



UNIVERSITY OF TRENTO - Italy

Department of Civil, Environmental  
and Mechanical Engineering

# Comparative assessment of methods to retrieve fine-scale ABL structures from airborne measurements

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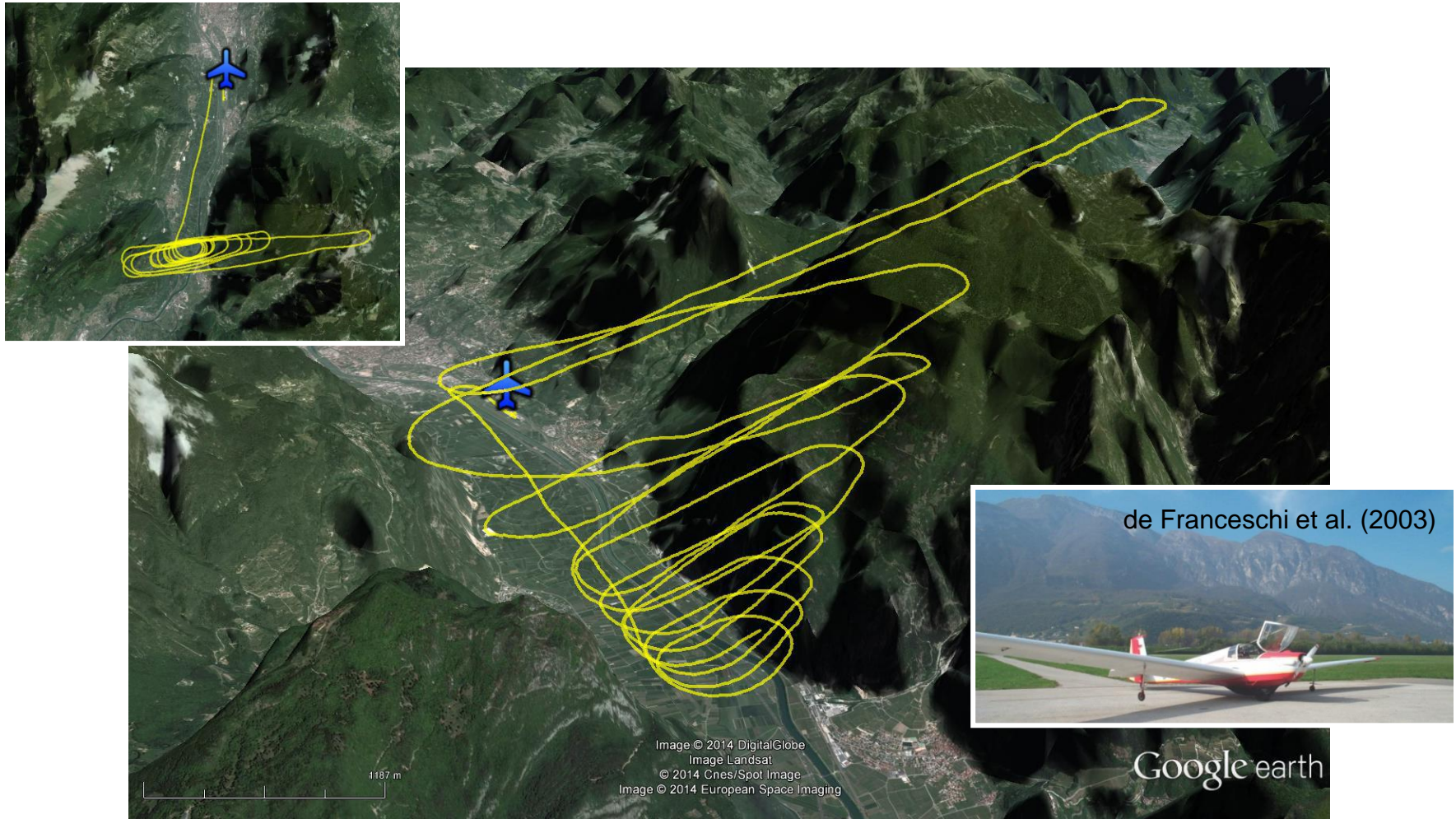
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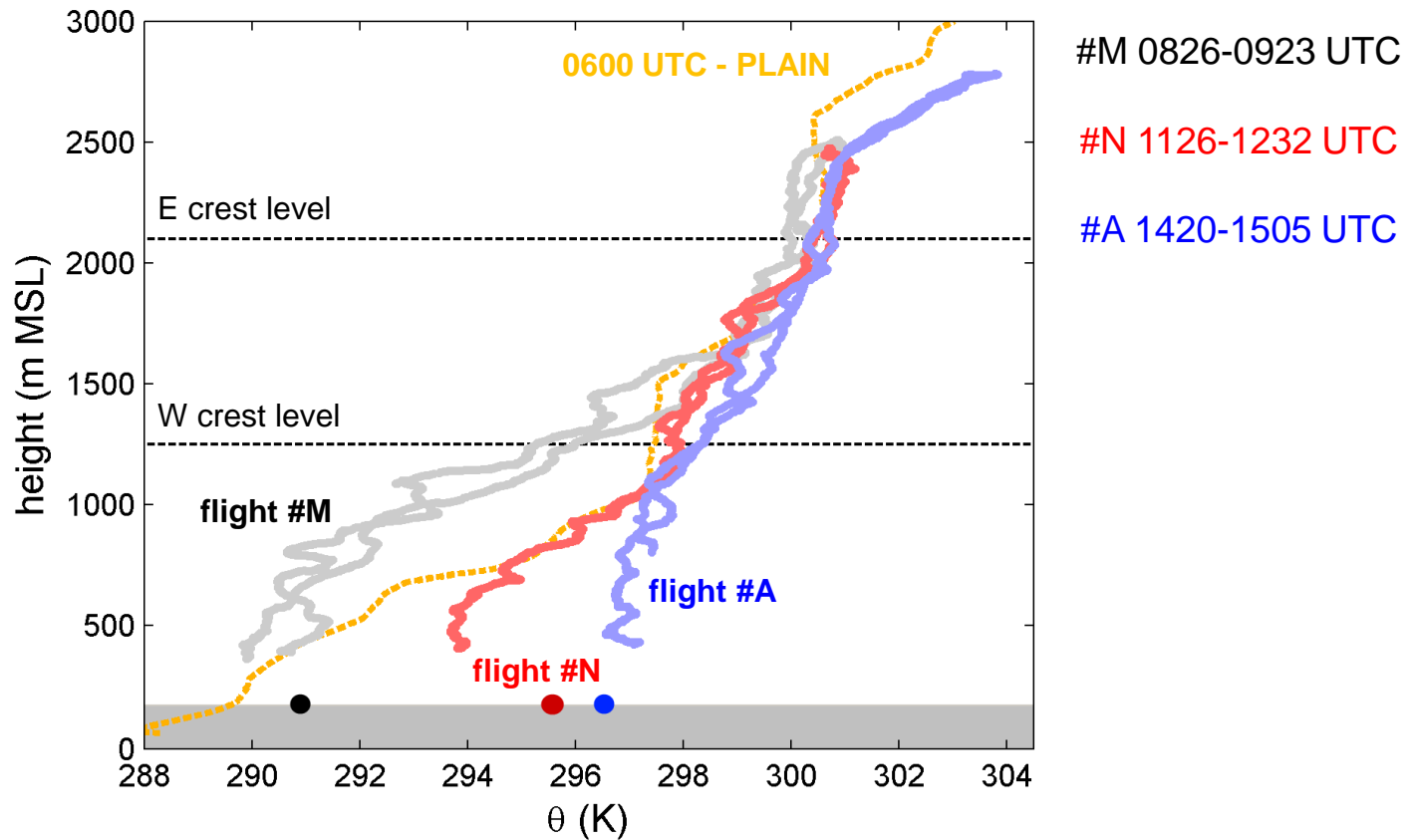
# Outline of the talk

1. Airborne measurements of the ABL in a valley
2. The interpolation methods
3. Residual Kriging (RK)
4. Interpolation results
5. Objective cross validation
6. Modified cross validation
7. Conclusions

