1. ABSTRACT

We previously developed a linear algebraic subgrid-scale stress [LASS] model. In this paper, we extend our previous work to be applicable to a range of atmospheric stability conditions for the dry atmosphere by adding a passive algebraic subgrid-scale [SGS] heat flux model, which includes scalar production and pressure redistribution terms. We apply this generalized LASS (GLASS) model to large-eddy simulations of idealized atmospheric convective and stable boundary layers. For both boundary layer cases, GLASS predicts the evolution of resolved and SGS turbulent quantities at least as well as the LESs with diffusion models, while including additional physics.

2. A COUPLED SGS TURBULENCE MODEL

Our linear algebraic subgrid-scale stress [LASS] model includes production, dissipation, and pressure redistribution terms (see Enriquez et al., 2010a; Enriquez et al., 2010b, for more details on our strategy and approach). Here, we extend our previous work to be applicable to a range of atmospheric stability conditions for the dry atmosphere by adding a passive algebraic subgrid-scale heat flux model, which includes scalar production and pressure redistribution terms. In what follows the SGS terms are:

1) stress, \( \overline{A}_{\kappa} = \overline{\bar{u}_i u_i} - \overline{\bar{u}} \bar{u} \), and

2) heat flux, \( \overline{\bar{a}} = \overline{\bar{u} \theta} - \overline{\bar{u}} \bar{\theta} \),

where, in the Carati et al. (2001) framework for LES, the overbar represents a spatial filter and the tilde represents a discretization filter. In this paper we focus only on the SGS model, ignoring reconstruction effects; see (Enriquez et al., 2010a) for an application with SGS and reconstruction models together.

2.1 The Linear Algebraic Subgrid-Scale Stress [LASS] Model

The evolution equation of the SGS stress, \( \overline{A}_{\kappa} \), includes: advection, diffusion, production, dissipation, pressure redistribution, buoyancy generation, and Coriolis terms. We simplify the equation to be modeled to include only production, dissipation, and pressure redistribution; neglected terms are assumed small. Production terms are not modeled, the dissipation term appears in its general isotropic form for high Reynolds number flows, and the pressure redistribution term is modeled with the Reynolds-averaged Navier-Stokes Launder et al. (1975) model adopted for LES.

\[
0 = - \overline{A}_{\kappa} \frac{\partial \overline{u}_i}{\partial x_k} - \overline{A}_x \frac{\partial \overline{u}_i}{\partial x_k} - \frac{\delta}{\tau} + \Pi_{i,LASS} \tag{1}
\]

\( \Pi_{i,LASS} \) is collectively a slow pressure-strain term, a rapid pressure-strain term, and a wall effects term that involves interactions of the first two terms and a surface:

\[
\Pi_{i,LASS} = -c_1 \frac{\tau}{\bar{e}} (\overline{A}_{\kappa} - \frac{\delta}{\tau} \overline{\bar{e}} \delta_i) \tag{2}
\]

With Equations (1) – (2) the following equations form the LASS model:

\[
P_i = -\left( \overline{A}_{\kappa} \frac{\partial \overline{u}_i}{\partial x_k} + \overline{A}_x \frac{\partial \overline{u}_i}{\partial x_k} \right) \tag{3}
\]

\[
P = -\overline{A}_i \frac{\partial \overline{u}_i}{\partial x_i} \tag{4}
\]

\[
\bar{S}_i = \left( \frac{\partial \overline{u}_i}{\partial x_k} + \frac{\partial \overline{u}_k}{\partial x_i} \right) \tag{5}
\]

\[
\bar{S}_i^2 = \frac{1}{2} \bar{S}_i \tag{6}
\]

\[
D_i = -\left( \overline{A}_{\kappa} \frac{\partial \overline{u}_i}{\partial x_k} + \overline{A}_x \frac{\partial \overline{u}_i}{\partial x_k} \right) \tag{7}
\]

\[
\Delta_y = \left( \Delta_x \Delta_y \Delta_z \right)^{\frac{5}{3}} \tag{8}
\]

\[
f(h) = 0.27 \frac{\Delta_y}{h} \tag{9}
\]

\[
\overline{\bar{e}} = \frac{1}{2} \overline{A}_i \tag{10}
\]

\[
\tau = 1.2 \frac{\overline{\bar{e}}^{\frac{1}{2}}}{\Delta_y} \tag{11}
\]

