The Thermal Phase Curve Offset on Tidally Locked and Non-Tidally Locked Planets: A Shallow Water Model

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Does relaxing the assumption of tidal-locking change the offset in the observed phase curve of an exoplanet?

If there is no heat transport present in the atmosphere of a planet, the hottest point (the *hotspot*) will be at the *substellar point*: the location on the planet receiving the most radiation from the host star. Phase curve observations of transiting exoplanets show that the hotspot can be offset from the substellar point by up to 50º degrees in longitude [1].

We use a numerical shallow water model to investigate how the hotspot offset from substellar point changes when a planet is not tidally-locked. **Figure 1** shows steady-state solutions of the model for planets where the substellar point is moving westwards **(a)**, stationary **(b)** or eastwards **(c)**.

Conclusions

- The hotspot offset from substellar point is sensitive to both planetary rotation rate and the speed of the substellar point across the surface. **Figure 2** shows the offset as a function of substellar velocity for three different planetary rotation rates.
- Retrograde (westward) offsets are more sensitive to substellar velocity than prograde because Rossby waves move at ~1/3 the speed of Kelvin & gravity waves.
- The radiative and frictional timescales become more important in determining the longitudinal offset once substellar velocity exceeds Kelvin wavespeed c.
- Given the observed phase curve of an exoplanet, measuring the offset could provide insight into the rotation rate of the planet.

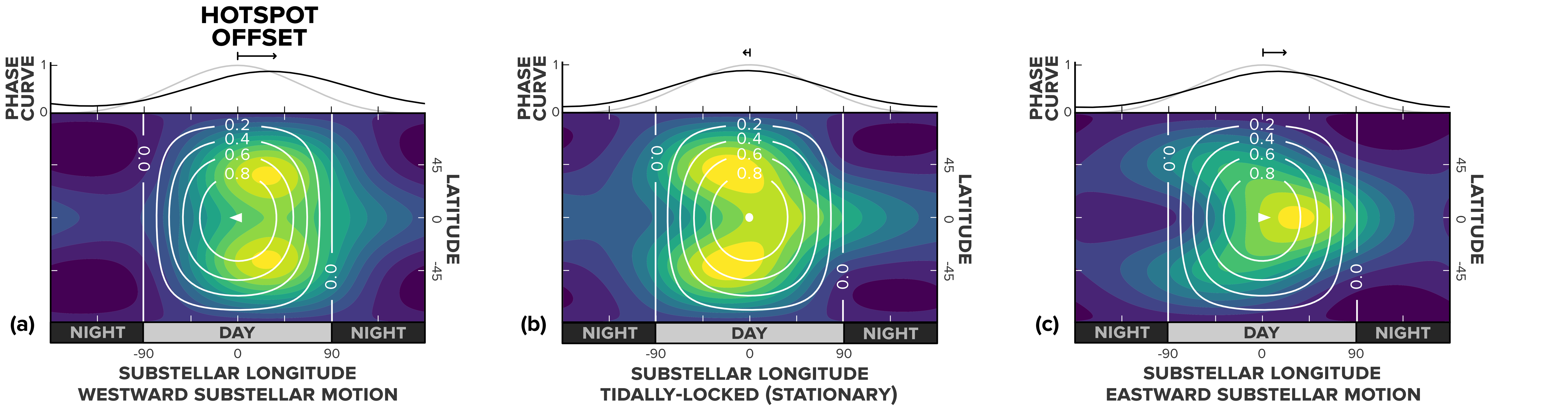

Figure 1. Left-to-right; steady-state solutions of three shallow-water experiments with the substellar point moving to the west, stationary and to the east.

Coloured contours show the height field, corresponding to temperature in the idealised model. Purple "coldest", yellow "hottest". White contours denote the position and direction of travel of the forcing. Phase curves are plotted above the maps. Black lines show the hemispheric integral of the height field, grey lines the integral of the forcing.

The rotation rate here corresponds to **MEDIUM** in Figure 2, the deformation radius is small enough that rotational effects are pronounced. **(a)** Westward substellar motion is dominated by Rossby gyres. **(b)** The tidally-locked case exhibits a steady-state analogous to a planetary scale Matsuno-Gill solution. **(c)** Eastward substellar motion response is dominated by a Kelvin wave-like feature.

Model

The forced shallow water equations we use are

$$\frac{\partial \boldsymbol{u}}{\partial t} + \boldsymbol{u} \cdot \nabla \boldsymbol{u} + \boldsymbol{f} \times \boldsymbol{u} = -g\nabla h - \frac{\boldsymbol{u}}{\tau},$$

$$\frac{\partial h}{\partial t} + \nabla \cdot (\boldsymbol{u}h) = \frac{h_{\text{eq}} - h}{\tau},$$

$$h_{\text{eq}} = \begin{cases} H + \Delta h \cos\phi \cos\xi & \cos\xi \geq 0, \\ H & \cos\xi < 0, \end{cases}$$

$$\xi = \lambda - \frac{\alpha c}{a} t, \qquad \boldsymbol{f} = 2\Omega \sin\phi \hat{\mathbf{k}},$$

where ϕ is latitude and ξ is the *substellar longitude*, the longitude relative to the centre of the forcing. In this simple model, height h is a proxy to temperature, as for other shallow water exoplanet studies [2].

