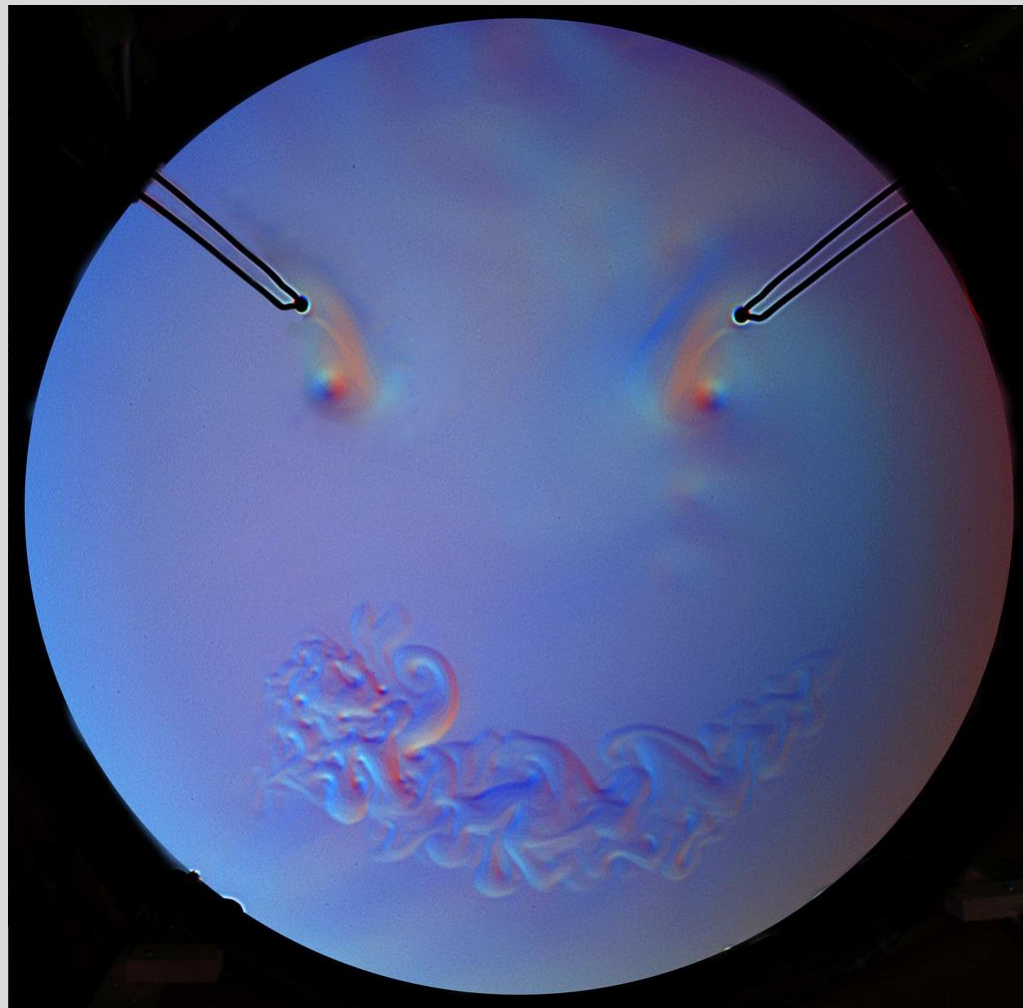
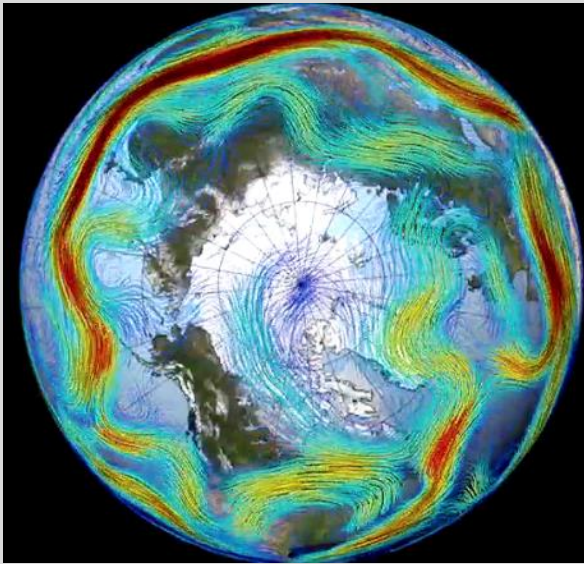


# Equilibrating baroclinic instability and zonal jets on the polar beta-plane: experiments with altimetry

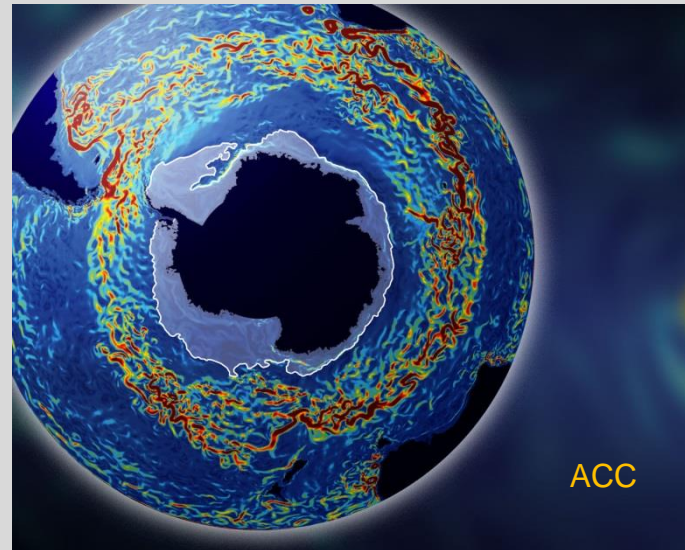
Yakov Afanasyev



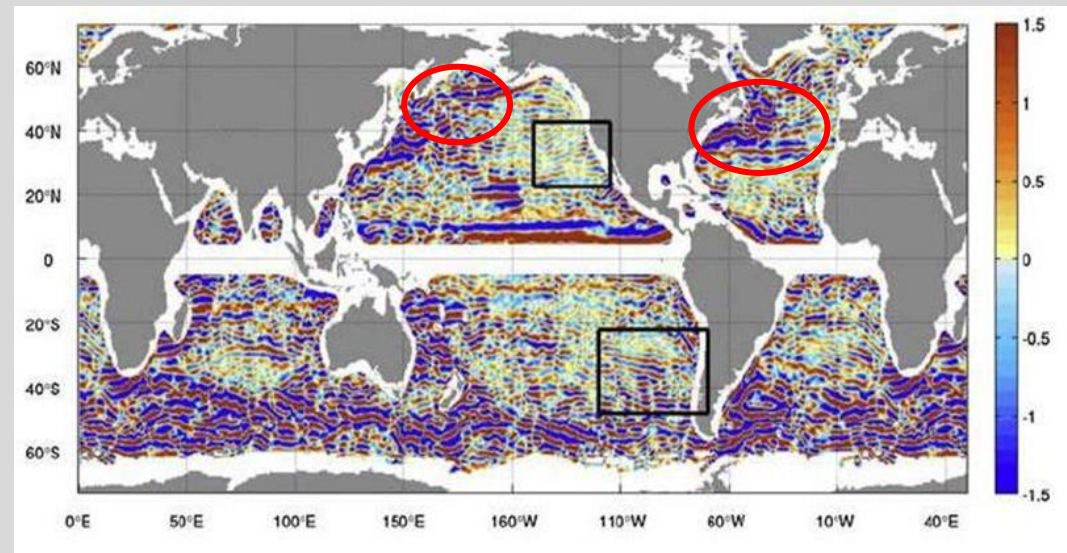
# Zonal Jets in the ocean and atmosphere: baroclinically unstable regions



