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Similarity Theory Based on Dougherty-Ozmidov Scaling

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SHEBA Site

Surface Heat Budget of the Arctic Ocean Experiment (SHEBA)

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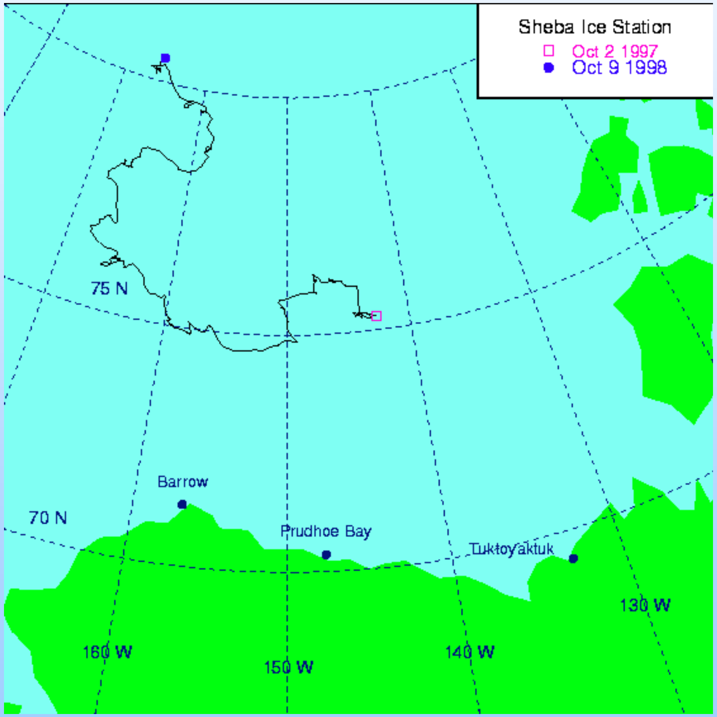
- The main SHEBA ice camp was deployed on the ice in the vicinity of the Canadian Coast Guard ice breaker *Des Groseilliers*, which was frozen into the Arctic ice pack north of Alaska from October 1997 to October 1998.
- During this period, the ice breaker drifted more than 1400 km in the Beaufort and Chukchi Seas, with coordinates varying from approximately 74° N and 144° W to 81° N and 166° W.



The SHEBA ice station drift from October 2, 1997 until October 9, 1998.

The SHEBA ice camp and the *Des Groseilliers*

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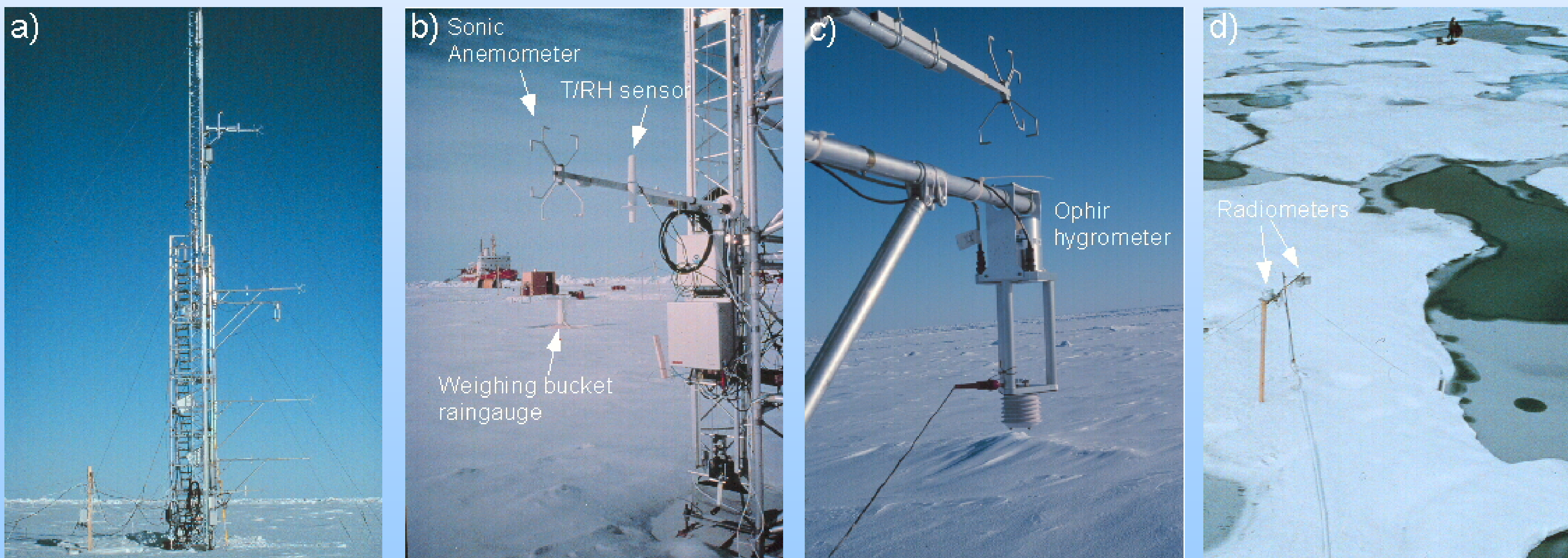
ASFG Instrumentation