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Model coefficients [Table 1] are based on suggestions from Launder et al. (1975), Morris (1984), Shabbir and Shih (1992), and Wallin and Johansson (2000). We strongly emphasize that a canopy wall model is 
\textit{not} used.

<table>
<thead>
<tr>
<th>Table 1. LASS model coefficients</th>
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<tbody>
<tr>
<td>$c_1$</td>
</tr>
<tr>
<td>1.8</td>
</tr>
</tbody>
</table>

Results of large-eddy simulations at various resolutions of a neutrally-stratified boundary layer flow over a flat, rough surface indicate that the LASS model is a more physically complete SGS stress turbulence model that provides near-wall anisotropies that eddy-viscosity models do not, and yields proper shear stress values in the logarithmic layer. We have also examined the combination of the LASS model with reconstruction, using the approximate deconvolution model, of the subfilter-scale stress (Chow et al., 2005). With that mixed model we are able to represent both anisotropy and energy backscatter (Enriquez et al., 2010a).

\textbf{2.2 The Generalized Linear Algebraic Subgrid-Scale Stress [GLASS] Model}

The SGS stresses are solved as a system of linear equations as above and are then coupled to the set of equations that model the SGS heat flux. The fully coupled active scalar version of the GLASS model includes a buoyant production term:

$$ 0 = -A_k \frac{\partial \overline{u}}{\partial x_k} - \frac{2}{3} \frac{\overline{U^3}}{\overline{U^2}} + \Pi_{i, \text{GLASS}} $$

and the rapid pressure strain term of Equation (2) has an additional corresponding buoyancy production term:

$$ \Pi_{i, \text{GLASS}} = \Pi_{i, \text{LASS}} - c_d \frac{g}{\Delta} \left( A_k \delta_i \delta_j - \frac{1}{3} A_0 \delta_i \delta_j \right) . $$

In this paper, the buoyant production terms are left out, as indicated by the crossed out terms above, and we are modeling SGS heat flux as a passive scalar, which is, however, influenced by the SGS stress acting on the mean temperature gradient.

The evolution equation of the SGS heat flux includes: advection, diffusion, gradient production, viscosity, pressure redistribution, buoyancy production, and Coriolis terms. We simplify the equation to be modeled to include only production, dissipation, and pressure redistribution. Buoyant production is neglected as Ramachandran and Wyngaard (2010) showed it to be small compared to other terms; other neglected terms are also assumed small. Production terms are not modeled, the dissipation term drops out, and the pressure redistribution term is modeled with the Reynolds-averaged Navier-Stokes Launder and Samaraweera (1979) model:

$$ 0 = -A_k \frac{\partial \overline{u}}{\partial x_k} + \Pi_{i, \text{GLASS}} $$

For this model, we account for stability and shear by modifying 1) the SGS TKE term and 2) the dissipation term by altering its length scale according to Deardorff (1980):

$$ \overline{\theta} = \begin{cases} \frac{1}{2} A_1 & \text{if } \frac{1}{2} A_1 > 0, \\ 0.0826 \sqrt[3]{S} & \text{if } \frac{1}{2} A_1 = 0 \text{ and } N^2 \leq 0.0. \end{cases} $$

$$ N^2 = \frac{g}{\theta} \frac{\partial \overline{\theta}}{\partial z} $$

$$ \Delta = \begin{cases} 0.76 \sqrt[3]{S} & \text{if } N^2 > 0.0, \\ \Delta_{\theta} & \text{if } N^2 \leq 0.0. \end{cases} $$

$$ \tau = 1.2 \overline{\theta}^{1.5} \Delta_{\theta} $$

We refer to the set of SGS stress-heat flux equations as the GLASS model. Model coefficients [Table 2] are based on suggestions from Launder and Samaraweera (1979), Chen and Jaw (1998), Wikstrom et al. (2000), and Craft and Launder (2001).