Jet stream in Southern hemisphere

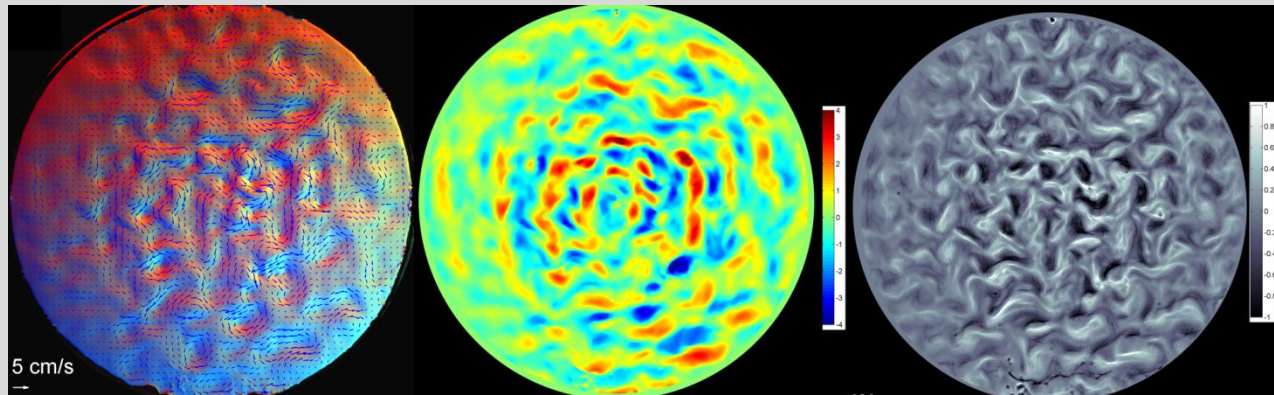


Satellite altimetry: mean surface geostrophic velocity. (Maximenko et al., 2008)



## Zonal jets: Mechanisms

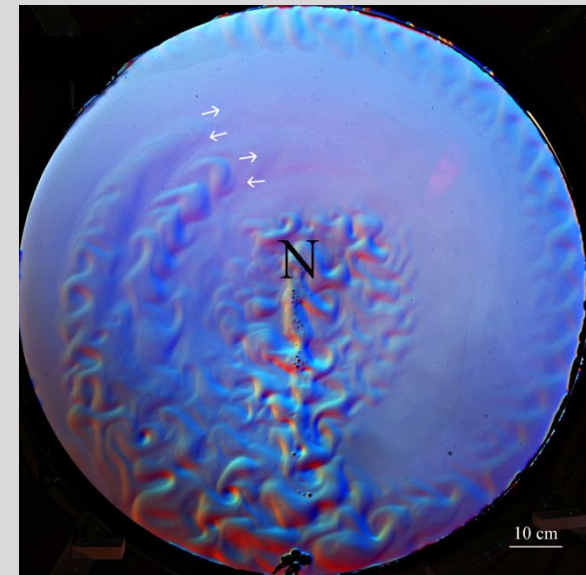
- “Rhines jets” occur due to transition between “eddies” and Rossby waves (Rhines 1975)



Barotropic turbulence  
generated by an array of  
magnets on polar  
 $\beta$ -plane

- “ $\beta$ -plumes” : radiation of Rossby waves by meanders, eddies (Afanasyev, O’Leary, Rhines, Lindahl 2012)

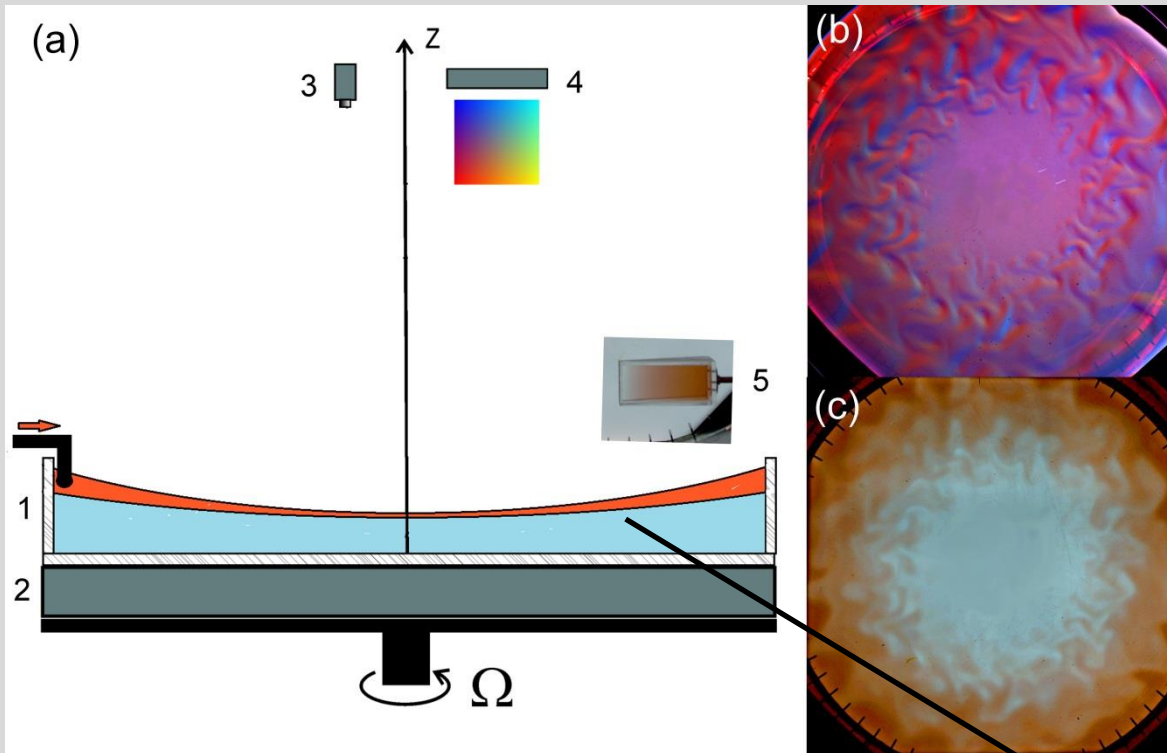
$\beta$ -plumes due to  
baroclinic meanders  
generated by a linear  
heat source