- The Atmospheric Surface Flux Group (ASFG) deployed a 20-m main micrometeorological tower, two short masts, and several other instruments on the surface located 280 – 350 m from the *Des Groseilliers* at the far edge of the main ice camp.
- Turbulent and mean meteorological data were collected at five levels, nominally 2.2, 3.2, 5.1, 8.9, and 18.2 m (or 14 m during most of the winter).
- Each level had a Väisälä HMP-235 temperature/relative humidity probe (T/RH) and identical ATI three-axis sonic anemometers/thermometers (resolution: wind speed 0.01 m/sec; sonic temperature 0.01°C).
- An Ophir fast infrared hygrometer was mounted on a 3-m boom at an intermediate level just below level 4 (8.1 m above ice).



Traditional SBL

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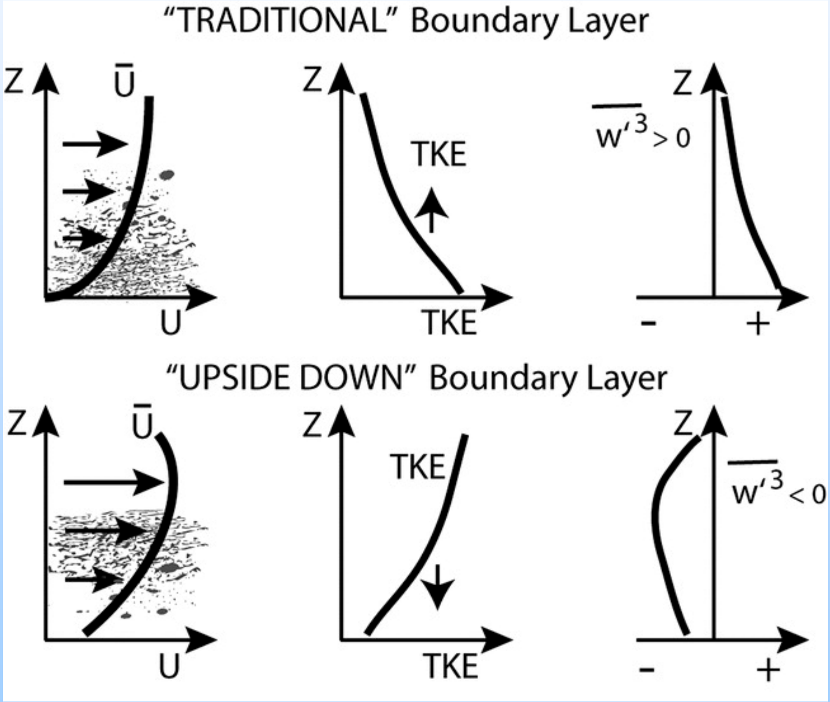
- The SHEBA site was located on Arctic pack ice, which had no large-scale slopes or heterogeneities; the site was a few hundred kilometers from land and thus provided almost unlimited and extremely uniform fetch. For these reasons, the SHEBA flux data are not generally contaminated by drainage (katabatic), strong local advective flows, or orographically generated gravity waves.
- Thus the SBL observed most often during SHEBA can be characterized as a traditional SBL layer.

The SHEBA ice camp and C-130



Schematic of structure of (top) traditional boundary layer versus (bottom) upside-down boundary layer (Banta et al (2006) JAS v.63(11) Fig.1)

Monin-Obukhov Similarity Theory

Surface Scaling

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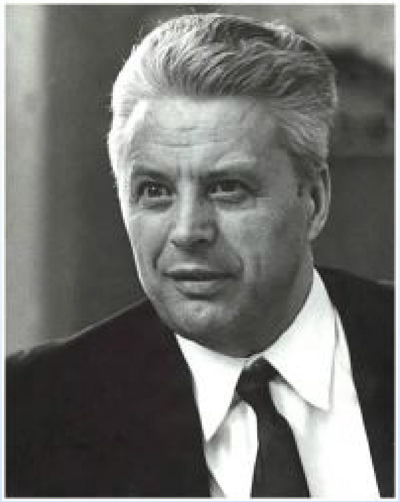

A. S. Monin

A. M. Obukhov and A. N. Kolmogorov

- **Obukhov (1946), Monin and Obukhov (1954)**
- **Obukhov length:** $L = -\dfrac{u_*^3 \, T_v}{\kappa g < w' T_v' >}$
- **Monin – Obukhov stability parameter:** $\zeta \equiv z / L$
- **Non-dimensional velocity and temperature gradients:** $\varphi_m(\zeta) = \dfrac{\kappa z}{u_*} \dfrac{dU}{dz}, \qquad \varphi_h(\zeta) = \dfrac{\kappa z}{T_*} \dfrac{d\theta}{dz}$
- **Non-dimensional standard deviations:** $\varphi_\alpha(\zeta) = \dfrac{\sigma_\alpha}{u_*}, \qquad \varphi_\theta(\zeta) = \dfrac{\sigma_\theta}{\theta_*} \qquad (\alpha = u, v, w)$