# 1. Airborne measurements of the ABL in a valley

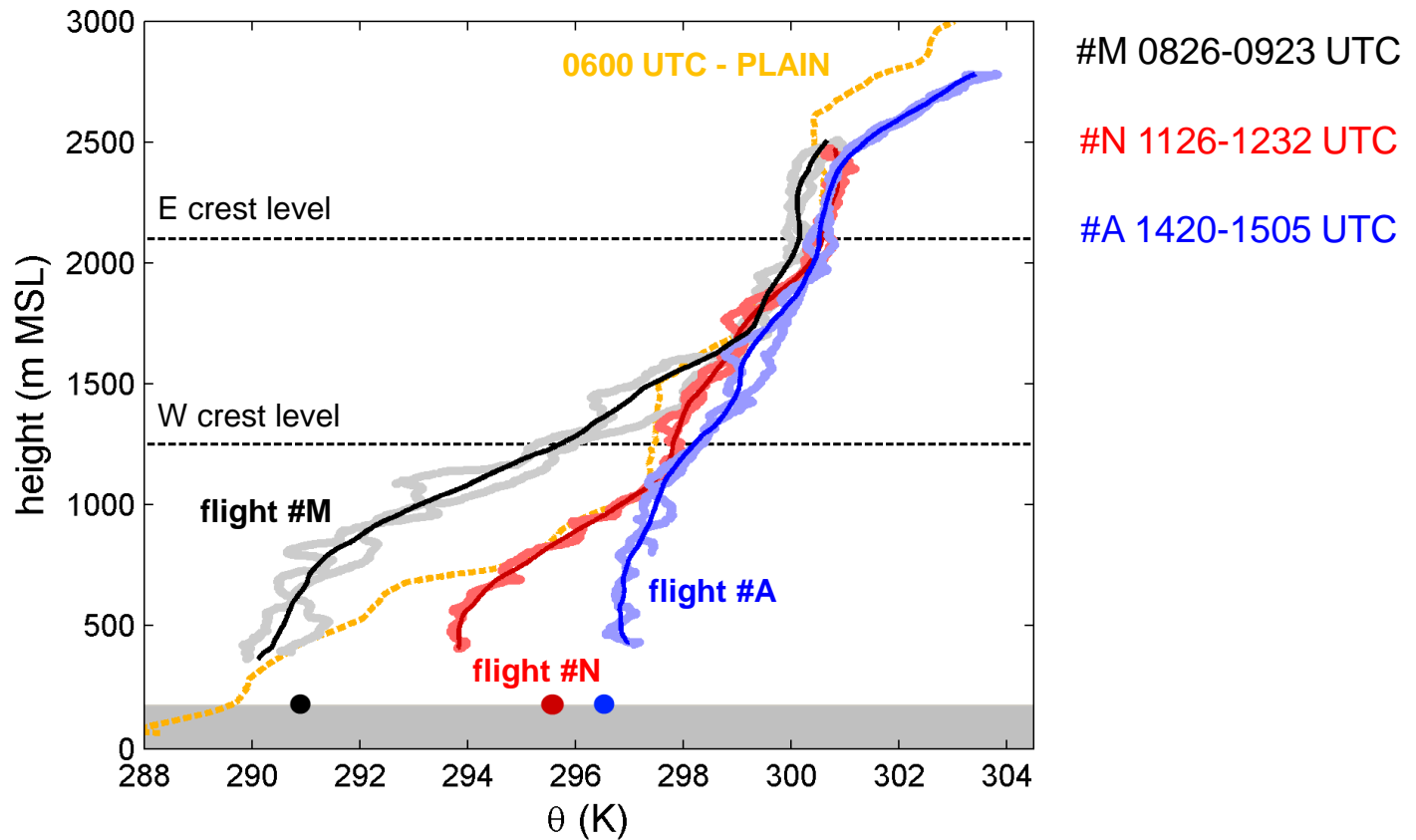


# 1. Airborne measurements of the ABL in a valley



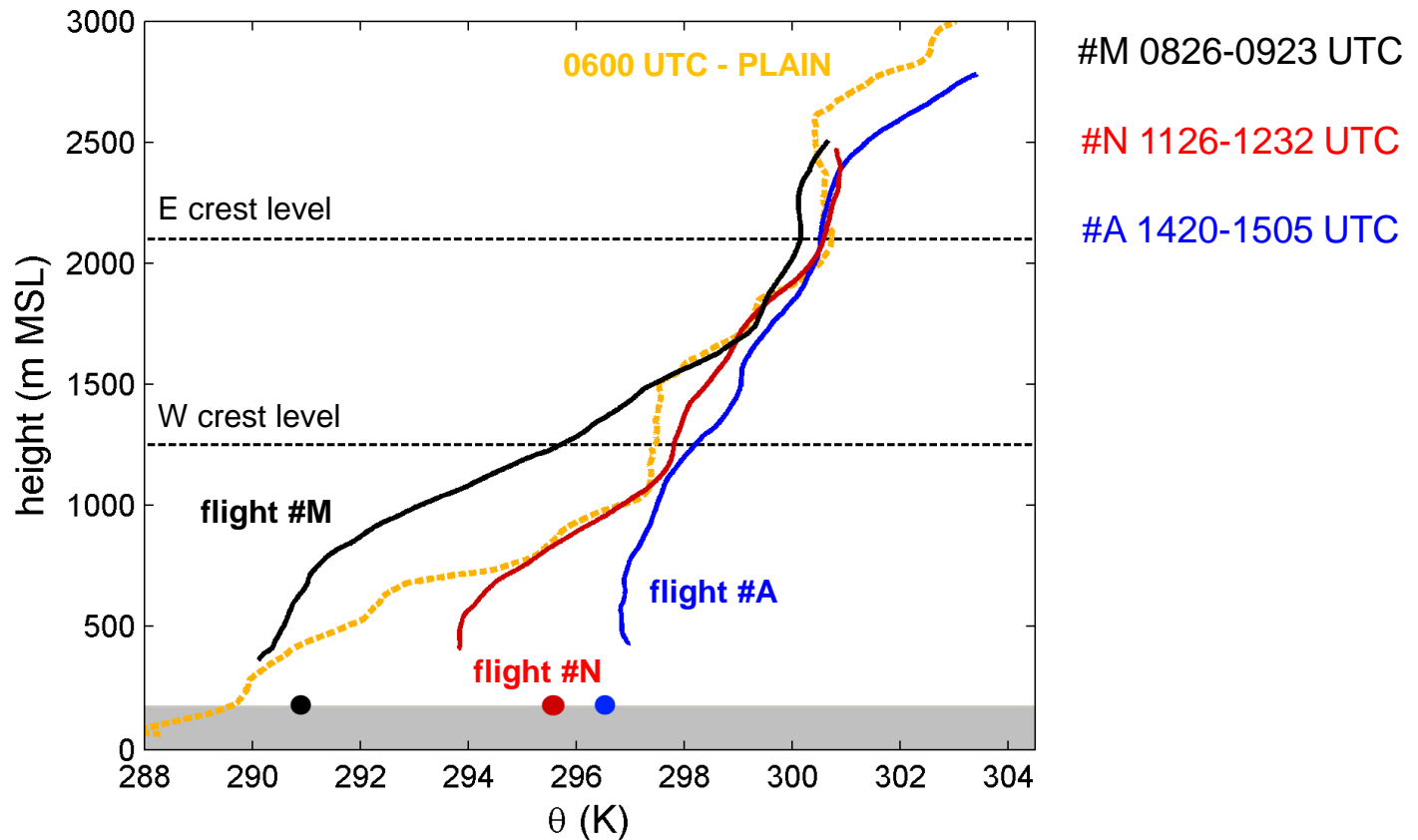
*ABL vertical structure*

# 1. Airborne measurements of the ABL in a valley



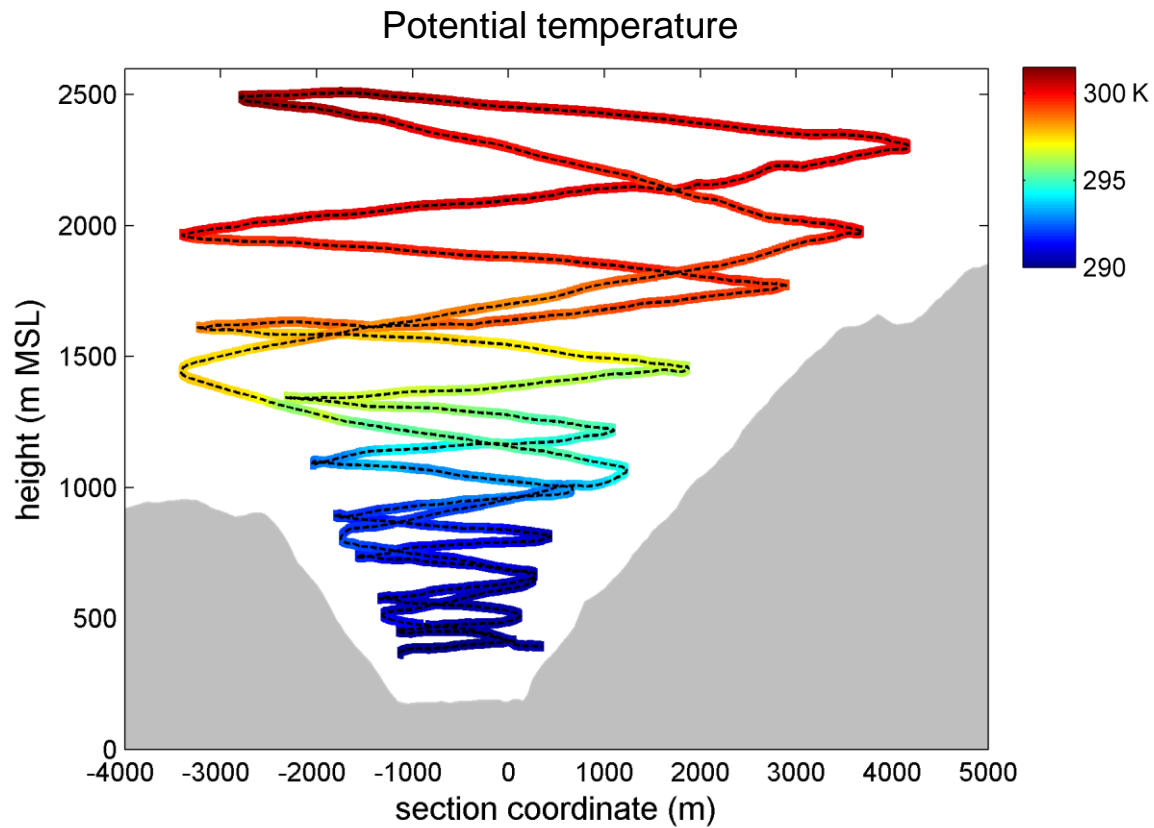
*ABL vertical structure*

# 1. Airborne measurements of the ABL in a valley



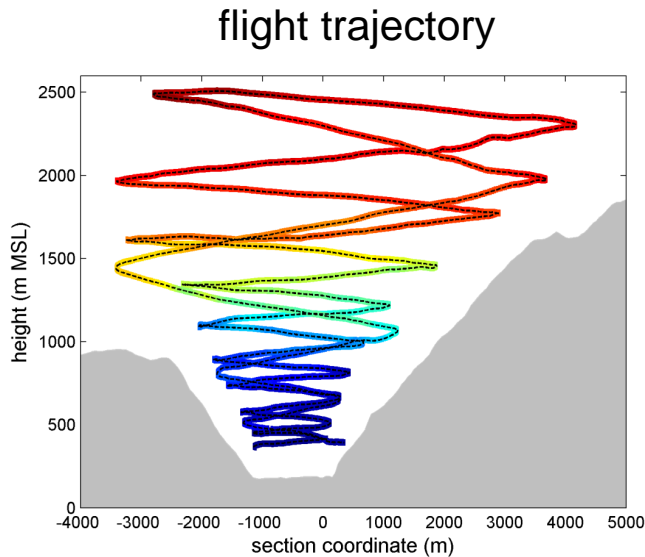
*ABL vertical structure*

# 1. Airborne measurements of the ABL in a valley



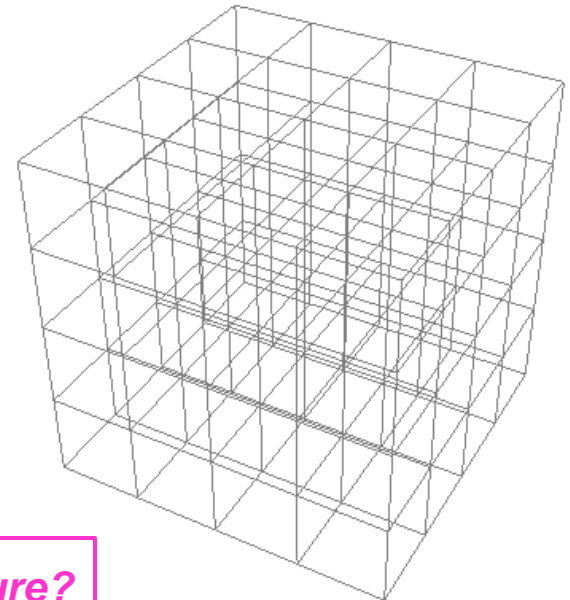
*ABL fine-scale 2D/3D structure?*

# 1. Airborne measurements of the ABL in a valley



interpolation

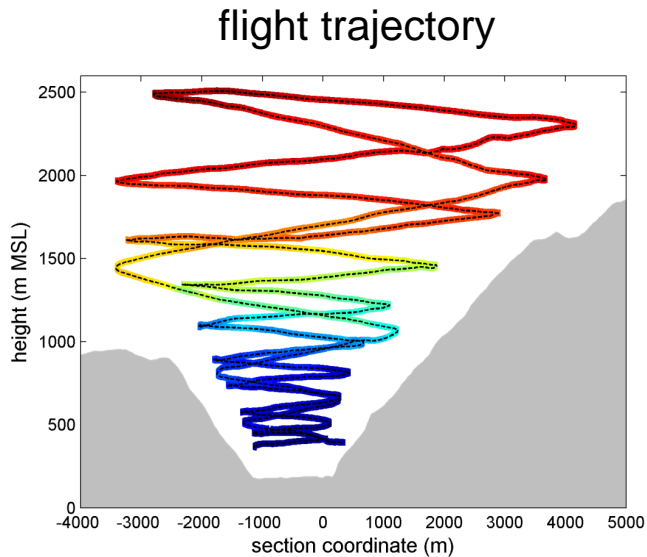
3D high-res regular grid



*ABL fine-scale 2D/3D structure?*

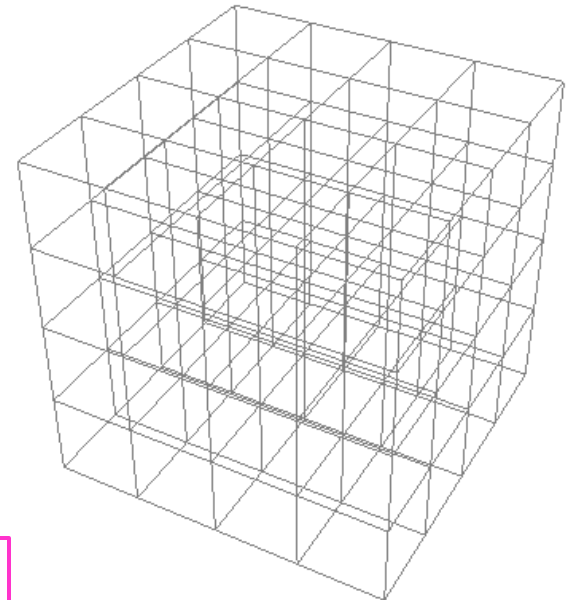


# 1. Airborne measurements of the ABL in a valley



interpolation

3D high-res regular grid



*interpolation method?*

## 2. The interpolation methods

Methods based on weighted averages:

$$\hat{\theta}(\mathbf{x}_g) = \sum_i \lambda_i \theta(\mathbf{x}_i) \quad \sum_i \lambda_i = 1$$