- “Noodle instability”: secondary instability of the baroclinically unstable flow (Berloff, Kamenkovich, Pedlosky 2009)

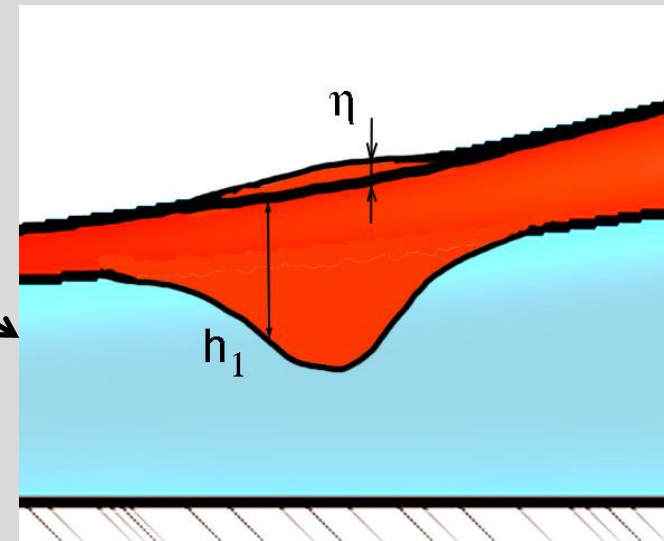


# Lab experiments: Altimetric Imaging Velocimetry (AIV) and Optical Thickness Velocimetry (OTV)



“Black and white” altimetry (Rhines, Lindahl and Mendez, JFM 2006)

Color altimetry (Afanasyev, Rhines and Lindahl, Exp. Fluids 2009)



$$h(r) = H_0 + \frac{\Omega_0^2}{2g} \left( r^2 - \frac{D^2}{8} \right) \quad \text{“Geoid”}$$

# AIV / OTV : velocity from the surface/interface slope

AIV measures the slope of the surface elevation field  $\eta$  in each pixel of the image of the flow

$$\nabla \eta = \left( \frac{\partial \eta}{\partial x}, \frac{\partial \eta}{\partial y} \right)$$

The velocity field is calculated using the geostrophic relation

$$\mathbf{V}_g = \frac{g}{f_0} (\mathbf{n} \times \nabla \eta) = \frac{g}{f_0} \left( -\frac{\partial \eta}{\partial y}, \frac{\partial \eta}{\partial x} \right)$$

Barotropic velocity (by AIV)

$$\mathbf{V}_{gbc} = -\frac{g'}{f_o} (\mathbf{n} \times \nabla h_1)$$

Baroclinic velocity (by OTV)

$$\mathbf{v}_1 = \mathbf{V}$$

Upper layer

$$\mathbf{v}_2 = \mathbf{V} + \mathbf{V}_{bc}$$

Lower layer

AIV / OTV: velocity from the surface slope

Full shallow water equation:

$$\frac{\partial \vec{V}}{\partial t} = -\nabla \left( \frac{1}{2} V^2 + g\eta \right) + \left( 2\vec{\Omega} + \nabla \times \vec{V} \right) \times \vec{V}$$



$$\vec{V} = \vec{V}_g - \frac{g}{f_0^2} \frac{\partial}{\partial t} \nabla \eta - \frac{g^2}{f_0^3} J(\eta, \nabla \eta)$$

geostrophic

unsteady

nonlinear

## AIV: Resolution

- 4 million velocity vectors at 5 – 10 frames per second
- typical experiment (30 min – 1 hour) generates hundreds of Gbytes of data
- spatial scales from 0.5 mm to 1.1 m
- velocity from 1 mm/s to 10 cm/s
- typical surface elevation 100  $\mu\text{m}$

## Dynamic similarity to polar $\beta$ -plane

$$f = 2\Omega \sin \varphi, \quad f \approx 2\Omega \sin \varphi_0 + \frac{2\Omega \cos \varphi_0}{a} y \quad \beta\text{-plane}$$

$$f \approx 2\Omega \left(1 - \frac{\phi^2}{2}\right) = 2\Omega - \frac{\Omega}{a^2} r^2 = f_0 - \gamma r^2 \quad \text{Near the pole, colatitude } \phi = 90^\circ - \varphi \rightarrow 0$$

- Rotating tank: conservation of potential vorticity (PV)

$$q = \frac{2\Omega_0 + \zeta}{h} = \frac{1}{H_0} \left[ \zeta + 2\Omega_0 \left( 1 - \frac{\Omega_0^2}{2gH_0} \left( r^2 - \frac{D^2}{8} \right) \right) \right]$$

- The polar  $\beta$ -plane ( $\gamma$ -plane)