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Monin-Obukhov Similarity Theory

Local Scaling

Frans Nieuwstadt (sketch by Z. Sorbjan)

- **Nieuwstadt F. T. M. (1984, JAS v.41) demonstrated that in the stable boundary layer (SBL) the assumption of height-independent fluxes is not necessary. He thus redefined Monin-Obukhov similarity in terms of local similarity (local scaling) for which the Obukhov length and the flux-profile and flux-variance relationships are based on the local fluxes at height *z* (i.e., *z*-dependent fluxes) rather than on the surface values.**

- **Sorbjan (1986, BLM v.34; 1988, BLM v.44) argued that the functional forms of the universal functions in the SBL are identical for both surface and local scaling assumptions. Thus, local scaling describes the turbulent structure of the entire SBL.**

Dougherty-Ozmidov Length Scale

Oceanic Vertical Mixing

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R. V. Ozmidov

- **In oceanography, the Brunt-Väisälä frequency *N*, and the dissipation rate of turbulent kinetic energy *ε* are traditionally used as the governing parameters to describe small-scale turbulence.**

- **A buoyancy length scale constructed from *N* and *ε* was originally suggested by Dougherty (1961) and independently by Ozmidov (1965) and herein is referred to as the Dougherty-Ozmidov length scale. It is also known as Ozmidov length and is widely used in oceanography to describe small-scale turbulence. Historically the term "Ozmidov length scale" was introduced by Carl H. Gibson in the oceanographic community.**

$$L_{N\varepsilon} = \sqrt{\varepsilon / N^3}$$

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Scaling Systems

Local Scaling

- **The Pi theorem used in MOST provides only a general methodology, and the choice of the primary governing variables is not unique. In fact, Nieuwstadt (1984) deprived the turbulent fluxes of their "privileged role" and paved the way to construct a local similarity theory in the SBL based on governing variables other than the fluxes.**

- **Monin and Obukhov (1954)**

$$\tau,\quad H,\quad \beta \qquad \Rightarrow \qquad L=\frac{\tau^{3/2}}{\kappa\,\beta H},\quad u_*=\sqrt{\tau},\quad \theta_*=\frac{H}{\sqrt{\tau}}$$

- **Smeets, Duynkerke, Vugts (2000, BLM v.97)**

$$\sigma_w,\quad H,\quad \beta \qquad \Rightarrow \qquad L_{wH}=\frac{\sigma_w^3}{\kappa\,\beta H},\quad U_{wH}=\sigma_w,\quad \theta_{wH}=\frac{H}{\sigma_w}$$

- **Sorbjan (2006, 2008, 2010), gradient-based scaling**

$$\sigma_w,\quad N,\quad \beta \qquad \Rightarrow \qquad L_{wN}=\frac{\sigma_w}{N},\quad U_{wN}=\sigma_w,\quad \theta_{wN}=\frac{\sigma_w N}{\beta}$$

$$\sigma_\theta,\quad N,\quad \beta \qquad \Rightarrow \qquad L_{\theta N}=\frac{\beta\sigma_\theta}{N^2},\quad U_{\theta N}=\frac{\beta\sigma_\theta}{N},\quad \theta_{\theta N}=\sigma_\theta$$

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Dougherty-Ozmidov Scaling System (1)

The N-ε scaling: Dimensional analysis

- **The N-ε scaling** $N, \quad \varepsilon, \quad \beta \quad \Rightarrow \quad L_{N\varepsilon} = \sqrt{\dfrac{\varepsilon}{N^3}}, \quad U_{N\varepsilon} = \sqrt{\dfrac{\varepsilon}{N}}, \quad \theta_{N\varepsilon} = \dfrac{\sqrt{\varepsilon N}}{\beta}$

$N = \sqrt{\beta(\partial\theta/\partial z)} = \sqrt{-g(\partial\rho/\partial z)/\rho}$ **is the Brunt-Väisälä frequency (or buoyancy frequency)**

- **According to Buckingham's Pi theorem, any properly scaled statistics of the small-scale turbulence are universal functions of a stability parameter defined as the ratio of a reference height *z* and the Dougherty-Ozmidov length scale :**

$$\xi = z / L_{N\varepsilon}$$

- **Non-dimensional relationships for *dU/dz*, momentum flux, and temperature flux**

$$Ri = \psi_R(\xi), \qquad \frac{\tau N}{\varepsilon} = \psi_m(\xi), \qquad \frac{\beta H}{\varepsilon} = \psi_h(\xi)$$

