A new equation for the inversion of Ertel's potential vorticity to be used for the model diagnosis of group-velocity-based energy flux associated with waves at all latitudes

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Auto-focus for all waves
Seamless at all latitudes

Single-layer equation system for linear waves without a mean flow

$$u'_t - fv' + p'_x = 0$$
$$v'_t + fu' + p'_y = 0$$
$$p'_t + c^2(u'_x + v'_y) = 0$$

$$E \equiv [u'^2 + v'^2 + (p'/c)^2]/2$$
$$q' \equiv v'_x - u'_y - (f/c^2)p'$$

All waves at all latitudes

mid-latitude IGWs
mid-latitude RWs
coastal KWs
equatorial IGWs
equatorial mixed RGWs
equatorial RWs
equatorial KWs

without relying on Fourier analysis nor Ray theory

Diagnosable expression for group-velocity-based energy flux

mid-latitude IGWs

$$\partial_t \overline{E} + \nabla \cdot \langle\langle \overline{u'p'}, \overline{v'p'} \rangle\rangle = 0$$

Longuet-Higgins (1964), Orlandki and Sheldon (1993)

mid-latitude RWs

$$\partial_t \overline{E} + \nabla \cdot \langle\langle \overline{u'p'} + [\overline{p'^2}/(2f)]_y, \overline{v'p'} - [\overline{p'^2}/(2f)]_x \rangle\rangle = 0$$

equatorial waves (previous studies)

$$\int_{-\infty}^{+\infty} c_g \overline{E}\, dy = \int_{-\infty}^{+\infty} \overline{u'p'}\, dy$$
$$c_g \overline{E} \neq \overline{u'p'}$$

Additional rotational flux (singular at the equator)

Aiki, Greatbatch and Claus (2017, Progress of Earth & Planetary Science)

$$q' \equiv v'_x - u'_y - (f/c^2)p'$$

equatorial waves (this study)

$$\nabla^2\varphi - (f/c)^2\varphi - (3/c^2)\varphi_{tt} = q'$$
$$\partial_t \overline{E} + \nabla \cdot \langle\langle \underbrace{\overline{u'p'} + (\overline{p'\varphi}/2 + \overline{u'_{tt}\varphi}/\beta)_y}_{=c_g\overline{E}}, \overline{v'p'} - (\overline{p'\varphi}/2 + \overline{u'_{tt}\varphi}/\beta)_x \rangle\rangle = 0$$

derived from analytical solution

approximation (trade-off between exactness and practical accessibility for model diagnosis)

$$\nabla^2\varphi^{app} - (f/c)^2\varphi^{app} = q'$$
$$\partial_t \overline{E} + \nabla \cdot \langle\langle \underbrace{\overline{u'p'} + (\overline{p'\varphi^{app}}/2)_y}_{\simeq c_g\overline{E}}, \overline{v'p'} - (\overline{p'\varphi^{app}}/2)_x \rangle\rangle = 0$$

Standard inversion of EPV

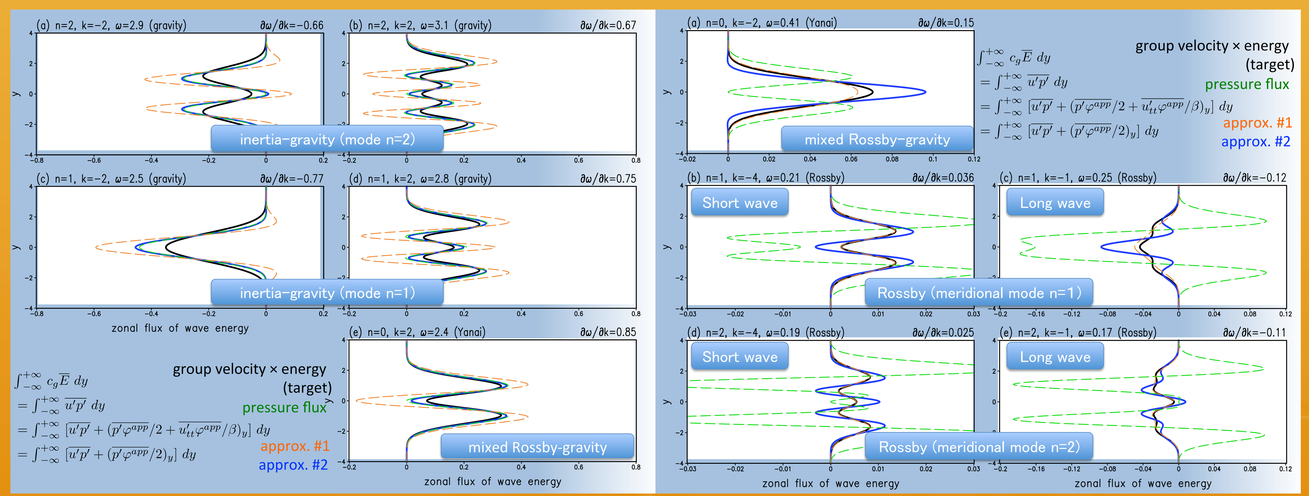
A new 3D inversion equation for EPV in a continuously stratified fluid