$$f = f_0 (1 - \gamma r^2)$$

Laboratory

$$\gamma = \frac{\Omega_0^3}{gH_0}$$

Earth

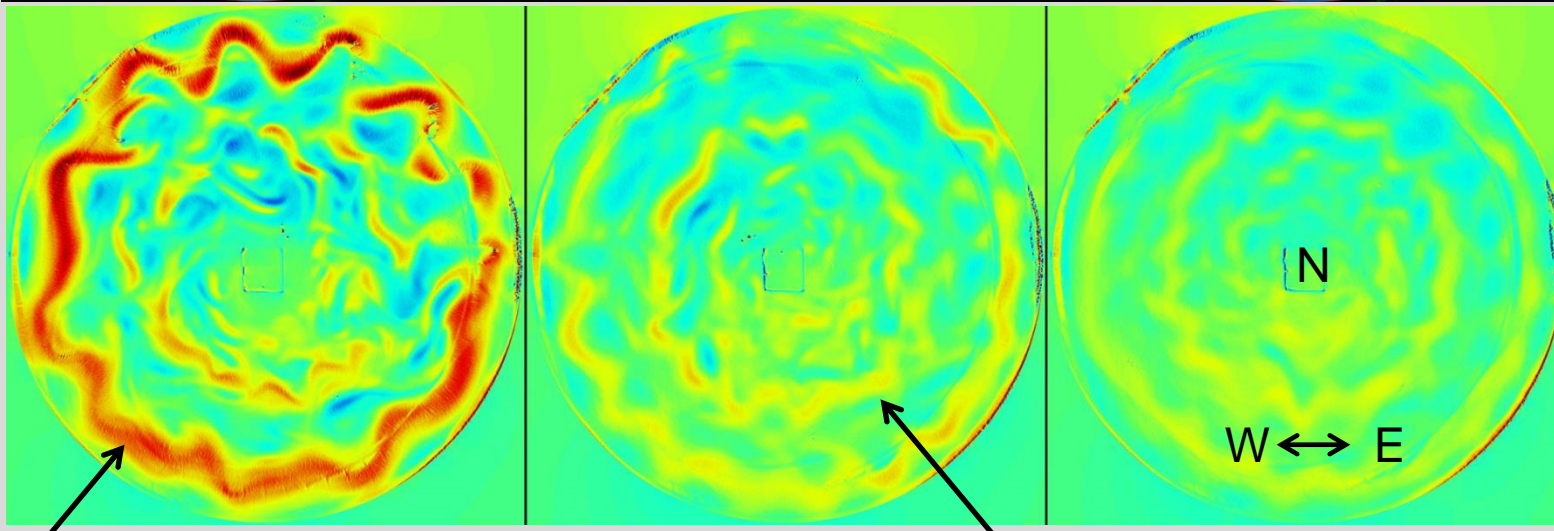
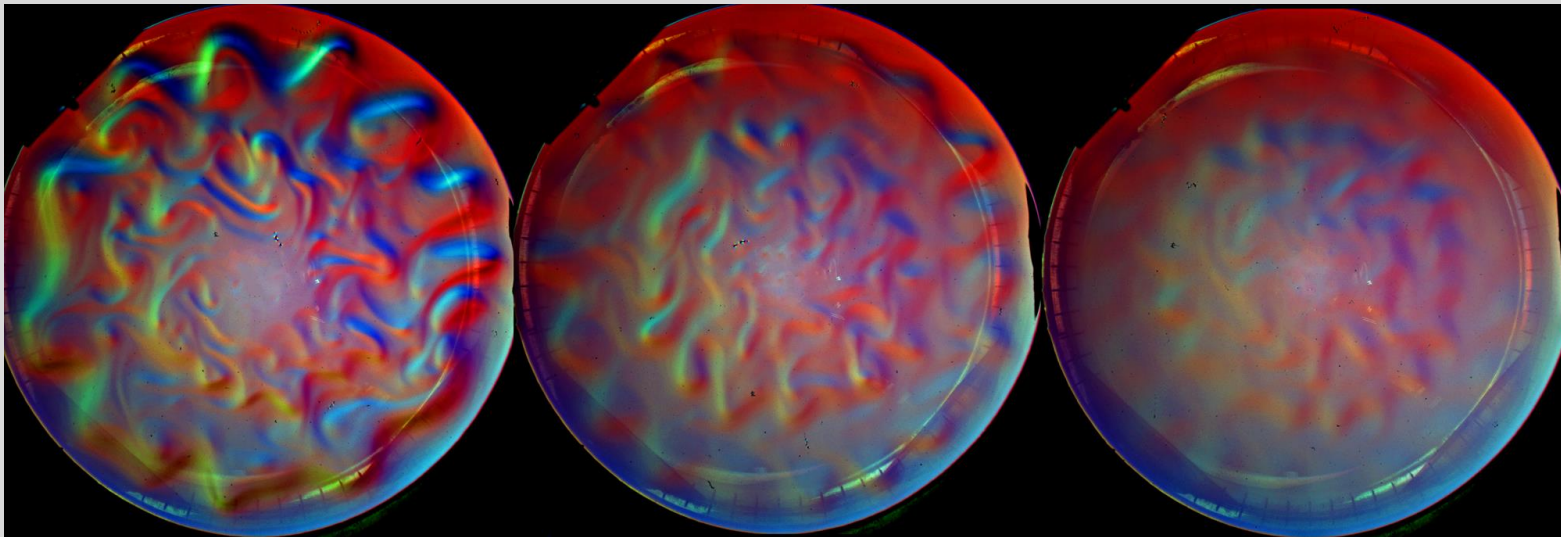
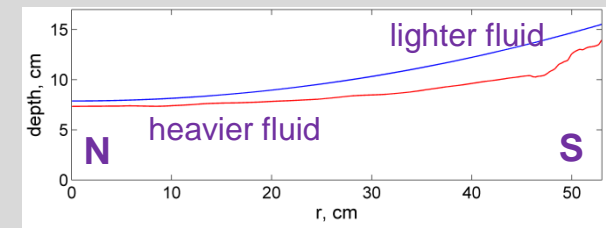
$$\gamma = \Omega_o / a^2$$