- **Non-dimensional relationships for standard deviations of wind speed components, temperature, turbulent viscosity and thermal diffusivity**

$$\frac{\sigma_\alpha}{\sqrt{\varepsilon / N}} = \psi_\alpha(\xi), \qquad \frac{\sigma_t \beta}{\sqrt{\varepsilon N}} = \psi_t(\xi) \qquad \frac{K_m N^2}{\varepsilon} = \psi_{Km}(\xi), \qquad \frac{K_h N^2}{\varepsilon} = \psi_{Kh}(\xi)$$

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Dougherty-Ozmidov Scaling System (2)

The asymptotic behaviour of the universal functions

• In the neutral case, various quantities become independent of the buoyancy parameter β (recall that β is included in N); that is, β is no longer a primary scaling variable. This requires that β cancels in the various relationships in the limit $\xi \rightarrow 0$; therefore,

$$\psi_R = a_R \xi^{4/3}, \quad \psi_m = a_m \xi^{2/3}, \quad \psi_{Km} = a_{Km} \xi^{4/3}, \quad \psi_{Kh} = a_{Kh} \xi^{4/3}, \quad \psi_\alpha = a_\alpha \xi^{1/3}, \quad \psi_t = a_t \xi$$

• In the very stable case, various dimensional variables become independent of z (z-less stratification) and the universal functions asymptotically approach constant values when $\xi >> 1$:

$$\psi_R = b_R, \quad \psi_m = b_m, \quad \psi_{Km} = b_{Km}, \quad \psi_{Kh} = a_{Kh}, \quad \psi_\alpha = b_\alpha, \quad \psi_t = b_t$$

where $\alpha = u, v, w$

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Dougherty-Ozmidov Scaling System (3)

Relationships between MOST and *N*-ε based scale systems

- **The Dougherty-Ozmidov length scale is a universal function of the Obukhov length scale:**

$$\xi = z / L_{N\varepsilon} = \frac{(\zeta \varphi_h)^{3/4}}{\kappa \varphi_\varepsilon^{1/2}}$$

- **Relationships between the universal functions:**

$$\psi_m = \frac{\sqrt{\zeta \varphi_h}}{\varphi_\varepsilon}, \quad \psi_{Km} = \frac{\zeta \varphi_h}{\varphi_m \varphi_\varepsilon}, \quad \psi_{Kh} = \frac{\zeta}{\varphi_\varepsilon}, \quad \psi_\alpha = \varphi_\alpha \left(\frac{\zeta \varphi_h}{\varphi_\varepsilon^2} \right)^{1/4}, \quad \psi_t = \varphi_t \left(\frac{\zeta^3}{\varphi_h \varphi_\varepsilon^2} \right)^{1/4}$$

- **Relationships for the universal functions can be rewritten as follows :**

$$\tau N / \varepsilon = \sqrt{Ri}, \quad K_m N^2 / \varepsilon = Ri, \quad K_h N^2 / \varepsilon = Rf, \quad \frac{\sigma_\alpha}{\sqrt{\varepsilon / N}} = \beta_\alpha Ri^{1/4}, \quad \frac{\sigma_t \beta}{\sqrt{\varepsilon N}} = \beta_t Rf / Ri^{1/4}$$

because: $Ri = \zeta \varphi_h / \varphi_m^2, \quad Rf = \zeta / \varphi_m, \quad \varphi_m \cong \varphi_\varepsilon = 1 + 5\zeta$

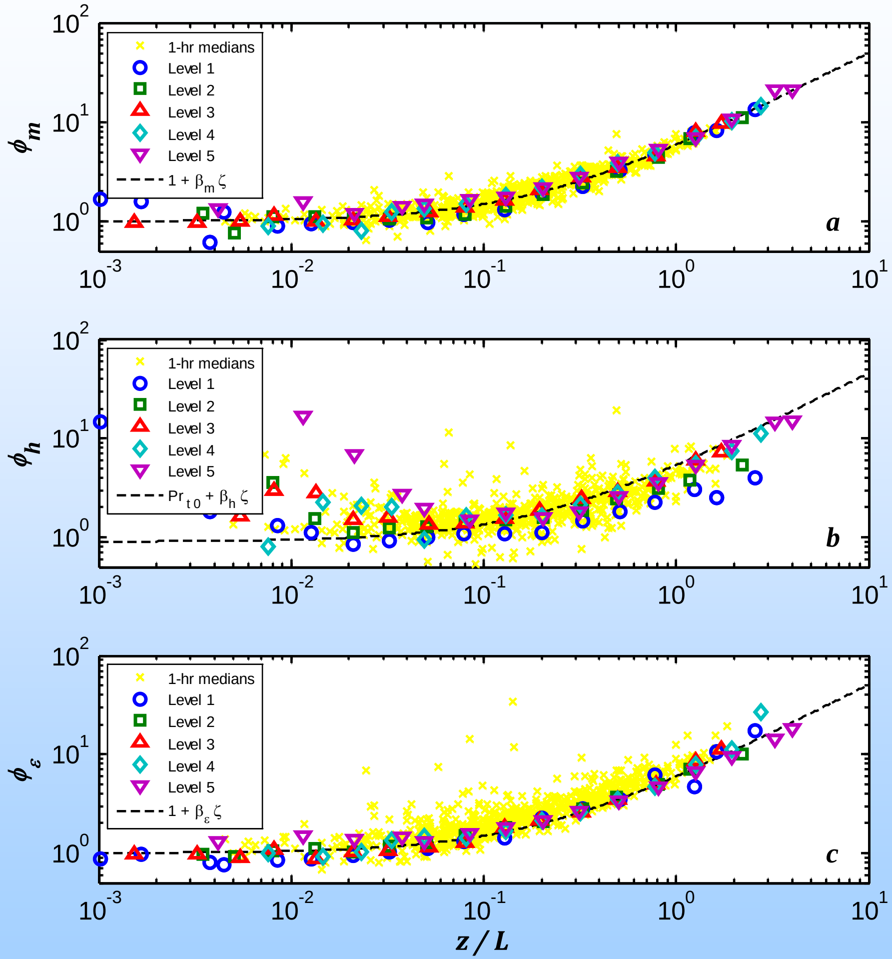
- **The applicability of the approach as well as MOST in stable conditions is limited by the inequalities :**