<table>
<thead>
<tr>
<th>Table 2. GLASS model coefficients</th>
</tr>
</thead>
<tbody>
<tr>
<td>$c_{10}$</td>
</tr>
<tr>
<td>3.0</td>
</tr>
</tbody>
</table>

\textbf{3. LES SETUP AND RESULTS}

We employ the Advanced Regional Prediction System [ARPS] in LES mode (cf., Chow et al., 2005; Enriquez et al., 2010a). ARPS is 3D, compressible, non-hydrostatic, and parallelized (Doyle et al., 2000; Xue et al., 2000; Xue et al., 2001; Chow et al., 2005). As usual in LES, the governing equations are filtered and ARPS generates the resolved-scale flow variables as influenced by the SGS terms discussed above, as well as the output from the SGS subroutines. See Enriquez et al. (2010a, 2010b) for more details of the setup and processes.

We test GLASS with different stability conditions with simulations of a convective and stable boundary layer. Details of the simulations and the results of the two simulations are described in the following subsections.
3.1 Convective Boundary Layer [CBL] Flow

We simulate a surface sheared CBL with a constant geostrophic wind of 20 m s$^{-1}$ (Fedorovich, 2004; Conzemius and Fedorovich, 2006; Fedorovich and Conzemius, 2008). Initial conditions of the simulation can be seen in Figure 1a and LES parameters are listed in Table 3. Simulations are carried out for 10,000 s, before the upper damping layer clearly impacts the CBL.

Table 3. Convective boundary layer LES parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Horizontal resolution, (\Delta x, \Delta y)</td>
<td>40 m</td>
</tr>
<tr>
<td>Vertical resolution, (\Delta z)</td>
<td>20 m average, 10 m minimum</td>
</tr>
<tr>
<td>Domain size</td>
<td>10.24 km x 10.24 km x 2 km</td>
</tr>
<tr>
<td>Time steps</td>
<td>0.5 s large, 0.05 s small</td>
</tr>
<tr>
<td>Reference temperature</td>
<td>300 °K</td>
</tr>
<tr>
<td>Surface heat flux</td>
<td>0.1 °K m s$^{-1}$</td>
</tr>
<tr>
<td>Coriolis parameter</td>
<td>(f[40^\circ N] = 0.9 \times 10^{-4} \text{ s}^{-1})</td>
</tr>
<tr>
<td>Lateral boundaries</td>
<td>Periodic</td>
</tr>
<tr>
<td>Bottom boundary</td>
<td>Rigid wall, semi-slip</td>
</tr>
<tr>
<td>Roughness length</td>
<td>0.01 m</td>
</tr>
</tbody>
</table>

Turbulence occurs at various scales in the CBL. Small-scale turbulence dominates in the surface layer, while large thermals play a key role in the mixed layer. Instantaneous resolved vertical velocity x-z slice at 10,000 s [Figure 2] shows that varied length scales can be seen, from meters by the surface to hundreds of meters near and above the inversion, in the simulation using the GLASS turbulence model.

In addition, in Figure 3 we compare the planar averaged ARPS SGS heat flux vertical profile produced by GLASS with the ARPS SGS heat flux vertical profile produced by a turbulent kinetic energy [TKE]-1.5 turbulence closure (Deardorff, 1980) at 10,000 s (Fedorovich, 2004). For this simulation, we see that these averaged SGS heat flux profiles are similar in magnitude and shape, and have a negative peak in close proximity to the inversion height. It is important to note that the SGS heat flux is a very small part of the total heat flux.