- Local  $\beta$ -plane can be introduced at “midlatitudes” at radius  $r_0$

$$\beta = 2\Omega^3 r_0 / gh(r_0)$$



# Baroclinically unstable flow visualized by the altimetry: evolution

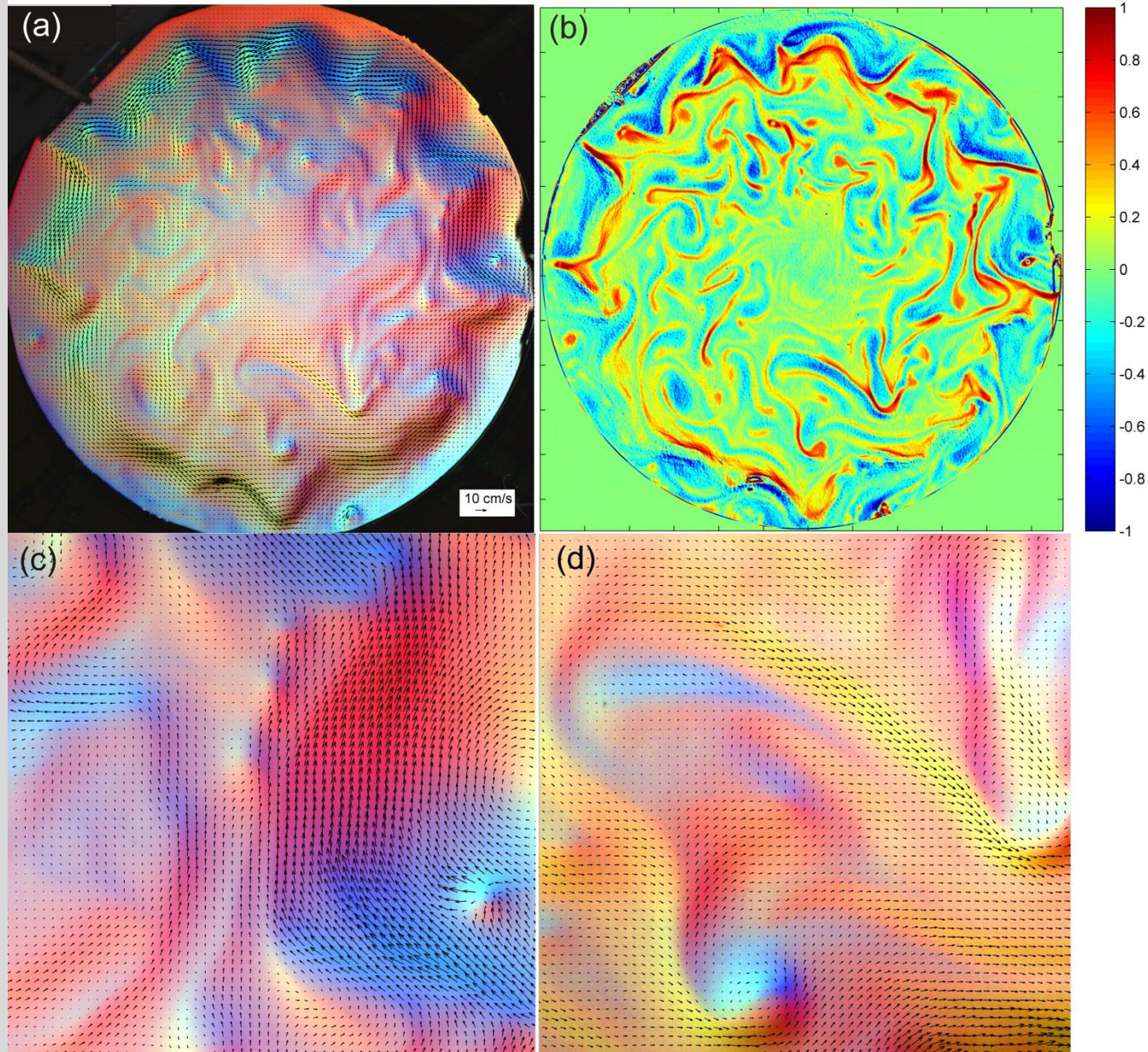


“Coastal” or “subtropical” jet

“Polar”, eddy-driven jet

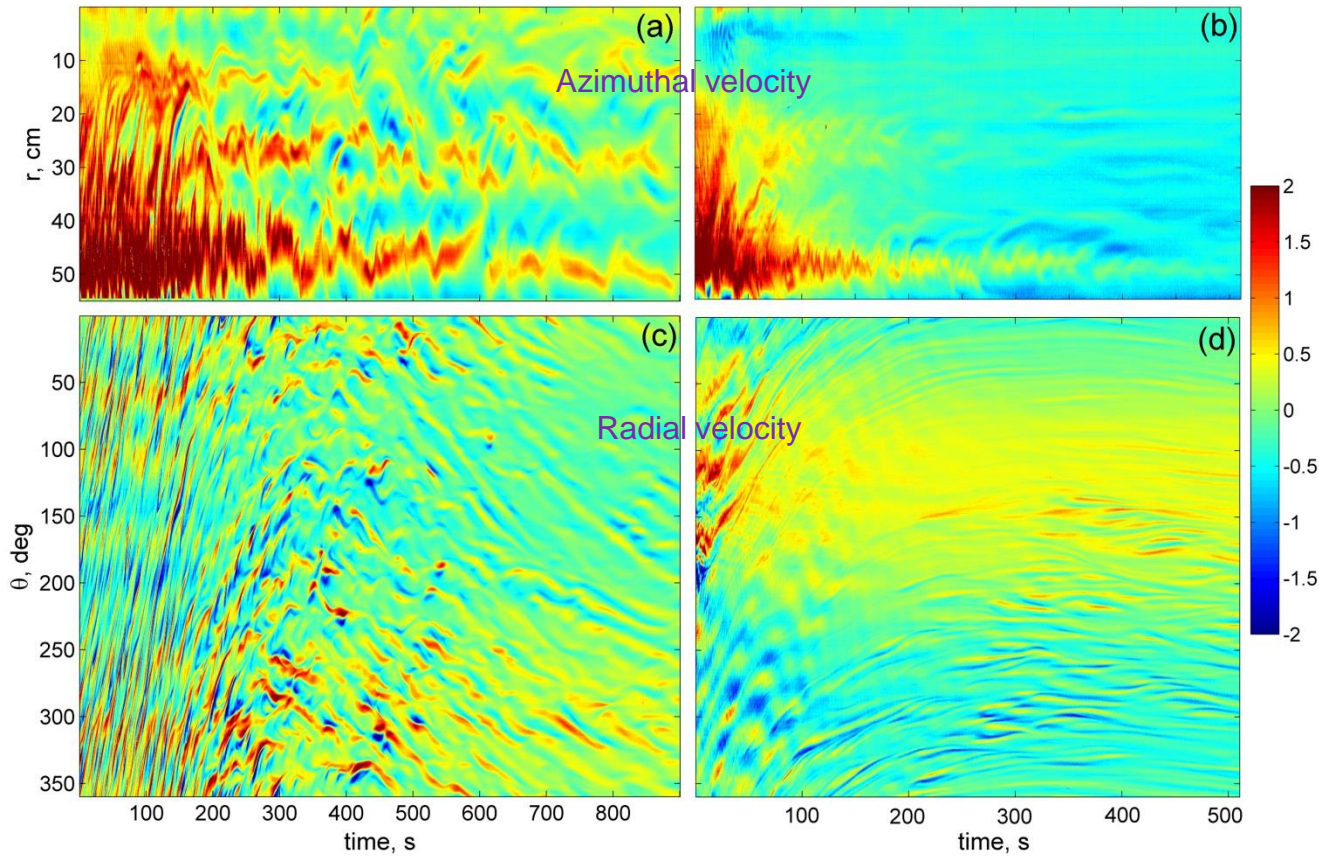


# Baroclinically unstable flow visualized by the altimetry: baroclinic meanders, filaments and eddies



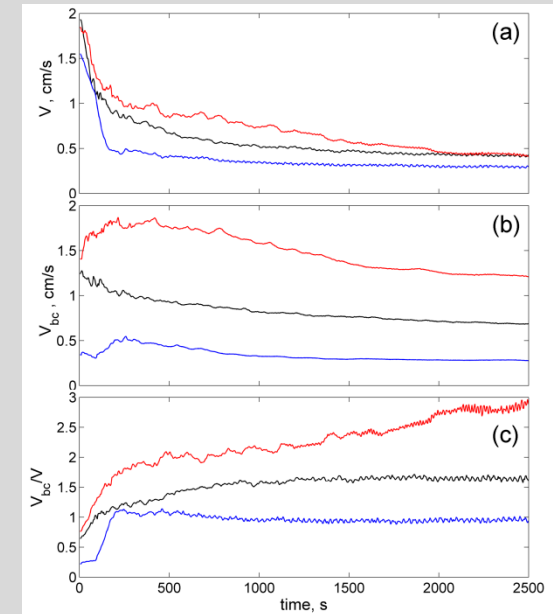


# Baroclinically unstable flow : evolution



$S = 30$  ppt and  $S = 10$  ppt

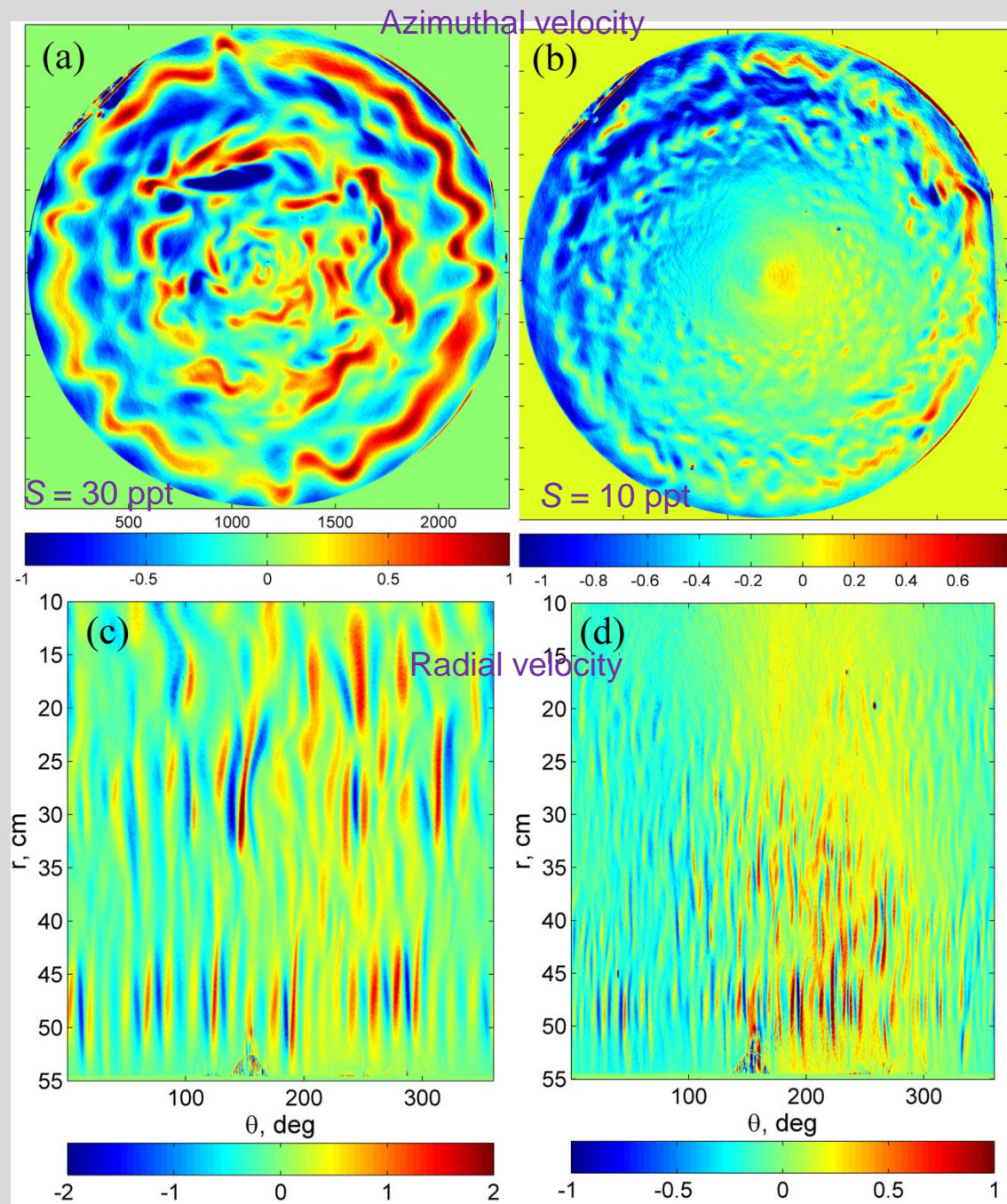