$$Ri < Ri_{cr}, \quad Rf < Rf_{cr}, \quad Ri_{cr}, Rf_{cr} \cong 0.20 - 0.25$$

(Grachev *et al*. 2013, BLM v.147)

MOST universal functions

SHEBA data



The bin-averaged non-dimensional universal functions (*a*) φ_m , (*b*) φ_h , and (*c*) φ_ε for five levels of the main SHEBA tower plotted versus $\zeta = z/L$ in the subcritical regime when prerequisites $Ri < Ri_{cr}$ and $Rf < Rf_{cr}$ with $Ri_{cr} = Rf_{cr} = 0.2$ have been imposed on the data. The dashed lines are based on $\beta_m = \beta_\varepsilon = 5.0$, $\beta_h = 4.5$ and $Pr_{t0} = \beta_h / \beta_m = 0.9$.

Dougherty-Ozmidov length scale

SHEBA data

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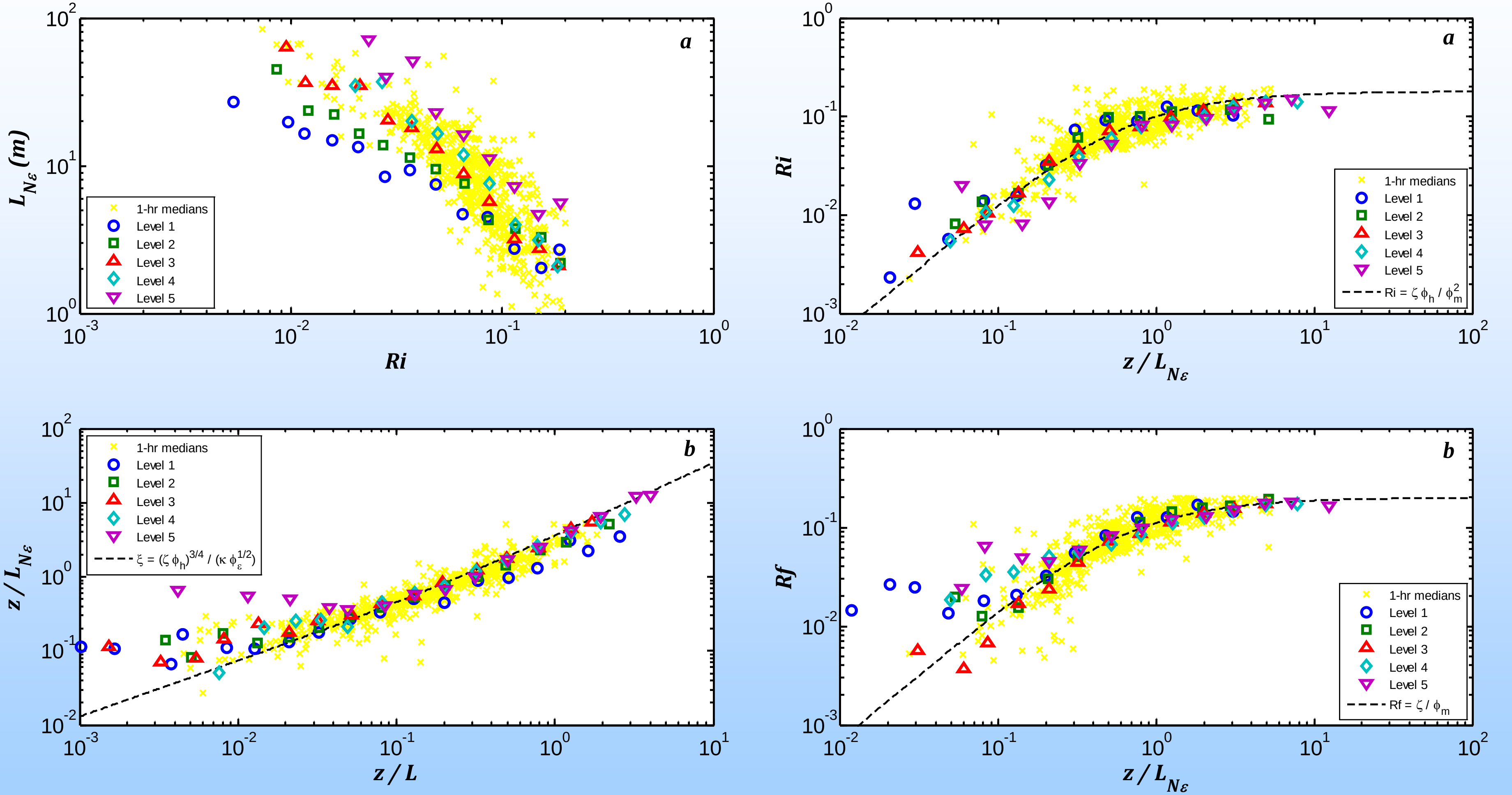