- **Distance weighting** **Inverse Distance Weighting IDW** *Egger (1983)*

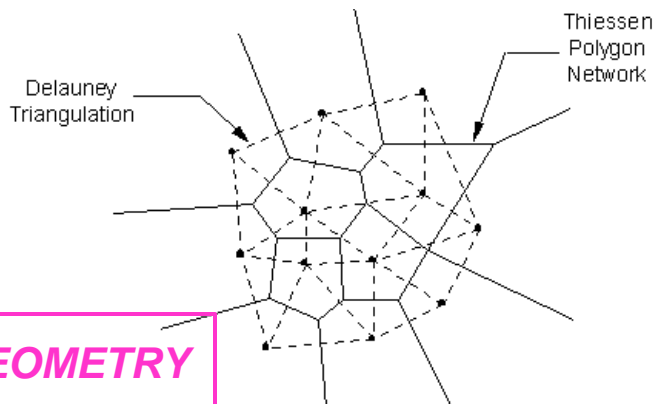
$$\lambda_i = \frac{(r_i)^{-1}}{\sum_{j=1}^N (r_j)^{-1}} \quad \text{for } \mathbf{x}_i \neq \mathbf{x}_g$$

- **Exponentially-Weighted Inverse Squared Distance- EWISD** *Hennemuth (1985)*

$$\lambda_i = \frac{\exp[-(r_i)^2]}{\sum_{j=1}^N \exp[-(r_j)^2]} \quad \text{for } \mathbf{x}_i \neq \mathbf{x}_g$$

**GEOMETRY**

- **Delaunay triangulation** **Natural Neighbors - NN** *Weigel and Rotach (2004)*



**GEOMETRY**

- **Geostatistical methods** **Residual Kriging - RK**

$$\sum_{j=1}^N \lambda_j \gamma(\mathbf{x}_i - \mathbf{x}_j) - q = \gamma(\mathbf{x}_g - \mathbf{x}_i)$$

$\gamma$  = semivariogram function

**GEOMETRY + COVARIANCE**

# 3. Residual kriging (RK)

see Laiti et al. (2013) Atmos. Sci. Lett.

target variable = drift + residuals

$$\theta(\mathbf{x}_i) = \mu(\mathbf{x}_i) + \delta(\mathbf{x}_i)$$

residual approach

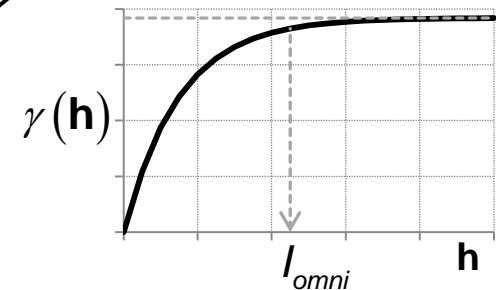
$$\hat{\delta}(\mathbf{x}_g) = \sum_i \lambda_i \cdot \delta(\mathbf{x}_i)$$

$$\hat{\mu}(\mathbf{x}_g)$$

interpolation by Ordinary Kriging (Cressie, 1993)

extraction of pseudo-soundings by moving-window vertical average

$$\text{semivariogram } \gamma(\mathbf{h}) = \frac{1}{N_h} \cdot \sum_{i=1}^{N_h} \frac{1}{2} \cdot [\delta(\mathbf{x}_i) - \delta(\mathbf{x}_i + \mathbf{h})]^2$$

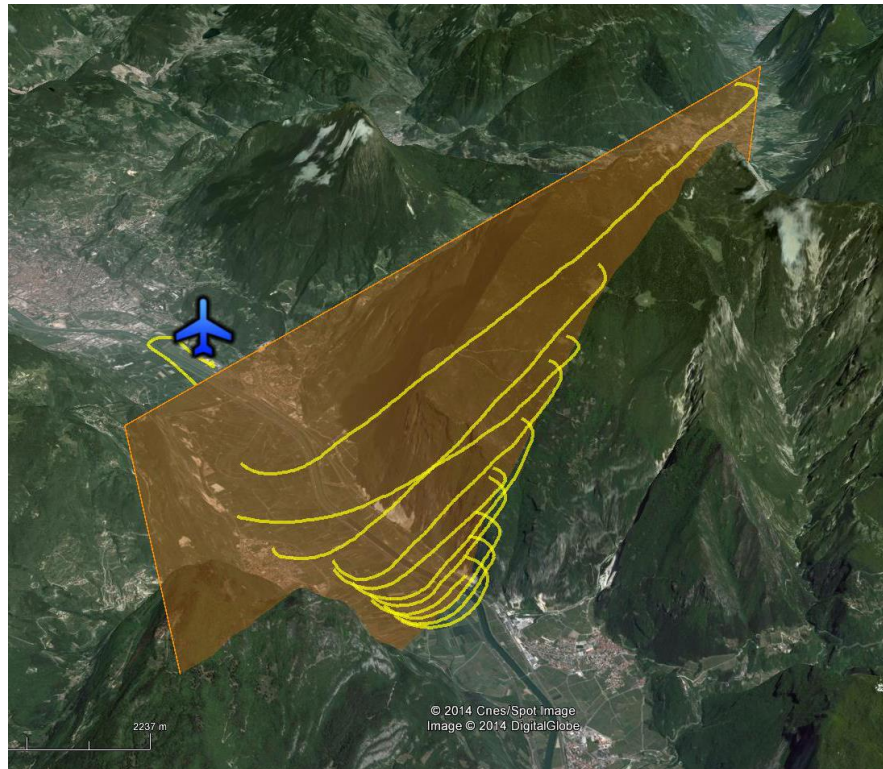


$$\hat{\theta}(\mathbf{x}_g) = \hat{\mu}(\mathbf{x}_g) + \hat{\delta}(\mathbf{x}_g)$$

byproduct: **variance of estimates**  $\hat{\sigma}^2(\mathbf{x}_g) = \gamma(0) - \sum_i \lambda_i \cdot \gamma(\mathbf{x}_i - \mathbf{x}_g) + q$

## 4. Interpolation results

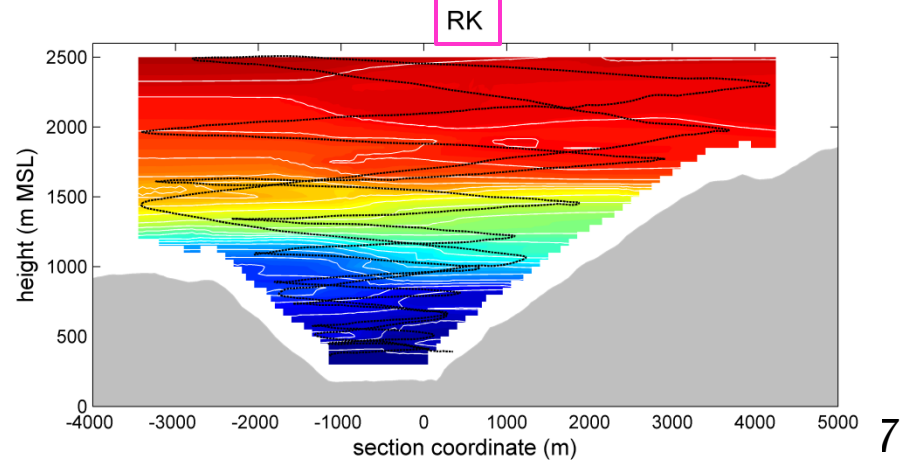
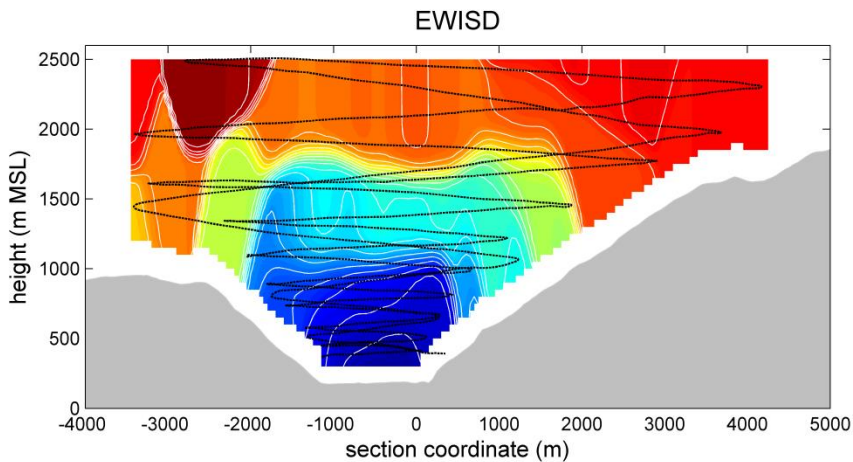
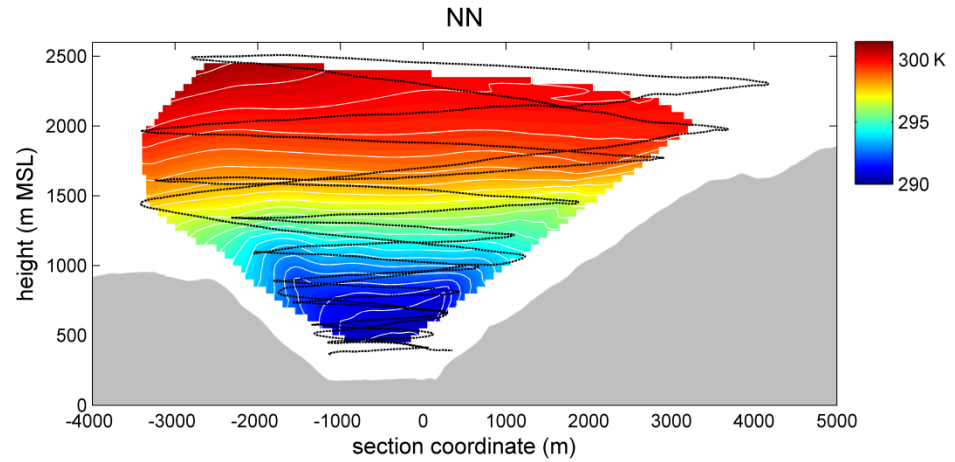
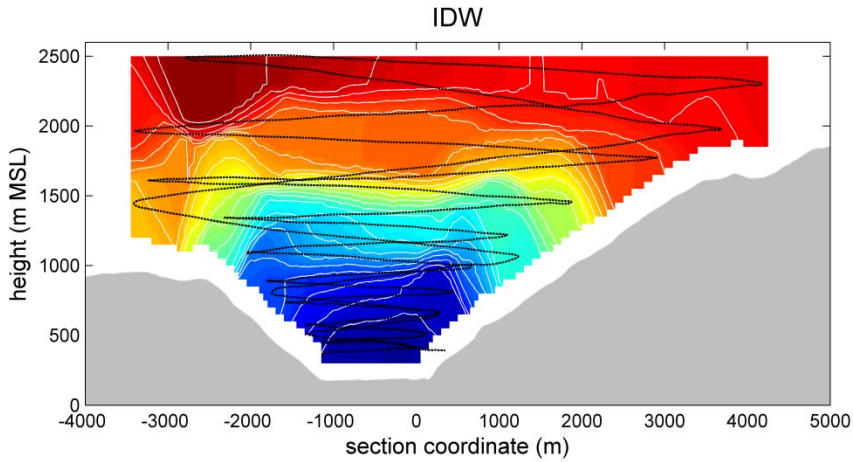
- interpolation grid:  $50 \times 50 \times 50 \text{ m}^3$