## RMS barotropic and baroclinic velocity



$S = 15$  (blue),  $45$  (black) and  $65$  ppt (red)

# Jet scaling

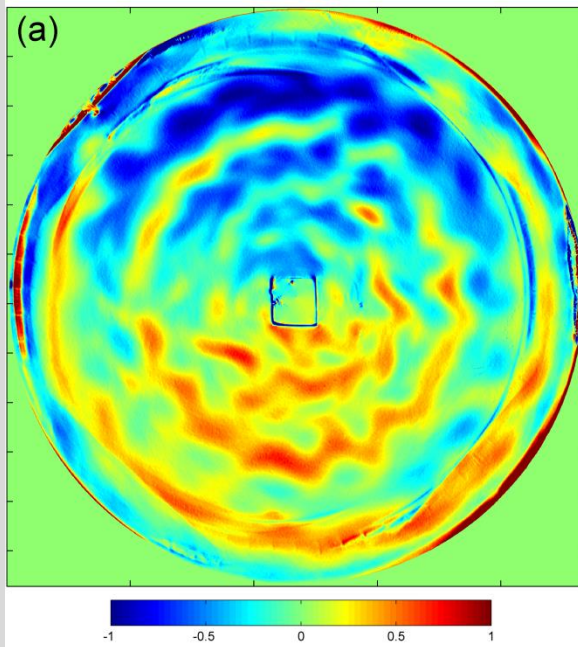
Radial perturbation velocity is coherent in the radial direction: meridional jets (a.k.a. “noodles”, Berloff et al. 2009) ?



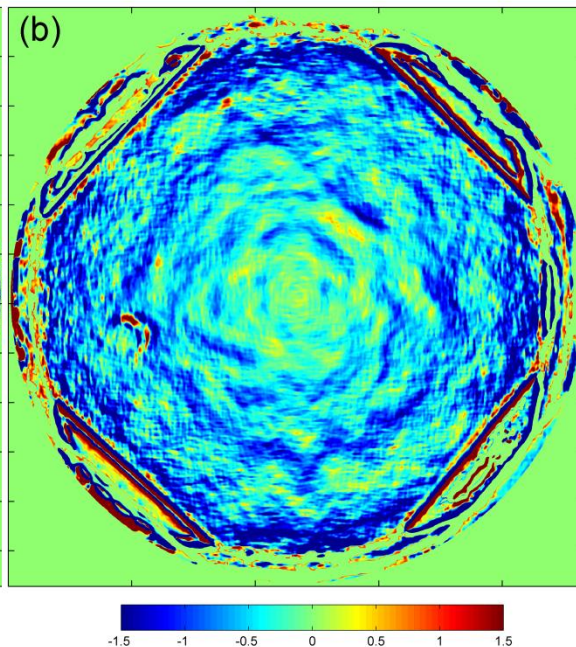


# Eddy forcing

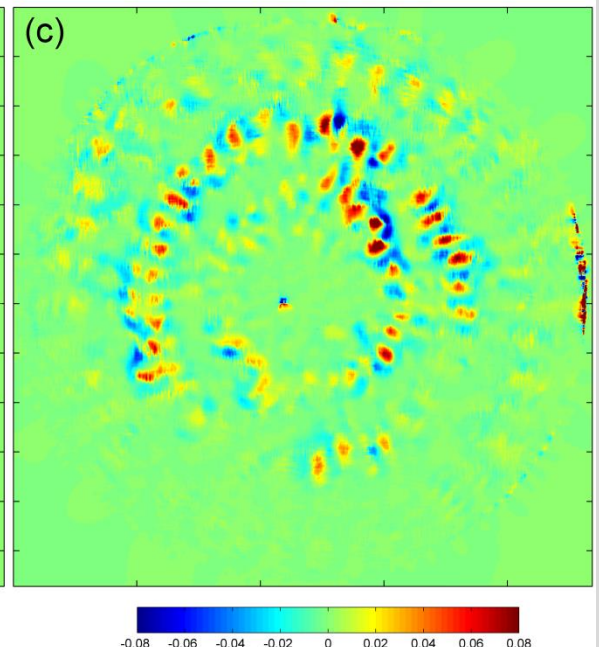
Barotropic velocity



Baroclinic velocity



Reynolds stress



$$\frac{\partial \bar{u}_n}{\partial t} \approx \begin{cases} -\frac{D_{1.5}}{\bar{h}_1} - \frac{\partial R_1}{\partial y}, & n=1 \\ \frac{D_{1.5}}{\bar{h}_2} - \frac{\partial R_2}{\partial y} - f_{bot}, & n=2 \end{cases}$$