Left panels: (*a*) Behaviour of the Dougherty-Ozmidov length scale observed in the SBL for SHEBA data plotted against the gradient Richardson number; (*b*) plot of the bin-averaged Dougherty-Ozmidov stability parameter versus the Monin-Obukhov stability parameter. ***Right panels***: Plots of (*a*) gradient Richardson number, ***Ri***, and (b) flux Richardson number, ***Rf***, versus the Dougherty-Ozmidov stability parameter.

Dougherty-Ozmidov universal functions (1)

SHEBA data

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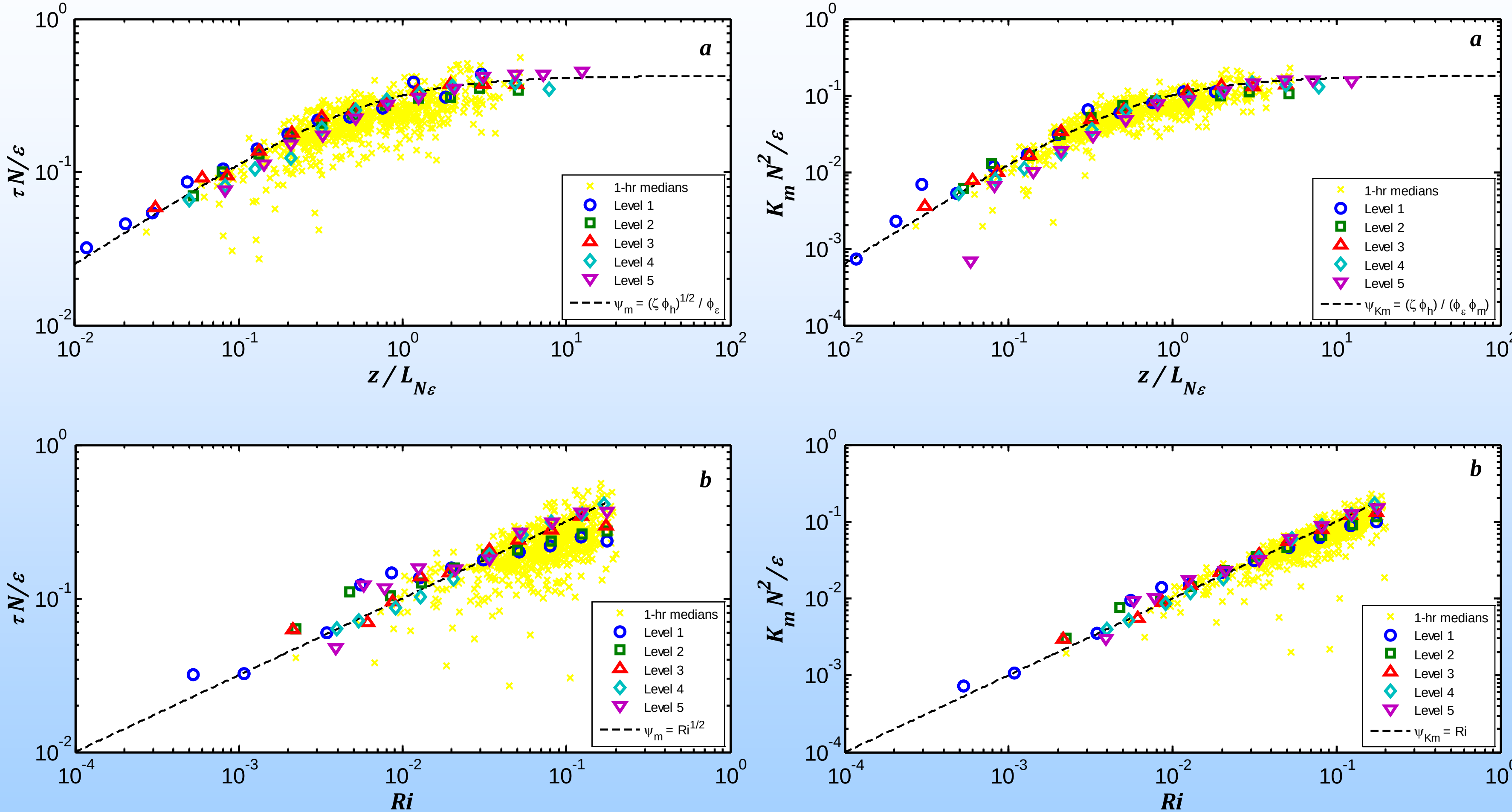