One of the most notable contributions of the LASS model is its ability to provide near-wall SGS stress anisotropy. In addition to near-wall stress anisotropy, GLASS represents the SGS heat flux anisotropy for wall-bounded flows that isotropic eddy diffusion models cannot because the SGS heat fluxes are not, in general, aligned with the resolved scalar gradient. The heat transport in the temperature field of wall-bounded shear flows is known to be anisotropic at inertial and dissipative scales (Warhaft, 2000). Figure 4 depicts the GLASS planar averaged SGS heat fluxes, at 10,000 s.
3.2 Stable Boundary Layer [SBL] Flow

We simulated a moderately stable atmospheric boundary layer driven by a geostrophic wind of 10 m s\(^{-1}\) at a coarse resolution (Zhou and Chow, 2011). Initial conditions of the simulation can be seen in Figure 1b and LES parameters are listed in Table 4. Similar simulations at higher resolutions have been conducted by Kosović and Curry (2000), Saiki et al. (2000), Basu and Porté-Agel (2006).

Table 4. Stable boundary layer LES parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Horizontal resolution, (\Delta_x, \Delta_y)</td>
<td>16 m</td>
</tr>
<tr>
<td>Vertical resolution, (\Delta_z)</td>
<td>16 m average, 5 m minimum</td>
</tr>
<tr>
<td>Domain size</td>
<td>640 m x 640 m x 640 m</td>
</tr>
<tr>
<td>Time steps</td>
<td>0.25 s large, 0.025 s small</td>
</tr>
<tr>
<td>Initial surface temperature</td>
<td>300 °K</td>
</tr>
<tr>
<td>Surface heat flux</td>
<td>-0.02 °K m s(^{-1})</td>
</tr>
<tr>
<td>Coriolis parameter</td>
<td>(f[45° N] = 1 \times 10^{-4} \text{ s}^{-1})</td>
</tr>
<tr>
<td>Lateral boundaries</td>
<td>Periodic</td>
</tr>
<tr>
<td>Bottom boundary</td>
<td>Rigid wall, semi-slip</td>
</tr>
<tr>
<td>Roughness length</td>
<td>0.1 m</td>
</tr>
</tbody>
</table>

We carry out our simulations for 50,000 s to demonstrate that GLASS sustains resolved turbulent kinetic energy at a coarse resolution. It is evident that at this coarse resolution of 16 m, the Dynamic Wong-Lilly (Wong and Lilly, 1994) SGS turbulence model [DWL] and GLASS sustain the resolved TKE throughout the simulation [Figure 5]. The resolved TKE decayed rapidly in similar simulations that used non-dynamic eddy-viscosity turbulence models, like the Smagorinsky and TKE-1.5 (Zhou and Chow, 2011). We use DWL results to compare the performance of GLASS to a dynamic eddy-viscosity model and to the results of Zhou and Chow (2011).

![Figure 4. GLASS planar and time averaged SGS heat fluxes, at 10,000 s.](image1)

![Figure 5. GLASS and Dynamic Wong-Lilly [DWL] simulations maintain a high level of vertically integrated SGS turbulent kinetic energy.](image2)

![Figure 6. Instantaneous resolved vertical velocity x-z slices (y = 320 m) of two different SGS turbulence models at 50,000 s.](image3)
4. CONCLUSIONS

The CBL and SBL simulations demonstrate that GLASS can 1) perform well at different stability regimes and 2) provide scalar and momentum flux anisotropies. Notably, GLASS overcomes the need to alter model coefficients for different positions in the flow, grid/filter aspect ratios, and atmospheric stabilities, etc. Future work includes adding the buoyancy production terms for two-way coupling and simulating a diurnal cycle to study the transitions from one stability case to another.

5. ACKNOWLEDGMENTS

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6. REFERENCES


Fedorovich, E., and R. Conzemius, 2008: Effects of wind shear on the atmospheric convective