$$D_{1.5} = -f_0 \overline{v'_{1.5} \eta'_{1.5}}$$

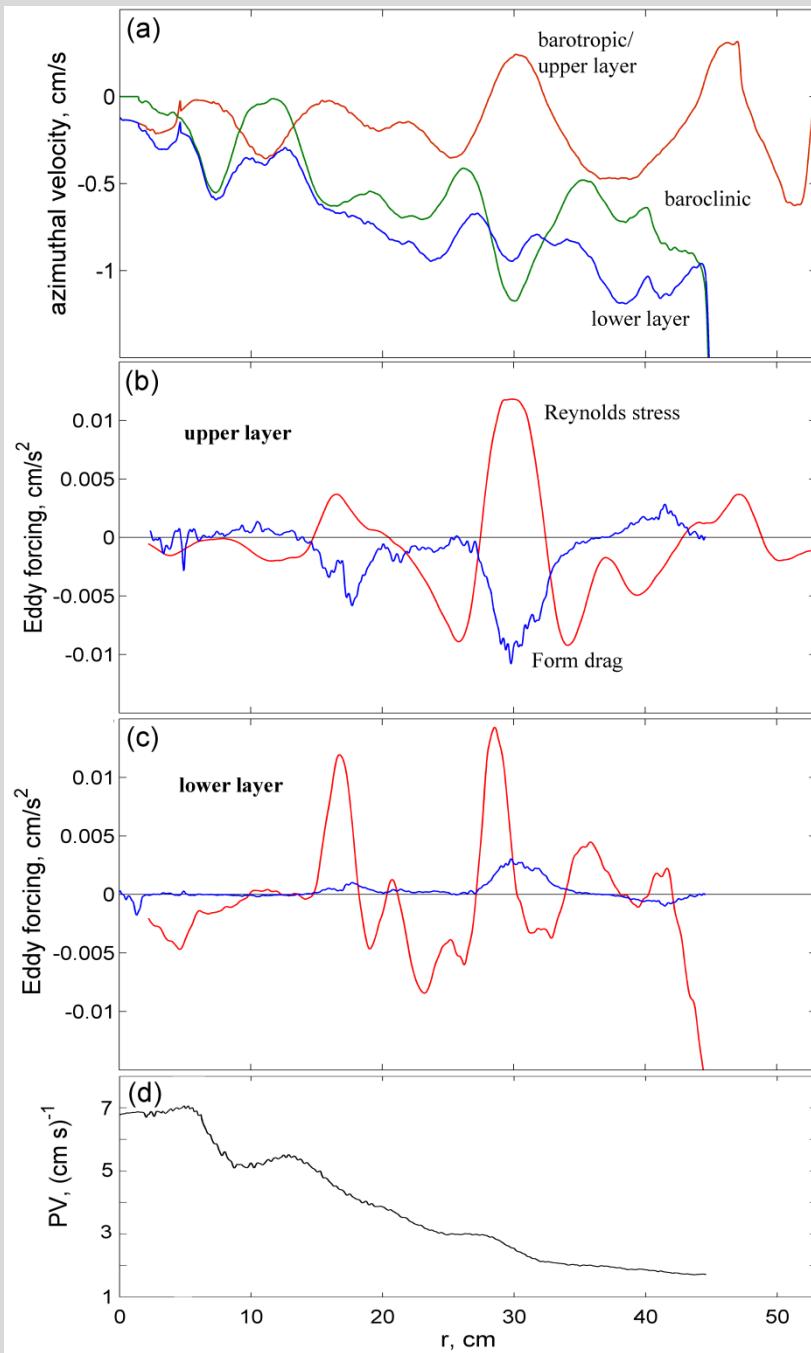
$$R_n = \overline{u'_n v'_n}$$

Form drag

Reynolds stress



## Eddy forcing



$$\frac{\partial \bar{u}_n}{\partial t} \approx \begin{cases} -\frac{D_{1.5}}{\bar{h}_1} - \frac{\partial R_1}{\partial y}, & n=1 \\ \frac{D_{1.5}}{\bar{h}_2} - \frac{\partial R_2}{\partial y} - f_{bot}, & n=2 \end{cases}$$