Left panels: **Plots of the bin-averaged non-dimensional momentum flux versus (*a*) the Dougherty-Ozmidov stability parameter; (*b*) the gradient Richardson number, *Ri*. *Right panels*: Plots of the non-dimensional turbulent viscosity versus (*a*) the Dougherty-Ozmidov stability parameter; (*b*) the gradient Richardson number, *Ri*.**

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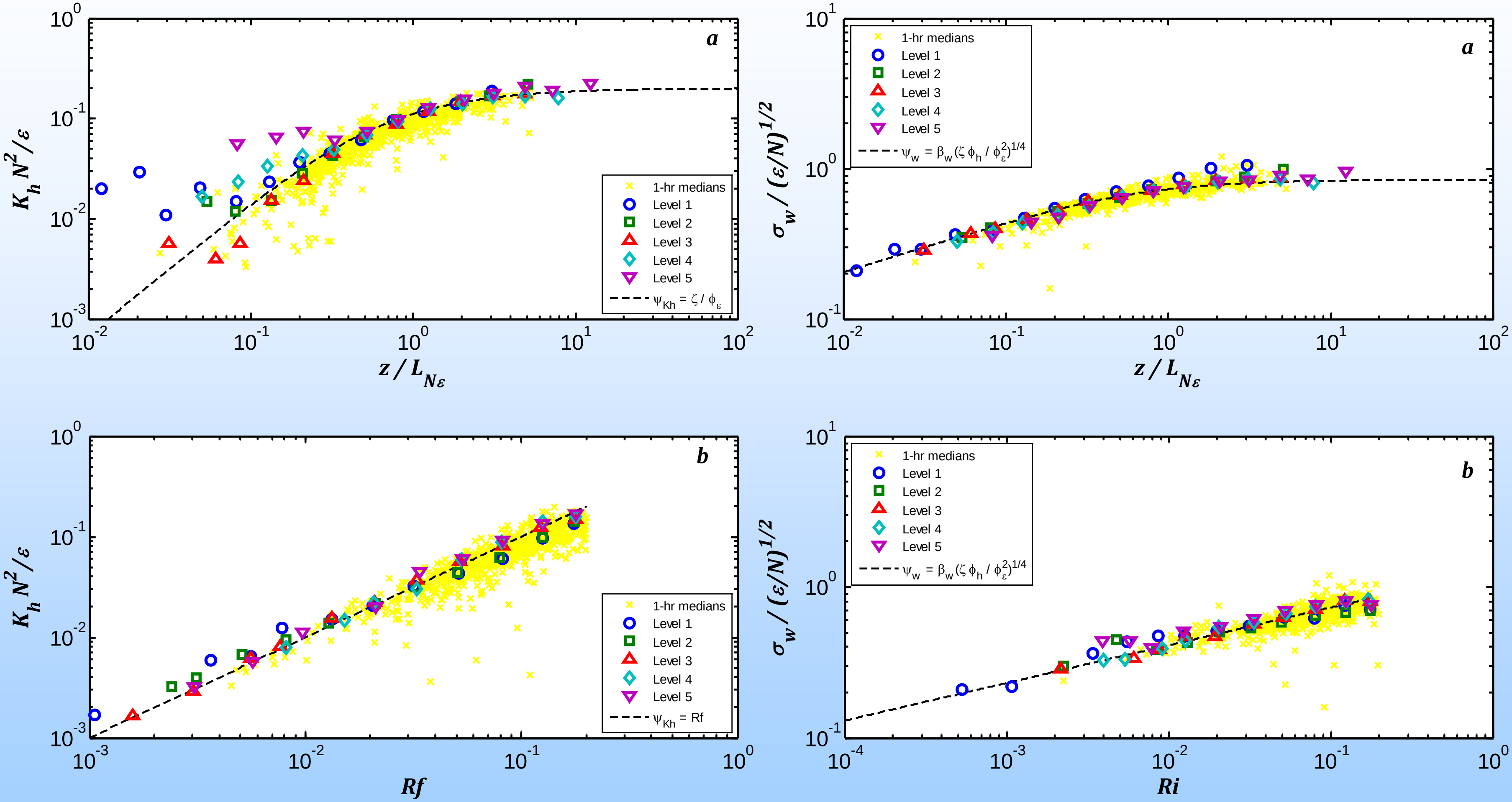
Dougherty-Ozmidov universal functions (2)