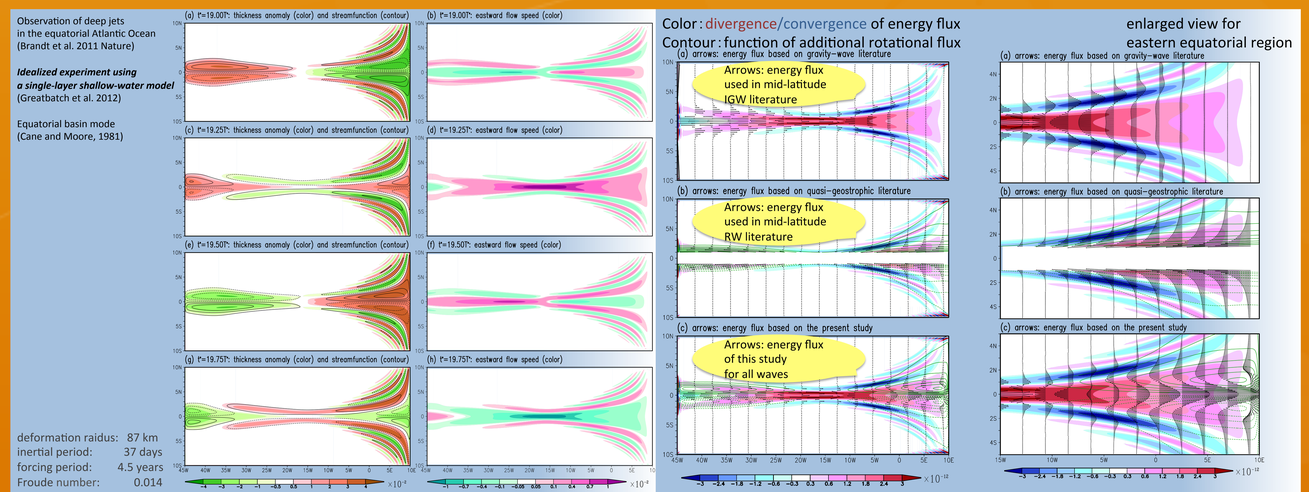
$$\frac{\partial^2}{\partial x^2}\varphi + \frac{\partial^2}{\partial y^2}\varphi + \frac{\partial}{\partial z}\left[\frac{(f^2 - 3\omega_b^2)}{N^2}\frac{\partial}{\partial z}\varphi\right] = \frac{\rho_z(f + v_x - u_y) - \rho_x v_z + \rho_y u_z}{\overline{\rho}_z} - f$$

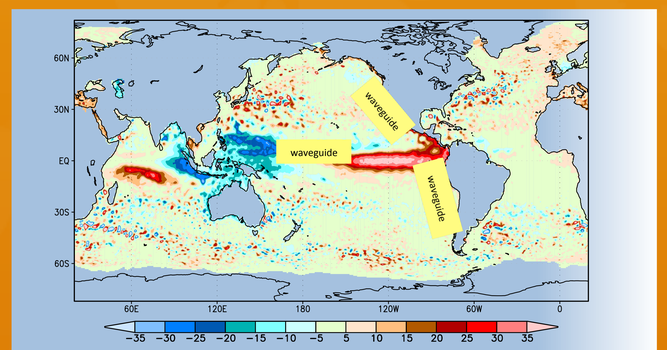
seamless at all latitudes

Fourier decomposition at each point in horizontal space

repeat EPV inversion to cover all frequency bands







- Model diagnosis in past 20 years: global mapping of energy conversion rates associated with eddies and external forcing
- Model diagnosis in future studies: global mapping of energy transfer (flux in physical space) by eddies and waves
- The energy flux of mid-latitude IGWs has already been estimated in oceanic studies

Classical Energy–based (CE) pseudomomentum equaiton

$$(E/c_p)_t + \nabla \cdot \langle\langle u'p'/c_p, v'p'/c_p, w'p'/c_p \rangle\rangle = 0,$$

$$\pi' \equiv \int^t p'dt,$$
$$u' - f\eta' = -\pi'_x,$$
$$v' + f\xi' = -\pi'_y,$$

$$E \equiv K + G$$
$$= (u'^2 + v'^2 + N^2\zeta'^2)/2$$
$$= (u'\xi'_t + v'\eta'_t - \zeta\pi'_{zt})/2,$$

Gauge tranformation (Aiki, Takaya and Greatbatch, 2015 JAS)

$$\overbrace{(-u'\xi'_x - v'\eta'_x + \zeta'\pi'_{zx})/2}^{E/c_p} = \zeta'_z u' + q'\eta'/2 + \nabla \cdot \langle\langle -v'\eta', u'\eta', \zeta'\pi'_x \rangle\rangle/2$$

CE pseudomomentum — IB pseudomomentum

$$\overbrace{(\xi'p'_x - u'\pi'_x)/2}^{\overline{u'p'}/c_p} = \overline{u'u'} - \overline{K} + \overline{G} + \underbrace{[(\xi'p')_x + (\eta'p')_y + (\zeta'p')_z]/2}_{\equiv\bar{\Lambda}}$$

CE flux — IB flux

Auto-focus feature: Lambda=-E (Rossby wave) Lambda=0 (gravity wave)

Impulse–Bolus (IB) pseudomomentum equation (Ripa, 1982; Haynes, 1988)

$$\partial_t(\zeta'_z u' + q'\eta'/2) + \nabla \cdot \langle\langle u'u' - K + G, v'u', \zeta'p'_x \rangle\rangle = 0,$$

Aiki, Takaya and Greatbatch (2015, JAS)