Simulation of atmospheric turbulence: Fractal turbulence

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Introduction

A new trend is to observe atmospheric turbulence fields by using scanning Doppler radars and/or lidars. See e.g. Chan (2011) for the retrieval of eddy dissipation rate (EDR) maps at the Hongkong International Airport.

To improve retrievals of turbulence we want to simulate turbulent wind fields.

Simulation of turbulence can be done with increasing complexity. At first instance, we have chosen the most simple approach: a synthetic signal that satisfies the Kolmogorov 5/3 power law.

Model	**synthetic clean signal**	**isotropic direct numerical simulation (DNS)**	**large eddy simulation (LES)**	**more advanced (e.g. LES forced with weather model)**	**real world**
Features	Kolmogorov power spectrum	Navier-Stokes equation	Inhomo-geneities	weather systems	
	simpel	→			**complex**

Models of turbulence in order of complexity

How does it look?

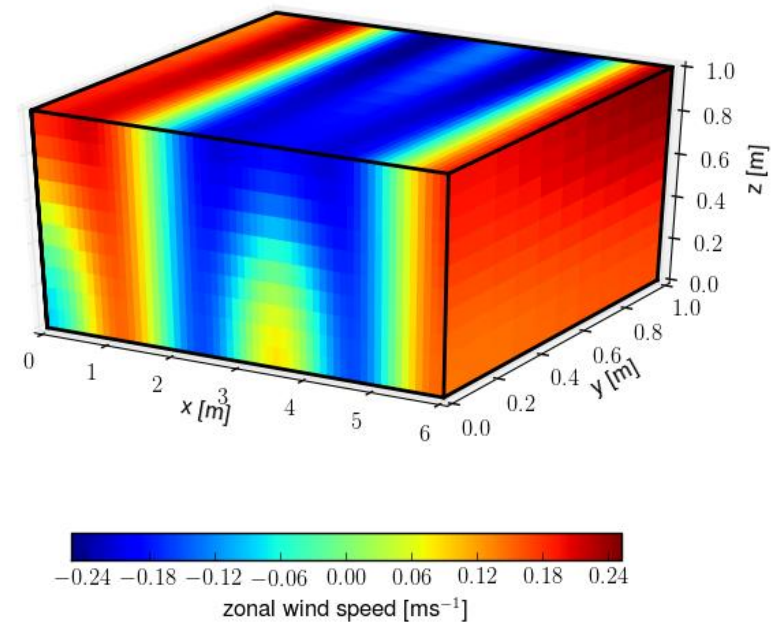
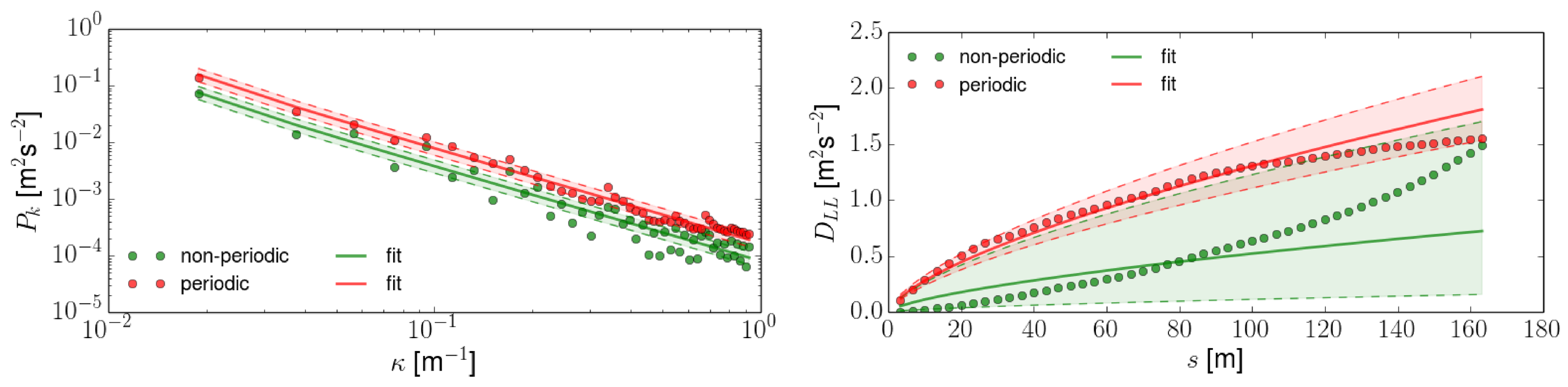


Illustration of a fractal turbulence building block of 7 x 2 x 2 points*. The zonal wind speed is shown in ms^{-1}. In the x-direction the signal satisfies the Kolmogorov -5/3 power law and the second order structure function. In the other directions the zonal wind is slightly correlated.*



Validation of the fractal turbulence model. *100 samples are taken with Δx= 3.33 m and ε = 1.10^{-3} m^{2}s^{-3} . In the figures the power spectrum (left) and second order structure function (right) are shown, with a fitted edr.*