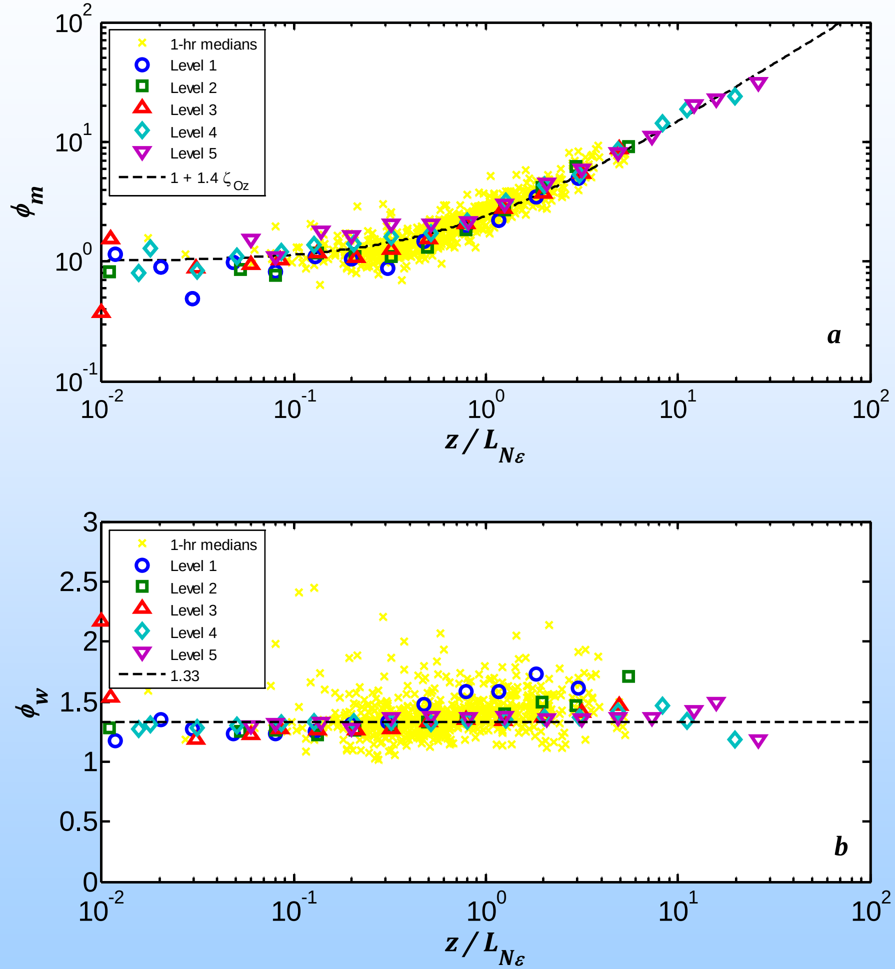
SHEBA data



***Left panels*: Plots of the bin-averaged non-dimensional turbulent thermal diffusivity versus (*a*) the Dougherty-Ozmidov stability parameter; (*b*) the flux Richardson number, *Rf*. *Right panels*: Plots of the non-dimensional standard deviation of the vertical wind speed component versus (*a*) the Dougherty-Ozmidov stability parameter; (*b*) the gradient Richardson number, *Ri*.**

MOST universal functions φ_m and φ_w

versus Dougherty-Ozmidov stability parameter – NO SELF-CORRELATION!



The bin-averaged non-dimensional universal functions (*a*) φ_m and (*b*) φ_w for five levels of the main SHEBA tower plotted versus the Dougherty-Ozmidov stability parameter in the subcritical regime when prerequisites $Ri < Ri_{cr}$ and $Rf < Rf_{cr}$ with $Ri_{cr} = Rf_{cr} = 0.2$ have been imposed on the data.

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Conclusions

• **We develop a local similarity theory for the SBL that is based on the Brunt-Väisälä frequency *N*, the dissipation rate of turbulent kinetic energy *ε*, instead of on turbulent fluxes as used in the traditional Monin-Obukhov similarity theory (MOST). A buoyancy length scale constructed from these two variables was originally suggested by Dougherty (1961) and independently by Ozmidov (1965) and herein is referred to as the Dougherty-Ozmidov length scale;**

• **Based on dimensional analysis, *N*, *ε*, and *β* can be considered as the governing variables that define other variables in the SBL at the height *z*. We show that any properly scaled statistics of the turbulence are universal functions of a stability parameter defined as the ratio of height *z* and the Dougherty-Ozmidov length scale;**

• **We also found that, in the framework of this approach, the non-dimensional turbulent viscosity is equal to the gradient Richardson number and the non-dimensional turbulent thermal diffusivity is equal to the flux Richardson number;**

• **The proposed approach is equivalent to traditional MOST and its applicability in stable conditions is limited by inequalities, when both gradient and flux Richardson numbers are below a "critical value" about 0.20–0.25.**

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Questions?