# 4. Interpolation results

- interpolation grid: 50x50x50 m<sup>3</sup>

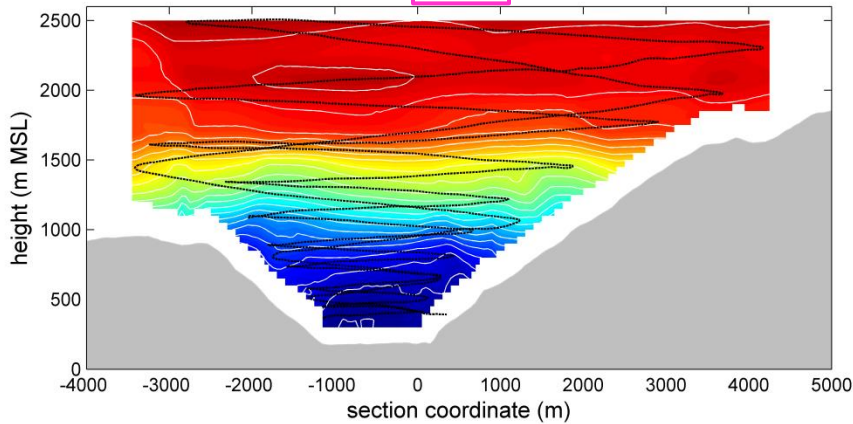
*residual approach*



# 4. Interpolation results

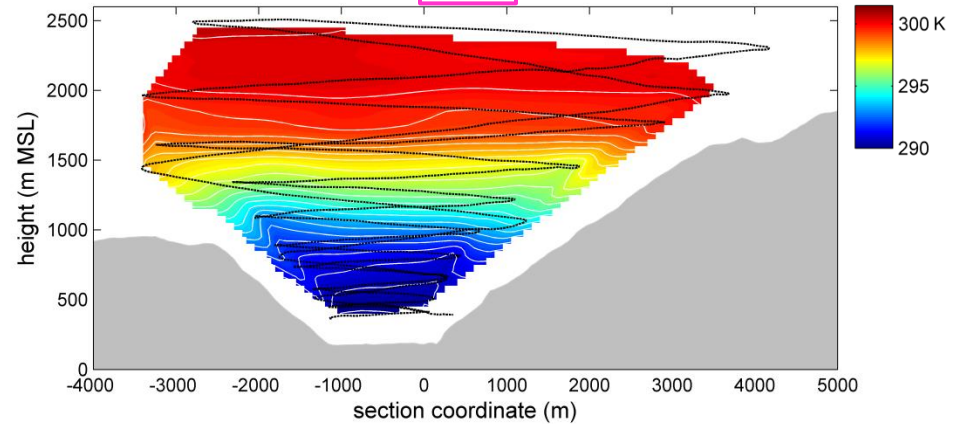
- interpolation grid: 50x50x50 m<sup>3</sup>

R-IDW

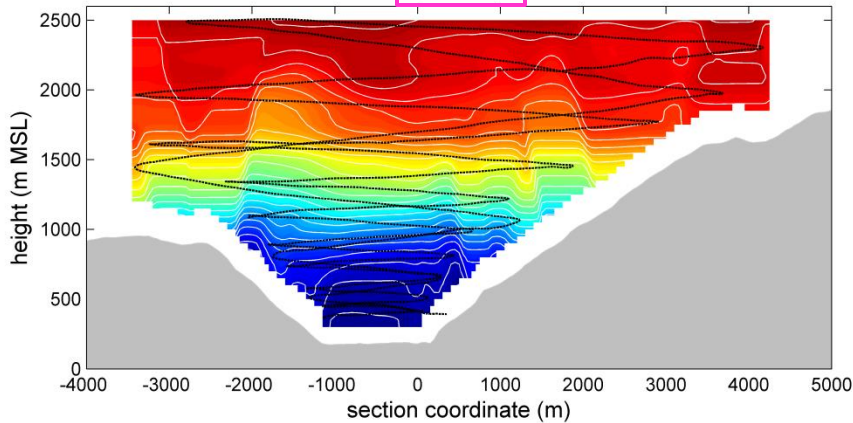


*residual approach*

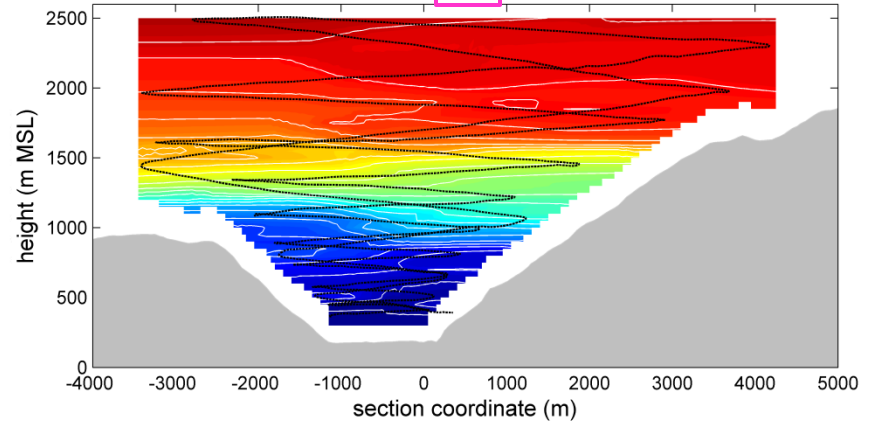
R-NN



R-EWISD

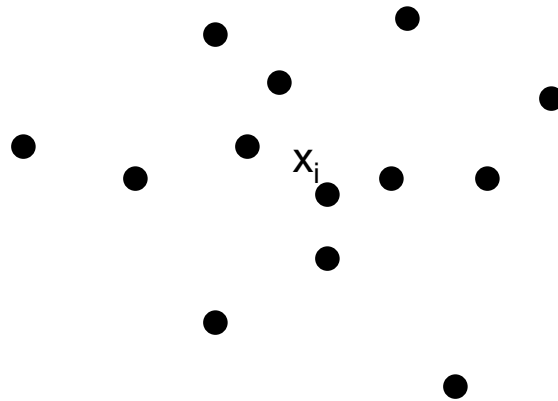


RK



# 5. Objective cross validation

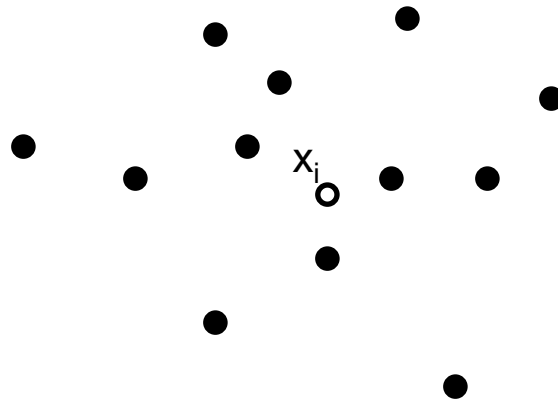
Leave One Out Cross Validation (LOOCV): exclusion of one observation at time





# 5. Objective cross validation

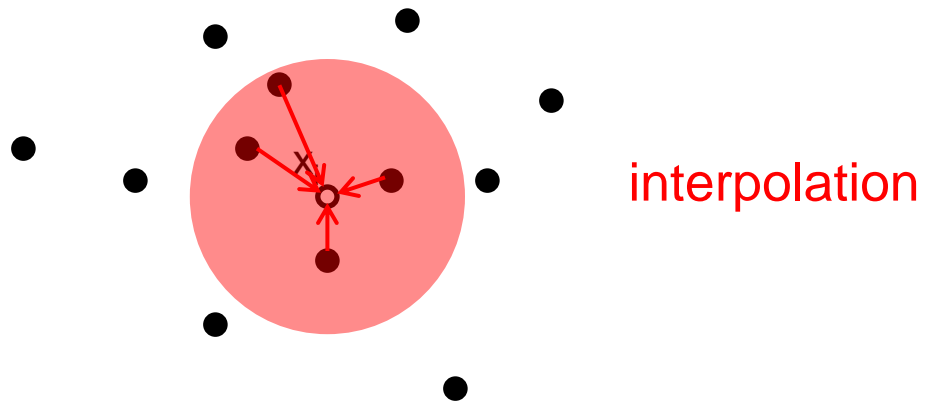
Leave One Out Cross Validation (LOOCV): exclusion of one observation at time