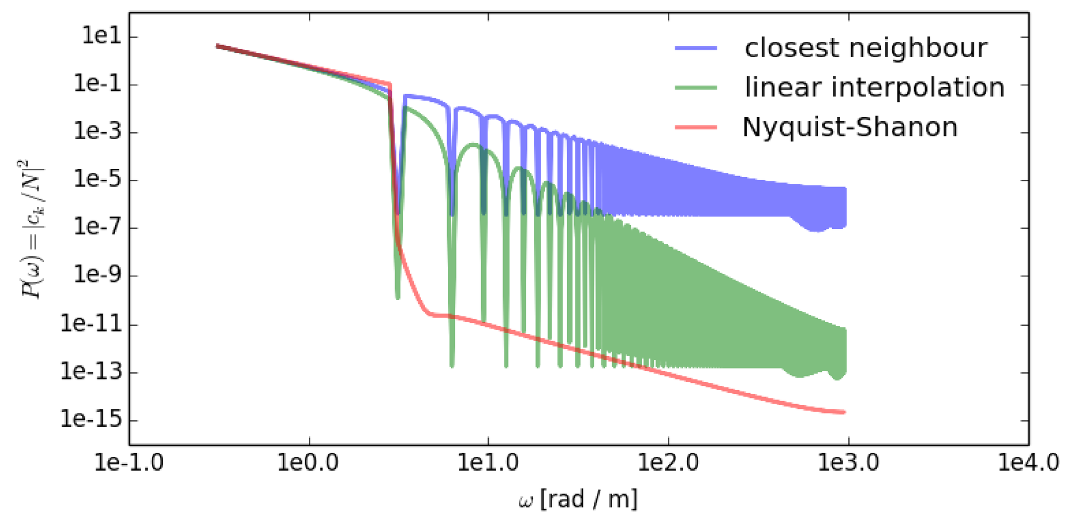
Fractal turbulence model

Here we propose a model of fractal turbulence that satisfies the Kolmogorov 5/3 power law for multiple scales, which is periodic and only needs a few samples as input.

Suppose we have N samples with an EDR of 1 m^{2}s^{-3}, then the unaliased continuous velocity signal with an eddy dissipation rate of ε on the domain [-∞,∞] is:

$$v_n(x) = \varepsilon^{1/3} 2^{5n/6} \sum_{i=0}^{N-1} a_i \Xi(2^{-n}x, i, N),$$

where 2^n is the scale, a_i are the samples and Ξ is the periodic Nyquist-Shanon sampling function. The periodic Nyquist-Shanon sampling function is an analytic expression of the sampling function with the assumption that $v(x + N) = v(x)$.



Periodic Nyquist-Shanon sampling. *Power spectra of several interpolation methods. Only with periodic Nyquist-Shanon the interpolated samples are unaliased.*

To obtain all frequencies in the fractal turbulence model, we add up the signals v_n for different scales 2^n. Rescaling is applied for each scale to satisfy the Kolmogorov power law.

$$v(x) = \sum_n v_n(x) = \varepsilon^{1/3} \sum_n 2^{5n/6} \sum_{i=0}^{N-1} a_i \Xi(2^{-n}x, i, N),$$

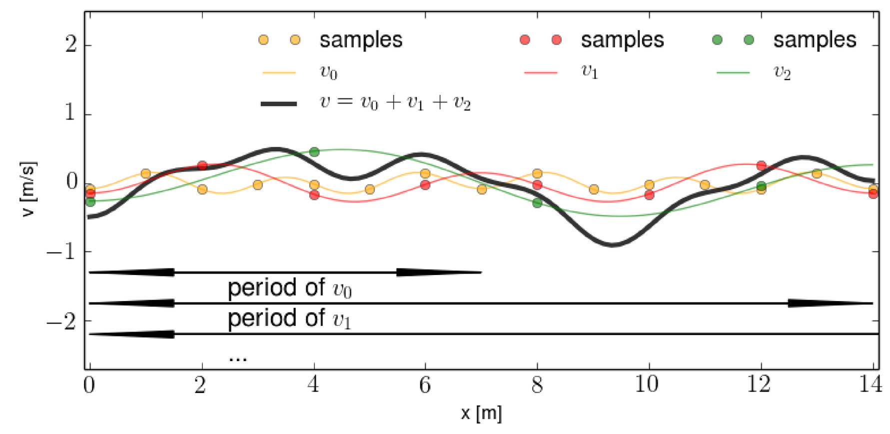


Illustration of the fractal turbulence model. *Seven samples are used for the x-direction. For each 2^n scale the samples are reused.*

Discussion

- Periodic Nyquist-Shanon sampling provides an unaliased signal.
- The fractal turbulence model is an elegant way to make a realization of a homogeneous isotropic turbulence wind field.
- It is possible to simulate an inhomogeneous EDR field, ε(x,y,z).
- With the model the effect of the response function of sonic anemometers and the footprint of radars and lidars can be simulated.

Acknowledgment This work has been done in the framework of the EU FP7 program, the UFO project.

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