The unforced shallow-water equations permit several waves: Gravity waves, equatorial Kelvin waves, and Rossby waves [3]. Kelvin waves are the fastest waves in the system and travel eastward at velocity

$$c = \sqrt{gH}.$$

We perform numerical simulations of these equations, varying rotation rate Ω and substellar velocity, $s = \alpha c$, integrating forward until a steady-state relative to the forcing is obtained. The *phase curve* of the simulation is calculated as the hemispheric integral of steady-state height at all observation longitudes

$$I(\delta) = \int_{\delta-\pi/2}^{\delta+\pi/2} \int_{-\pi/2}^{\pi/2} a^2 h(\xi, \phi) \cos\xi \cos^2\phi \, d\phi \, d\xi.$$

Phase curves are calculated and the longitude of the peak (corresponding to the planetary hotspot) is measured from the substellar point. Figure 1 shows the steady-state height field and corresponding phase curves for three experiments. Figure 2 shows the relationships between measured offsets for several experiments with different Ω and α.

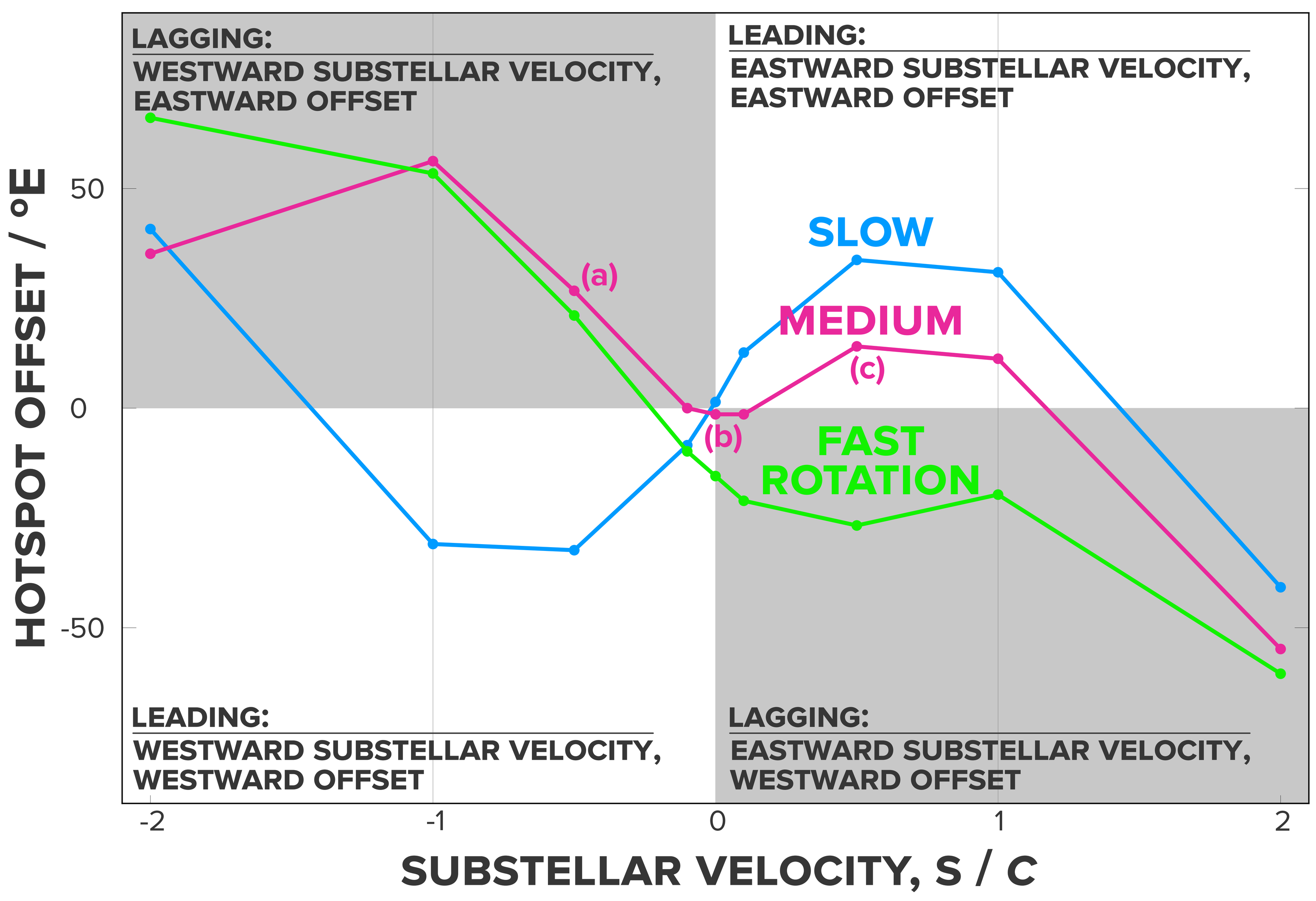

Figure 2. Hotspot offset as a function of substellar velocity (abscissa, denoted s, measured in multiples of wavespeed c) for three characteristic planetary rotation rates (coloured lines).

When |s| > c, all curves tend to a lagging limit determined by the frictional timescales τ. When |s| < c, the offset is flow dependent. For **SLOW** rotation (deformation radius larger than planetary radius), the hotspot leads ahead of the substellar point. When rotation is **FAST** the hotspot lags behind. Sample height fields and phase curves that correspond to point data labelled (a), (b), (c) on the **MEDIUM** curve can be seen in Figure 1.

The transition from slow to fast regime is different for prograde (eastward) or retrograde (westward) substellar progression. On fast rotating planets Rossby waves form. These travel west at approximately 1/3 the speed of Kelvin and gravity waves. Total westward wave action is therefore slower than eastward and the hotspot lags at a lower substellar velocity.

References
1. B.-O. Demory et al. (2016). A map of the large day–night temperature gradient of a super-Earth exoplanet. Nature, 532 (7598), 207–209.
2. A. P. Showman & L. M. Polvani (2010). The Matsuno-Gill model and equatorial superrotation. GRL, 37 (18)
3. G. K. Vallis (2017). Atmospheric and Oceanic Fluid Dynamics. Cambridge University Press.

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