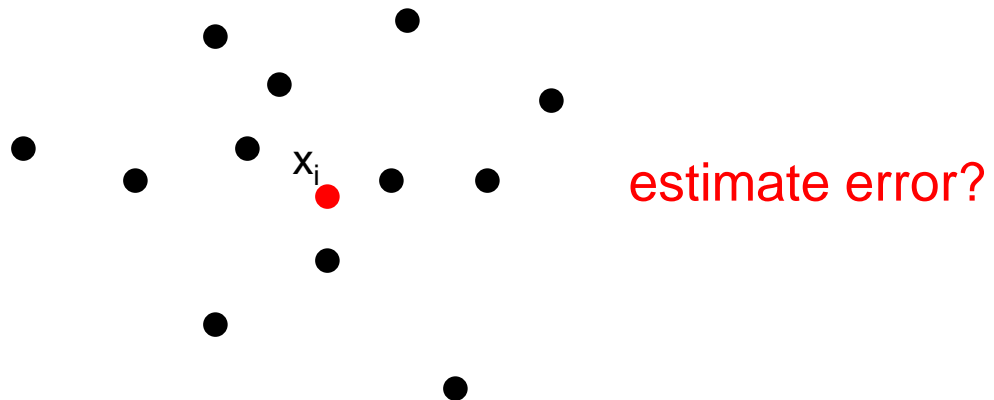
# 5. Objective cross validation

Leave One Out Cross Validation (LOOCV): exclusion of one observation at time



# 5. Objective cross validation

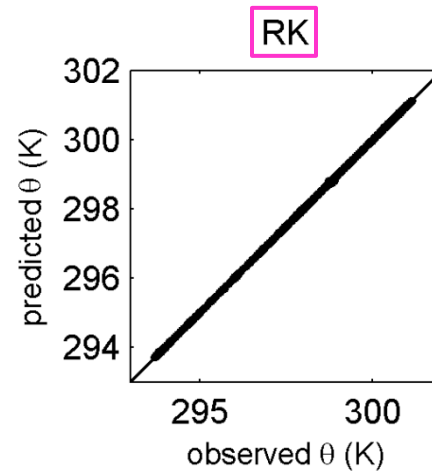
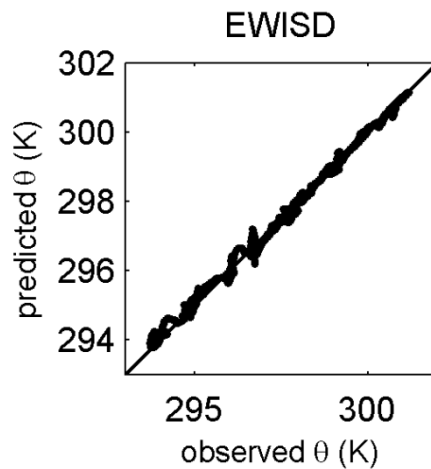
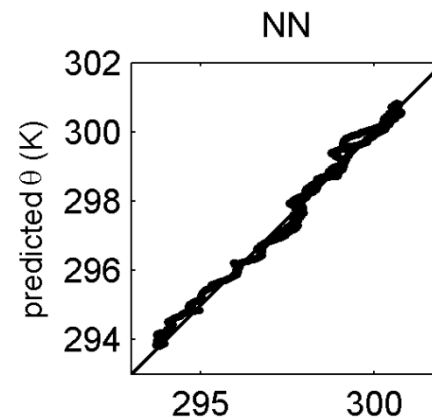
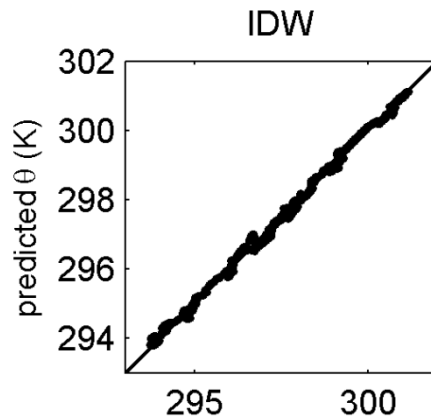
Leave One Out Cross Validation (LOOCV): exclusion of one observation at time



# 5. Objective cross validation

LOOCV

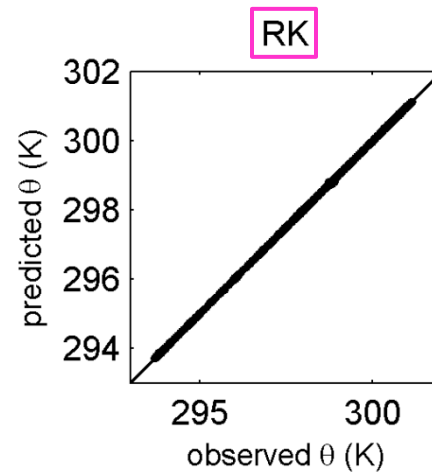
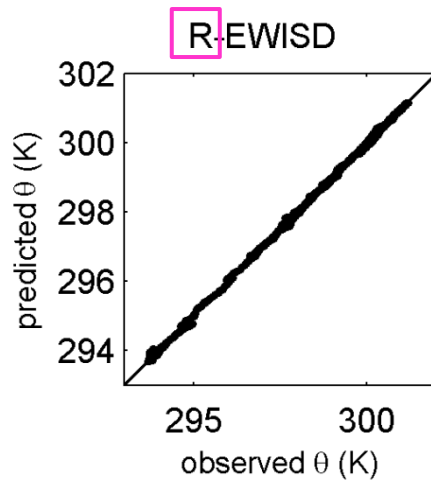
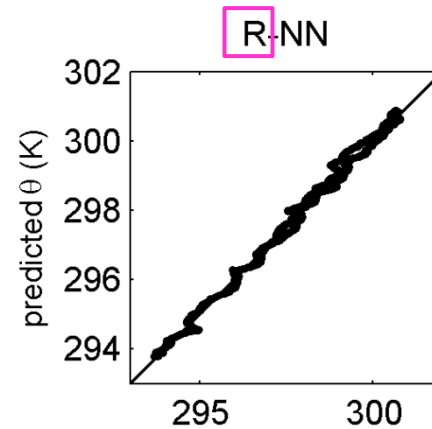
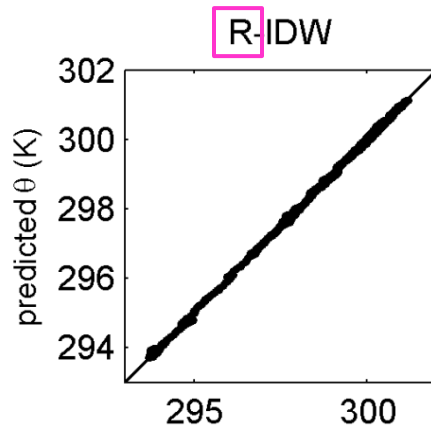
*residual approach*



# 5. Objective cross validation

LOOCV

*residual approach*



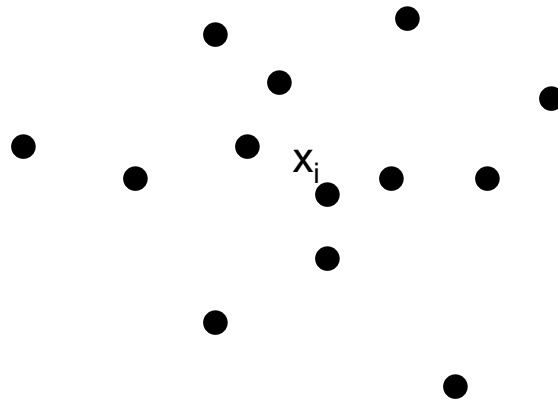
# 5. Objective cross validation

LOOCV

Method	MBE (K)	MAE (K)	RMSE (K)
IDW	-2.71E-02	1.49E-01	2.05E-01
EWISD	-8.90E-03	1.25E-01	1.93E-01
NN	-1.38E-01	3.27E-01	4.23E-01
R-IDW	-4.20E-04	1.02E-01	1.39E-01
R-EWISD	-8.05E-04	8.45E-02	1.29E-01
R-NN	-9.72E-02	2.63E-01	3.58E-01
RK	-5.90E-06	9.80E-03	2.26E-02

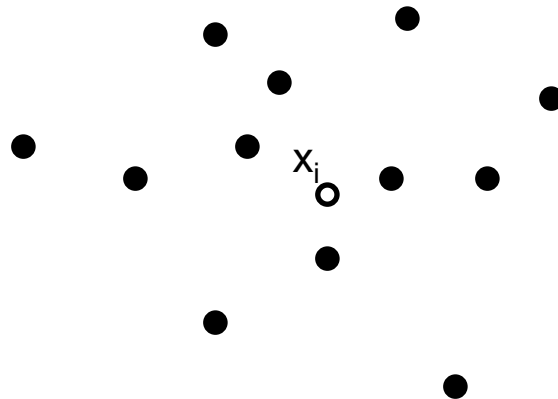
## 6. Modified cross validation

Modified LOOCV: exclusion of subsets of observations of increasing radius



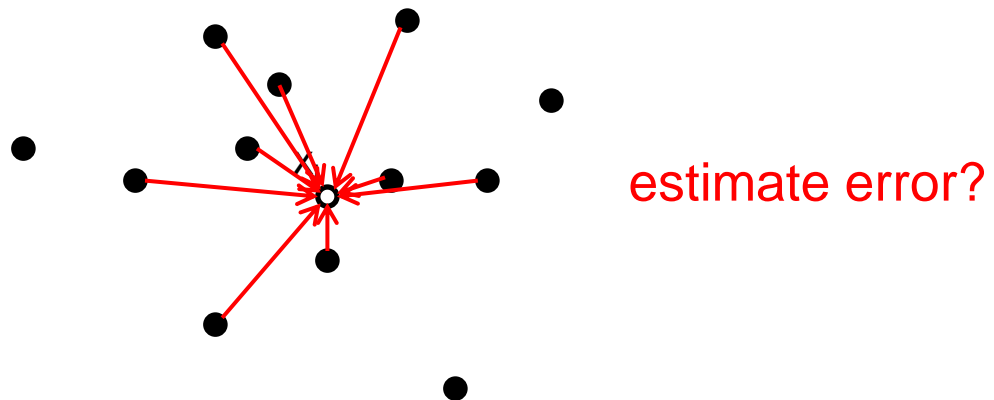
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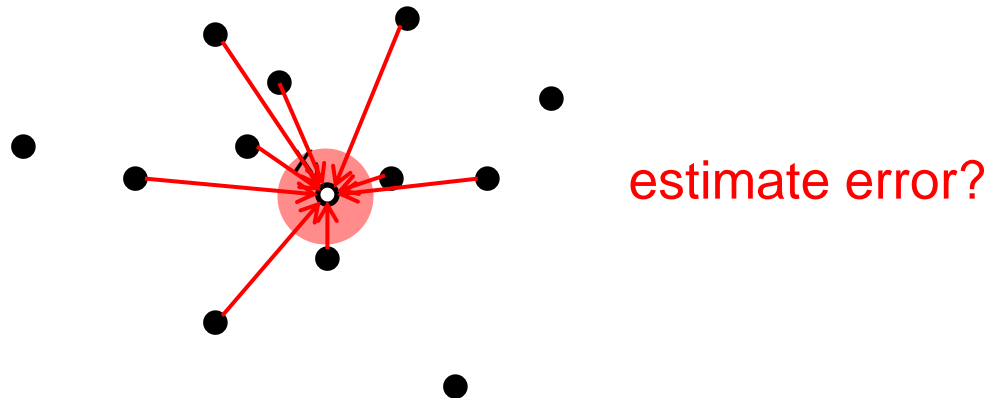
Modified LOOCV: exclusion of subsets of observations of increasing radius





