism for large aspect ratio rolls
 - Mechanism remains to be proven by experiment
- Future work
 - Does this mechanism explain variability in detection of 2 km rolls in TCs?
 - Extend to non-co-linear waves (string of pearls)
 - Cold-air outbreaks
 - Higher-order truncations
 - Understand QL Landau Coefficients

Extra Slides

Zach Gruskin (grad student) and Prof. Greg Tripoli, Univ. Wisconsin (*pers. comm.*)



133 km box15 m Divergence field

333 m resolution numericalmodel of TC-like flow10 km organized convergence isconsistent feature in the simulations

dicular to the rolls. Radar volumes were updated every 300 s;

these intervals were too long to permit esti-





Fig. 3. Large-scale Doppler velocity structure at 23:30:19 UTC, as measured by the DOW radar. Strong easterly flow peaking at \sim 60 m s⁻¹ is evident both off- and onshore. The eye of the hurricane is at the edge of radar visibility to the south. Visibility was severely limited by attenuation. Pink curved arrows illustrate average wind flow. Scan is at 5° elevation.

Fig. 5. Schematic representation of observed shear- and wind-parallel boundary layer rolls. High-momentum air (red) is brought to the surface in the downward legs of the rolls, while air slowed near the surface is brought aloft in the upward legs.

~30 m/s mean +/- 15 m/s across-roll variation in low-level wind



Fig. 4. High-resolution image of Doppler velocity field to the east of Wilmington at 23:58:17 UTC. Sub-kilometer-scale streaks caused by boundary layer rolls modulate the mean easterly flow. Near the radar (left) at altitudes of ~100 m agl, peak and trough wind speed values are ~40 m s⁻¹ and ~10 m s⁻¹, respectively. Further from the radar (right), peak and trough wind speed values alternate from ~25 to ~55 m s⁻¹. Azimuthal shear values are (~30 m s⁻¹/~300 m) \approx 0.1 s⁻¹ across many of the rolls. Scan is at 2° elevation.



Altitude (m agl)

Fig. 6. Altitude dependence of peak wind speeds as observed by DOW and National Weather Service KLTX radars. DOW-measured peak speeds at 100 m agl are nearly as high as those at 1000 m agl as a result of momentum transport in the rolls and agree closely with surface peak wind observations. KLTX-measured peak speeds are smaller at low altitude because of poorer resolution and possibly because of longer overland trajectories.

Wurman and Winslow (1998) *Science*, **280**, 555-557

Transfer Moderate (~3 km) to Small (sub-km) Scales



FIG. 16. Example of superposition of scales of motion. (a) Vertical cross section of residual radial velocity from Hurricane Frances. The solid line indicates the height at which the data presented in (b) were extracted. (b) The thin line represents the individual residual velocity data at 350 m AGL, and the dashed line outlines the larger scales of motion superimposed on the signal.

1000

2000

3000

An Observational Study of Hurricane Boundary Layer Small-Scale Coherent Structures

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(Manuscript received 4 June 2007, in final form 7 January 2008)

ABSTRACT

An Observational Case for the Prevalence of Roll Vortices in the Hurricane

Boundary Laver*

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Boundary-Layer Meteorol (2008) 128:173–189 DOI 10.1007/s10546-008-9281-2

ORIGINAL PAPER

Effects of Roll Vortices on Turbulent Fluxes in the Hurricane Boundary Layer

Jun A. Zhang · Kristina B. Katsaros · Peter G. Black · Susanne Lehner · Jeffrey R. French · William M. Drennan

AUGUST 2005

JOURNAL OF GEOPHYSICAL RESEARCH, VOL. 113, D17104, 16 PP., 2008 doi:10.1029/2007JD009643

Simulation and parameterization of the turbulent transport in the hurricane boundary layer by large eddies

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AUGUST 2008

VOLUME 62

LORSOLO ET AL.

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JOURNAL OF GEOPHYSICAL RESEARCH, VOL. 115, D06205, 13 PP., 2010 doi:10.1029/2009JD011819

Helical circulations in the typhoon boundary layer

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An Observational Study of Hurricane Boundary Layer Small-Scale Coherent Structures

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(Manuscript received 4 June 2007, in final form 7 January 2008)

ABSTRACT

Why Rolls are Prevalent in the Hurricane Boundary Layer

JOURNAL OF THE ATMOSPHERIC SCIENCES

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(Manuscript received 5 April 2004, in final form 1 December 2004)

ABSTRACT

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(Manuscript received 5 April 2004, in final form 12 December 2004)

 λ_{α} = 1.05 (km); λ_{β} = 1.51 (km); λ_{γ} =3.51 (km)

W (shading) and ψ (contour); time = 0120 (min)





Wavelength: ~ 6km; orientation : - 16°

How do rolls fit into overall TCBL momentum balance?