$$D_{1.5} = -f_0 \overline{v'_{1.5} \eta'_{1.5}}$$

Form drag

$$R_n = \overline{u'_n v'_n}$$

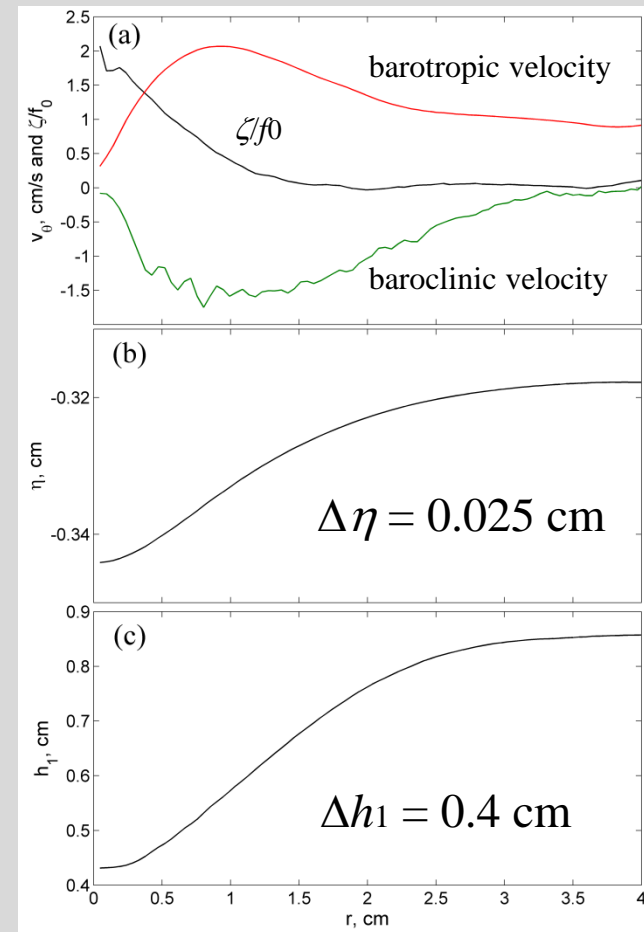
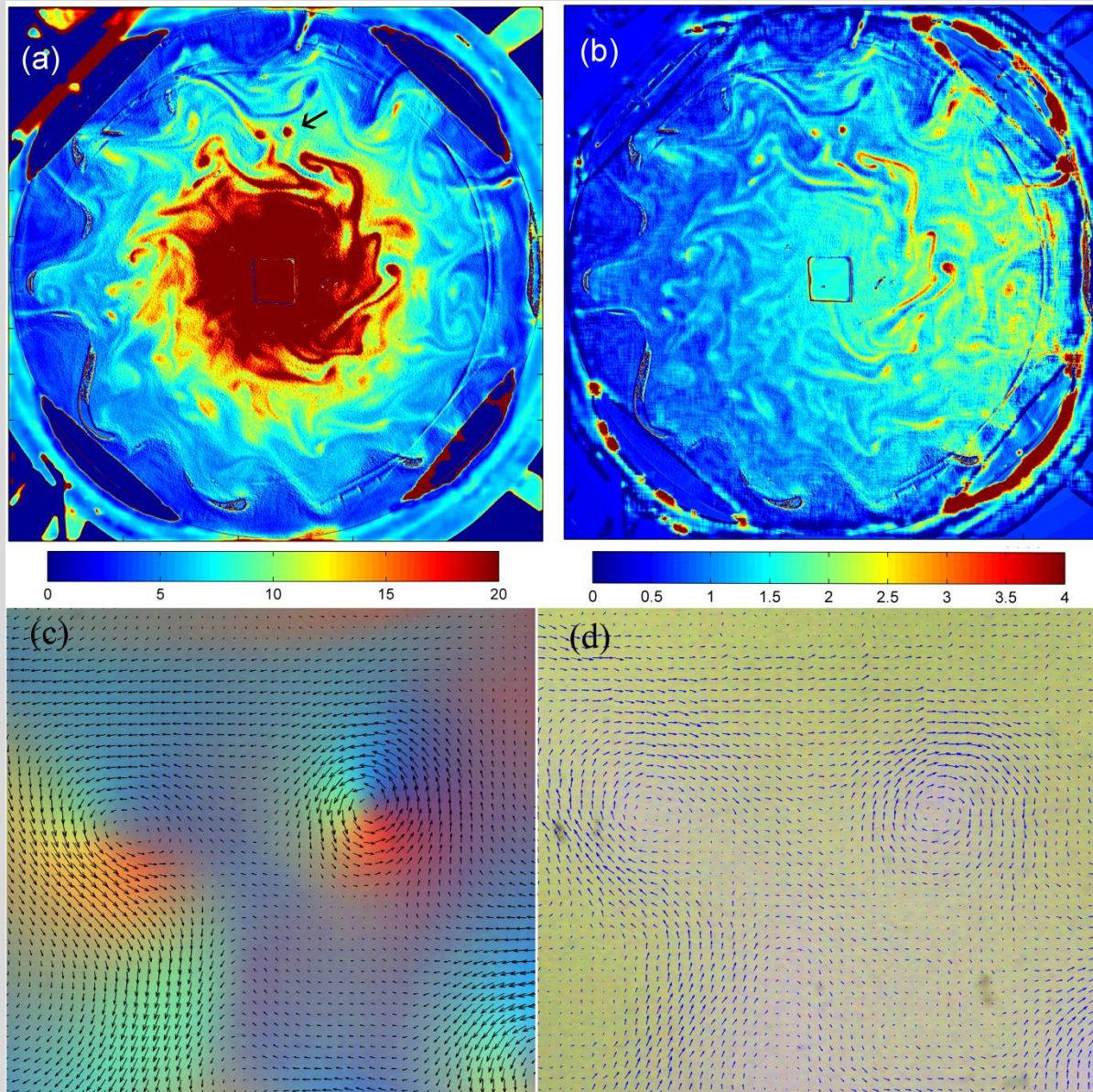
Reynolds stress

$$v_{1.5} = (v_1 + v_2) / 2$$

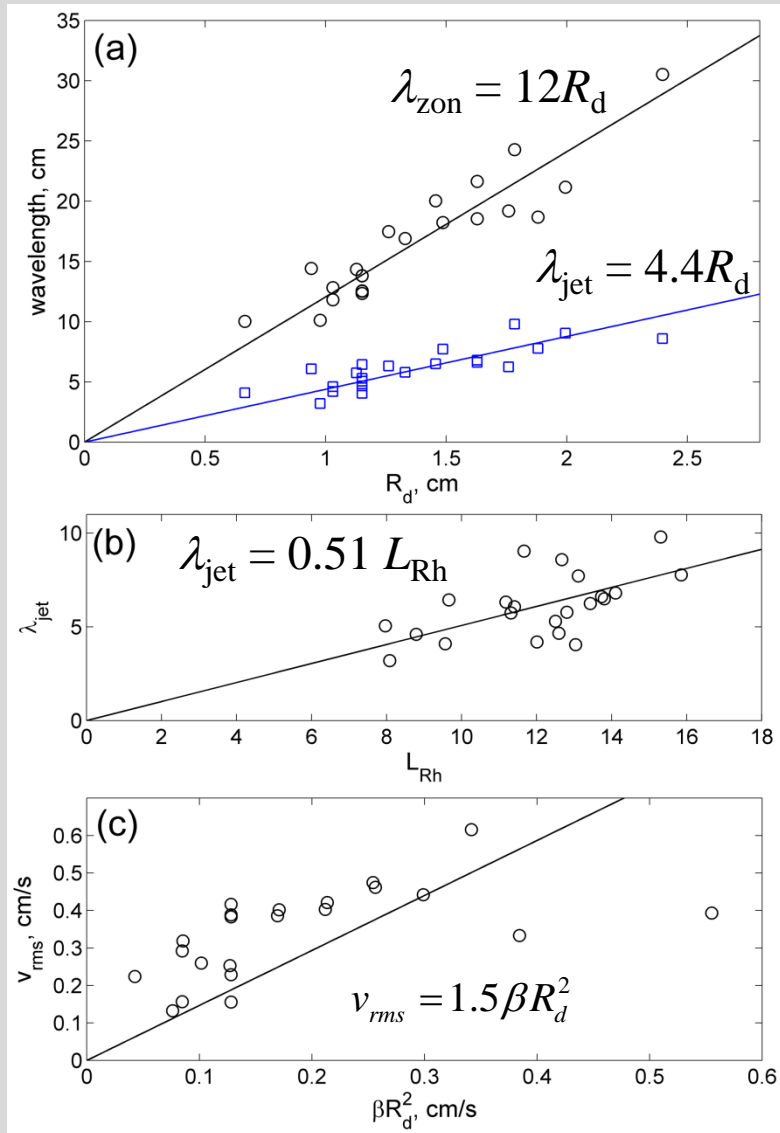
$$\eta'_{1.5} = \bar{h}_1 - h_1$$

# Eddies

- All eddies are cyclonic
- Lifetime  $\sim 2 - 10$  “days”
- Strongly baroclinic, “surface intensified”



# Jet scaling



Rhines scale

$$L_{Rh} = 2\pi(V_{rms}/\beta)^{1/2}$$

Radius of deformation

$$R_d = (g'H_0/2)^{1/2}/f_0$$

## Conclusions

- Zonal jets are created in a baroclinically unstable flow.
- Jets are driven by eddy forcing which is provided by baroclinic meanders and filaments on the  $\beta$ -plane.
- The meridional wavelength of the jets varies linearly with the baroclinic radius of deformation.
- The jet wavelength is in good agreement with the (modified) Rhines scale.
- rms radial velocity in an equilibrated baroclinically unstable flow is proportional to  $\beta R_d$ .



# Flow generated by a distributed thermal forcing