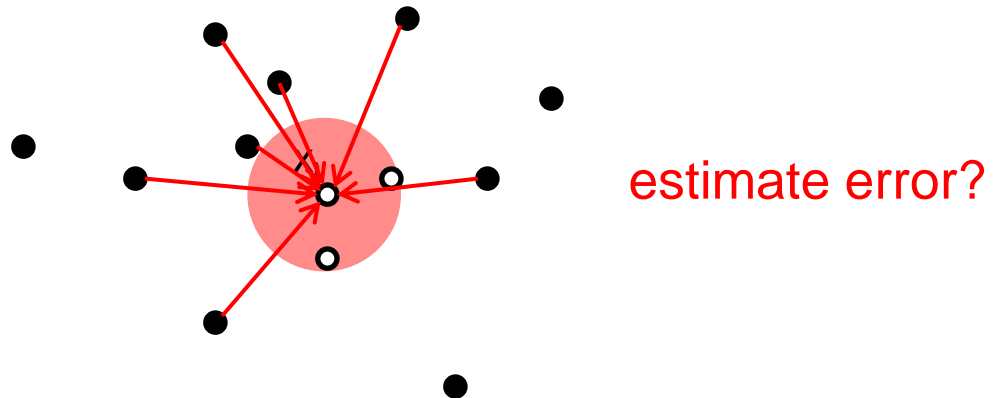
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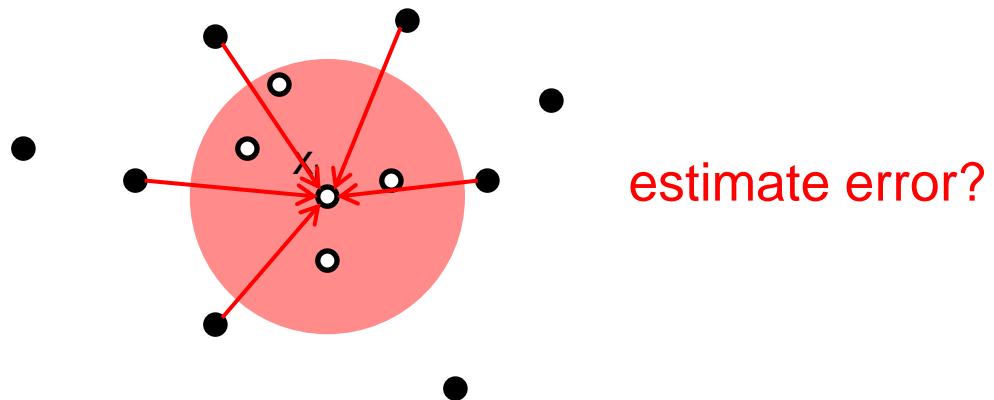
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Modified LOOCV: exclusion of subsets of observations of increasing radius



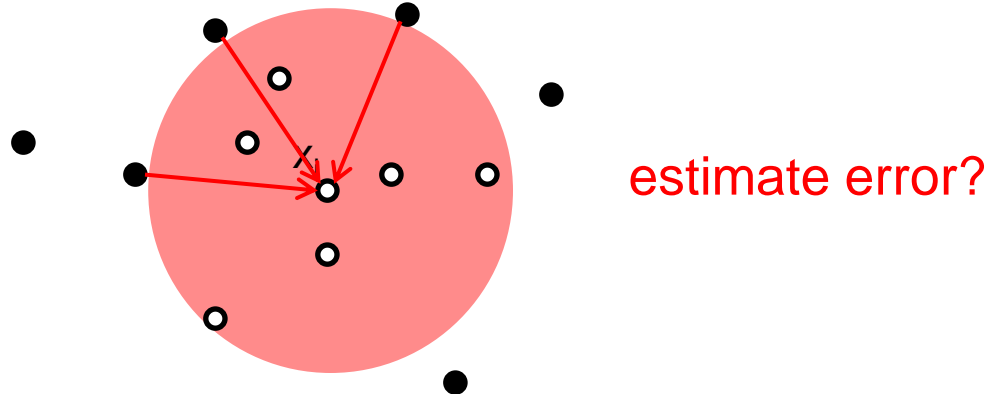
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## 6. Modified cross validation

Modified LOOCV: exclusion of subsets of observations of increasing radius



# 6. Modified cross validation

## Modified LOOCV

flight #M

$$r_{MBE} = \frac{|MBE_{method}|}{|MBE_{RK}|}$$

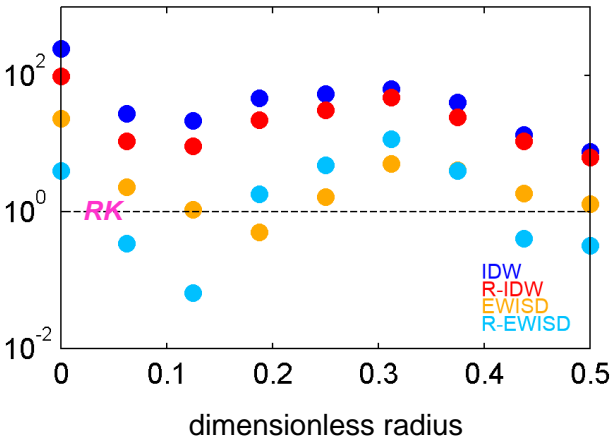
$$r_{MAE} = \frac{MAE_{method}}{MAE_{RK}}$$

$$r_{RMSE} = \frac{RMSE_{method}}{RMSE_{RK}}$$

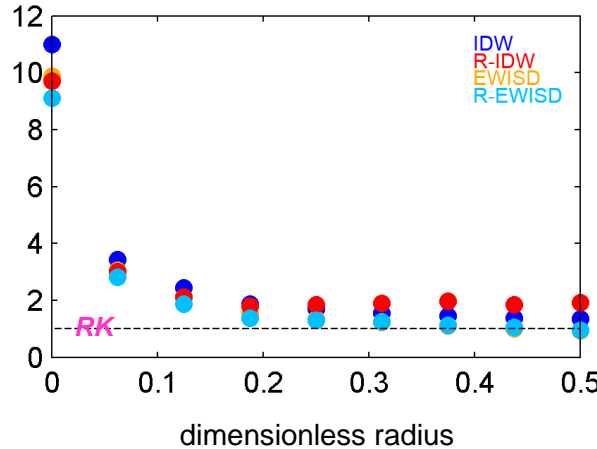
$$\text{dimensionless radius} = \frac{\text{radius}}{I_{omni}}$$

$I_{omni}$  = max distance of correlation

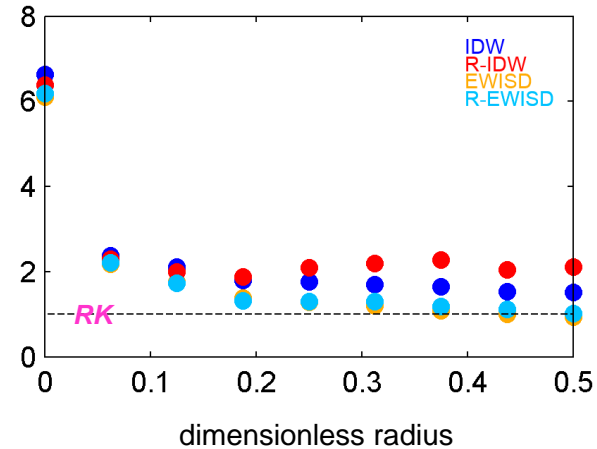
$r_{MBE}$



$r_{MAE}$



$r_{RMSE}$



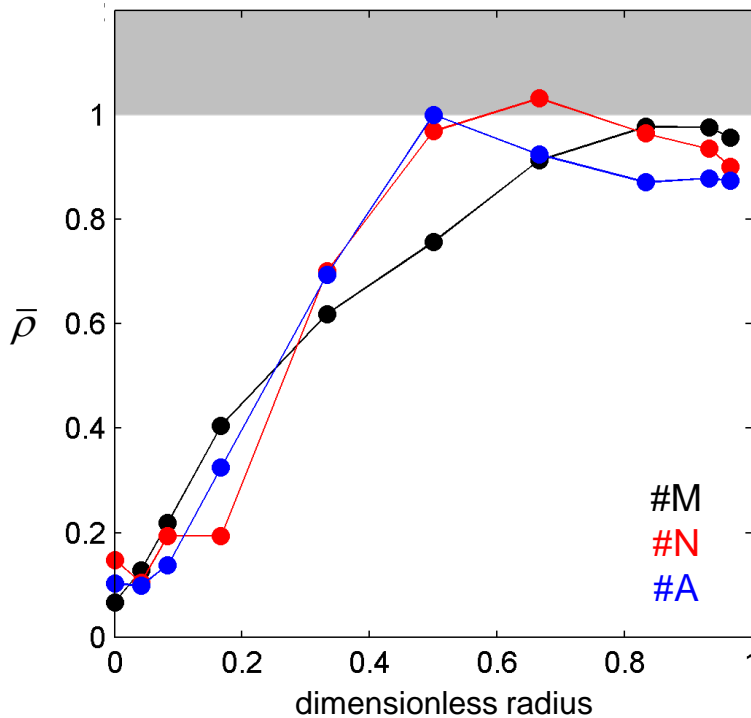
# 6. Modified cross validation

Is RK-predicted variance a good estimate of the interpolation error?

flight #M

$$\bar{\rho} = \frac{1}{N} \sum_{i=1}^N \rho(\mathbf{x}_i) = \frac{1}{N} \sum_{i=1}^N \frac{[\hat{\theta}(\mathbf{x}_i) - \theta(\mathbf{x}_i)]^2}{\hat{\sigma}^2(\mathbf{x}_i)}$$

RK estimate variance

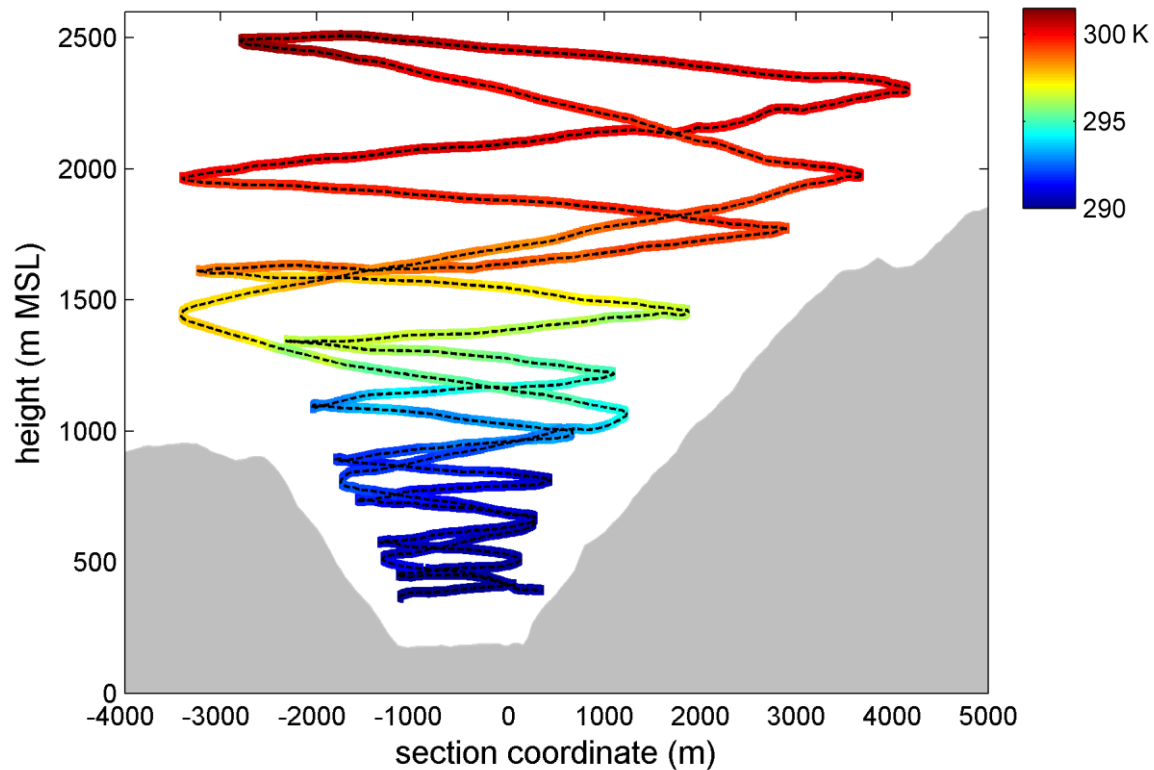


$$\text{dimensionless radius} = \frac{\text{radius}}{I_{\text{omni}}}$$

$I_{\text{omni}}$  = max distance of correlation

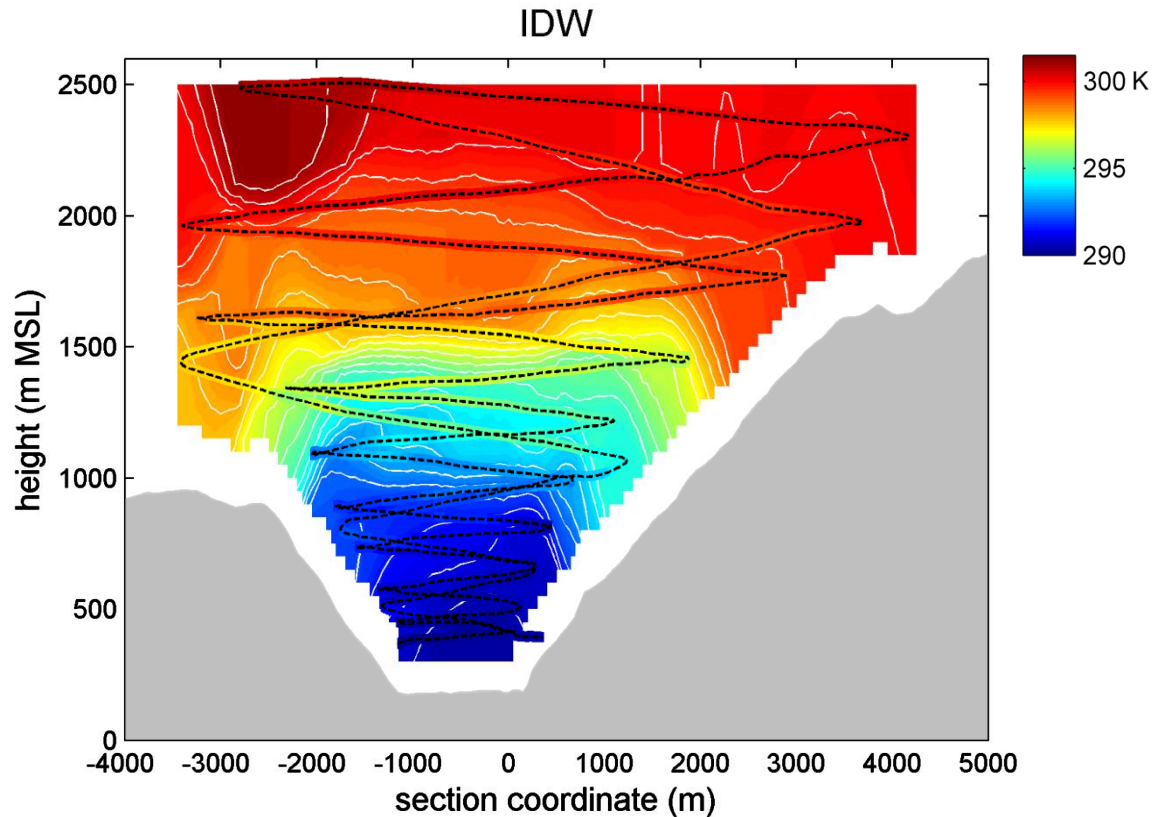
# 7. Conclusions

- The performances of 4 interpolation methods for the retrieval of fine-scale ABL structures from airborne measurements were compared



# 7. Conclusions

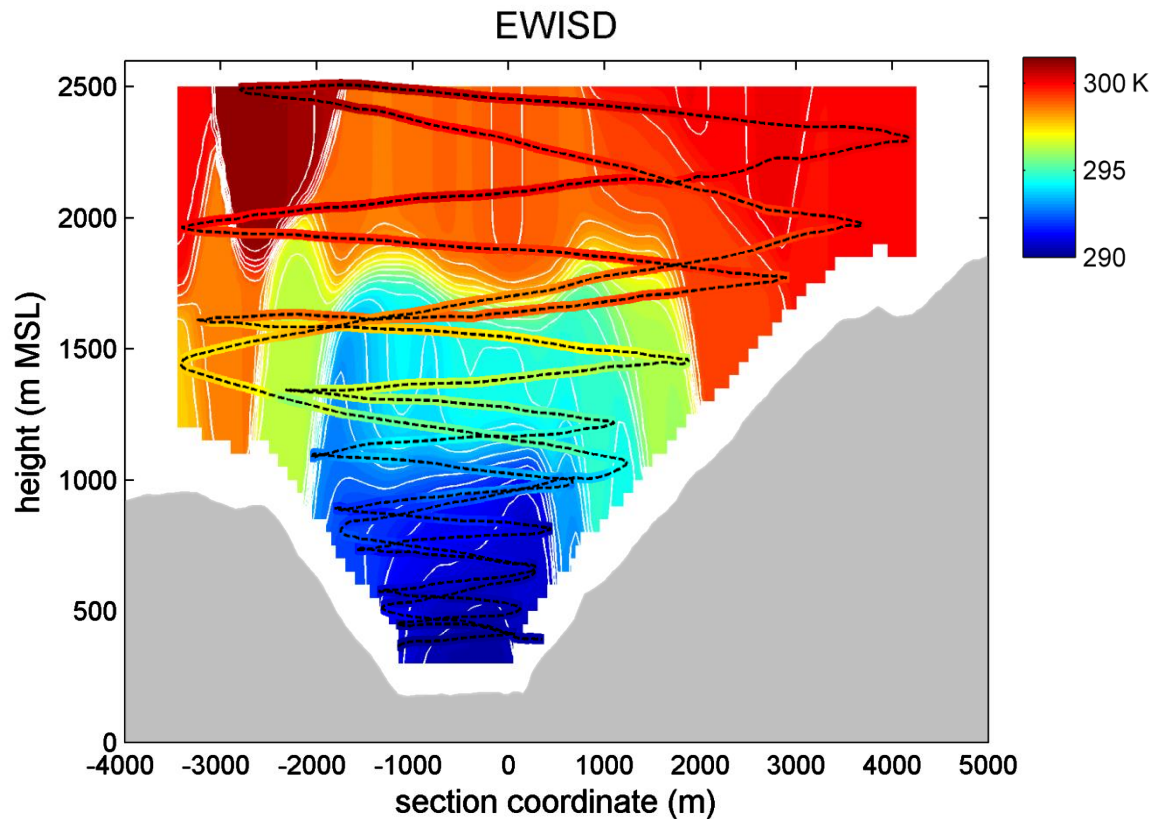
- The performances of 4 interpolation methods for the retrieval of fine-scale ABL structures from airborne measurements were compared





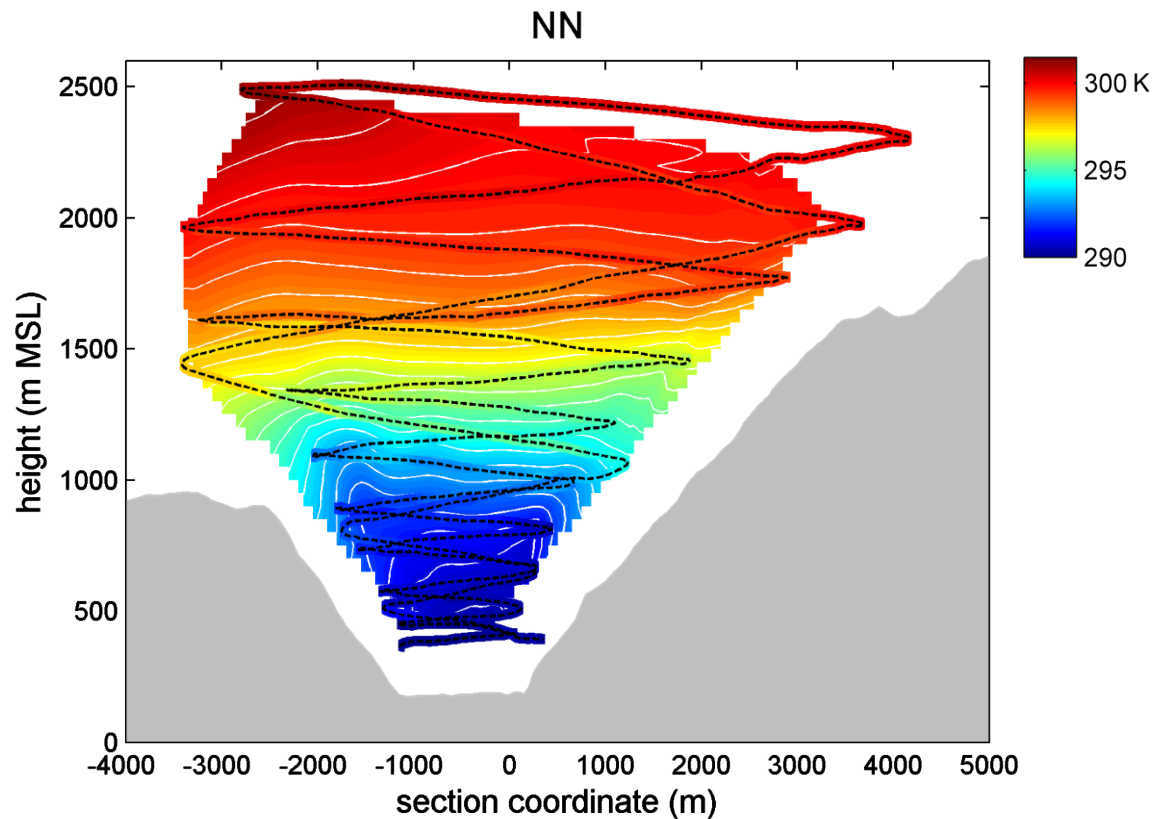
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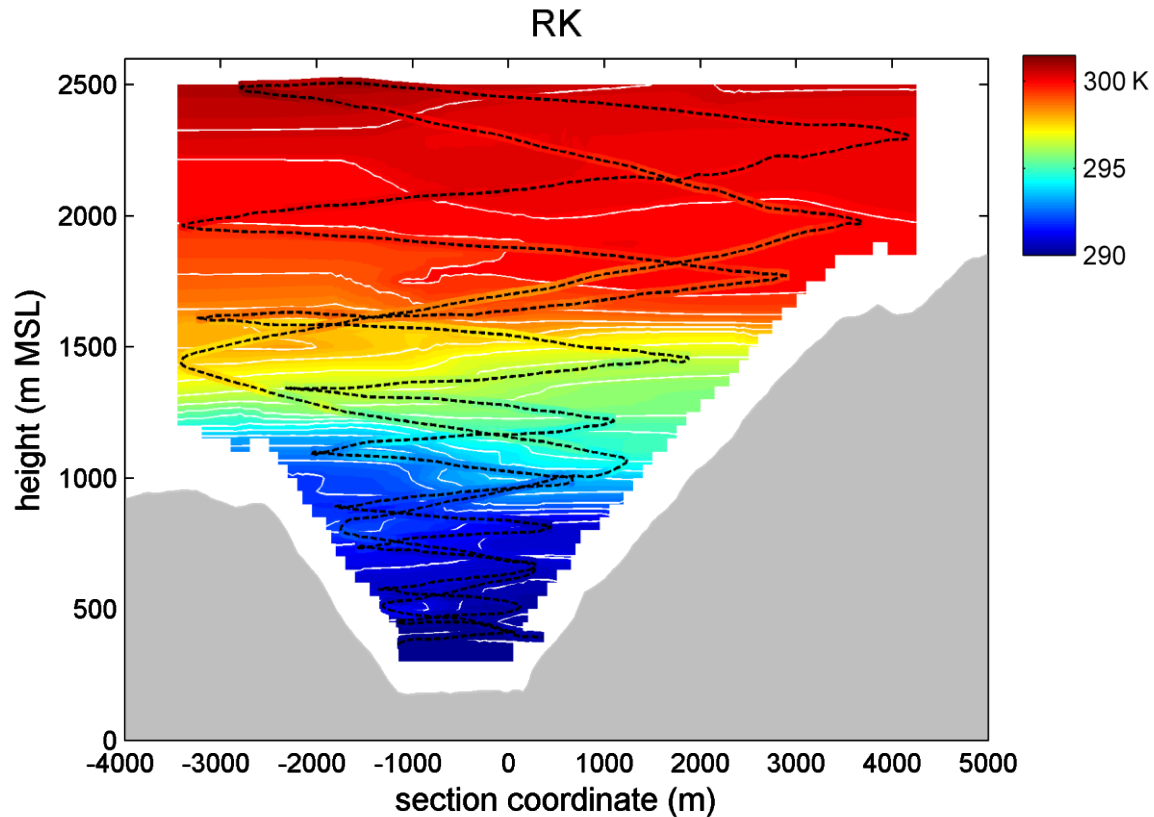
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# 7. Conclusions

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# 7. Conclusions

- The performances of 4 interpolation methods for the retrieval of fine-scale ABL structures from airborne measurements were compared
- LOOCV:
  - RK performs better than the other methods (IDW, EWISD, NN)
  - the residual approach improves the performance of all methods (especially IDW and EWISD)
- modified CV:
  - RK performs better than the other methods when observations are clustered (i.e. along a flight trajectory)
  - RK variance overestimates the interpolation error for distances shorter than  $\frac{1}{2}$  correlation range
- The interpolated fields provide an ideal benchmark for comparison with high-resolution numerical model output



Thank you for your attention

The Adige Valley from Castel Beseno.

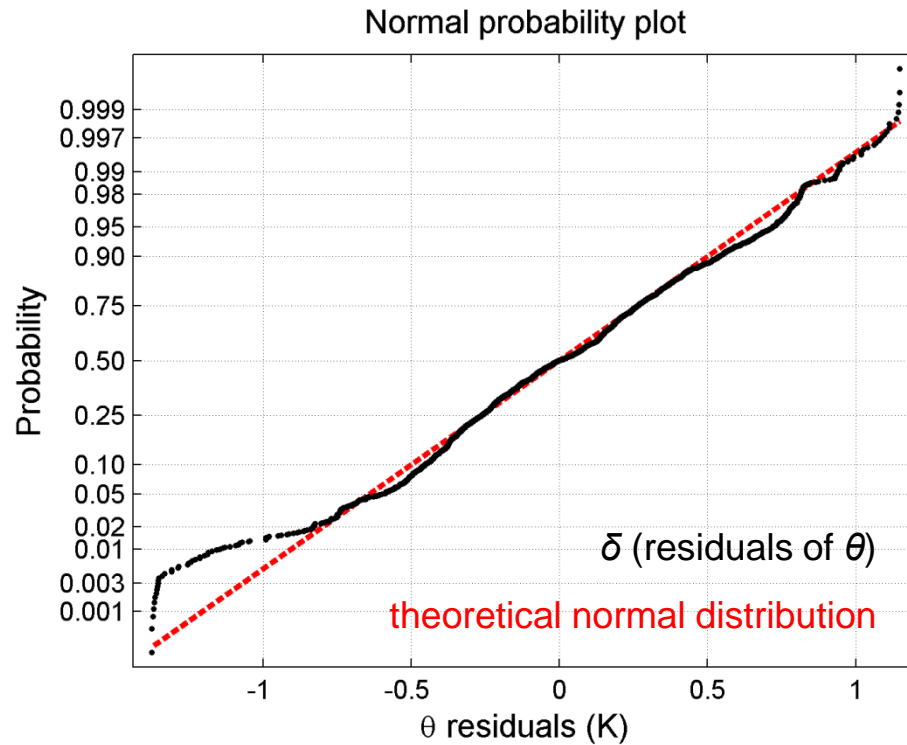
[www.gommoneclubverona.com](http://www.gommoneclubverona.com)

# References

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# Remarks about kriging - 1

The target variable (i.e.  $\theta$  residuals) must be normally distributed (random process)

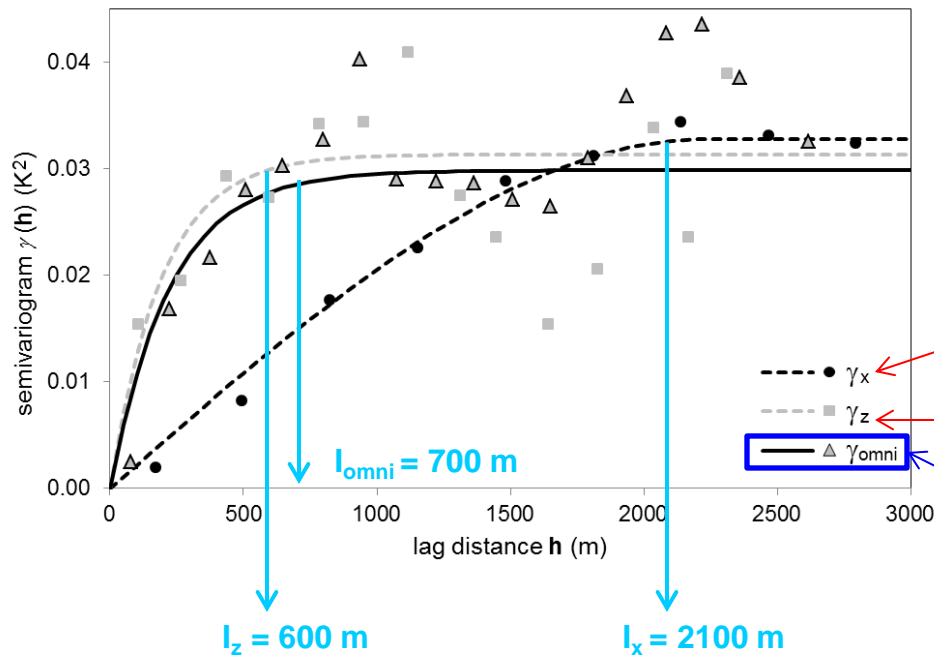


# Remarks about kriging - 2

To deal with the anisotropy of the target field:

1. the **directional semivariograms** were estimated to get the anisotropy scales (**semivariogram ranges**, i.e. correlation ranges)
2. the spatial coordinates were rescaled by anisotropy ratios
3. the **omnidirectional semivariogram** was estimated and finally used in RK

$$\gamma(\mathbf{x}_i - \mathbf{x}_j) = \gamma(\mathbf{h}) = \frac{1}{N_h} \sum_{i=1}^{N_h} \frac{1}{2} [\delta(\mathbf{x}_i) - \delta(\mathbf{x}_i + \mathbf{h})]^2$$



$$x' = \frac{x}{I_x} \cdot I_z$$

$$y' = \frac{y}{I_y} \cdot I_z$$

$$z' = z$$

horizontal semivariogram

vertical semivariogram

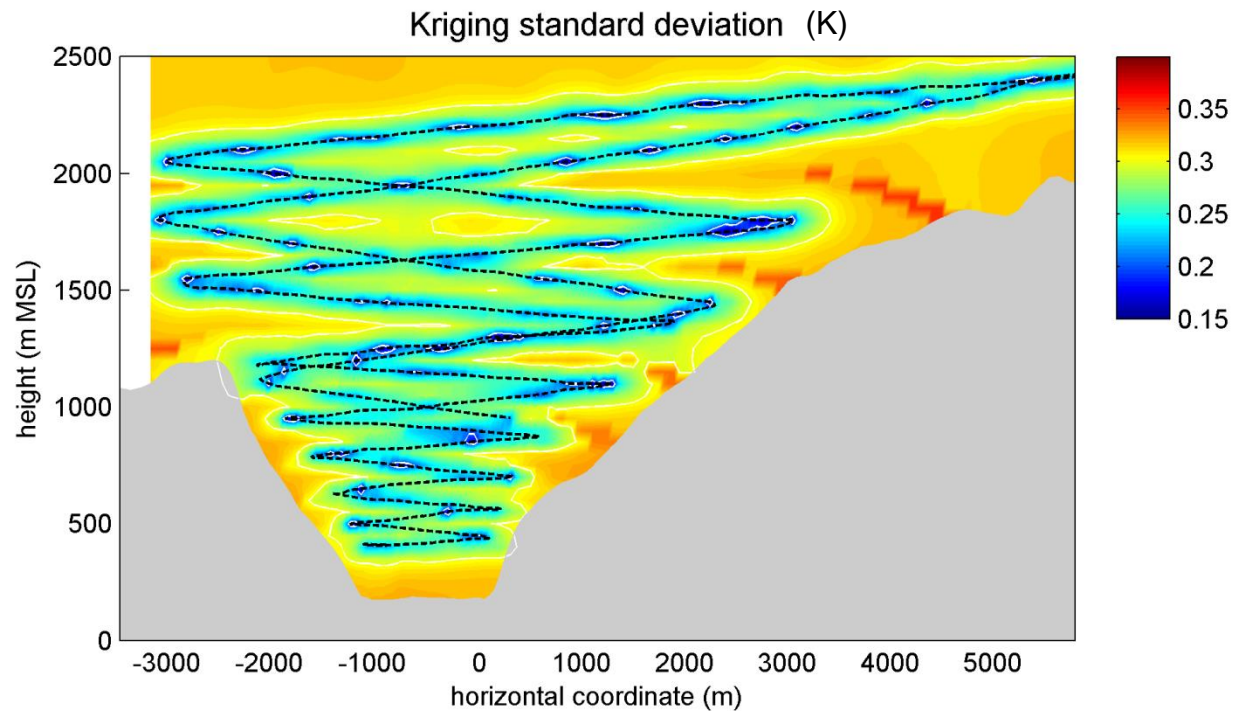
omnidirectional semivariogram



# Remarks about kriging - 3

Kriging also provides an estimate of the interpolation error, i.e. the kriging variance (or kriging standard deviation)

$$\hat{\sigma}^2(\mathbf{x}_g) = \gamma(0) - \sum_i \lambda_i \gamma(\mathbf{x}_i - \mathbf{x}_g) + q$$



# 6. Modified LOOCV results

Modified LOOCV: exclusion of subsets of observations of increasing radius

flight #M

$$\bar{\rho} = \frac{1}{N} \sum_{i=1}^N \rho(\mathbf{x}_i) = \frac{1}{N} \sum_{i=1}^N \frac{[\hat{\theta}(\mathbf{x}_i) - \theta(\mathbf{x}_i)]^2}{\hat{\sigma}^2(\mathbf{x}_i)}$$

*RK estimate variance*

$$\text{dimensionless radius} = \frac{\text{radius}}{I_{\text{omni}}}$$

$I_{\text{omni}}$  = max distance of correlation

