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Turbulence spectra in the stable atmospheric boundary layer

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Background and Motivation

- Stratification (Lilly 1983) generates anisotropy.

- In the mesoscale regime, a scaling close to -5/3 is still observed (Nastrom and Gage, 1985). Direct energy cascade hypothesis (Lindborg, 2006).

- Monin-Obukhov similarity theory does not apply in very stable cases (Mahrt, 1998) .

➢To find out how stratification influences turbulence spectra of the universal equilibrium range in atmospheric boundary layer.

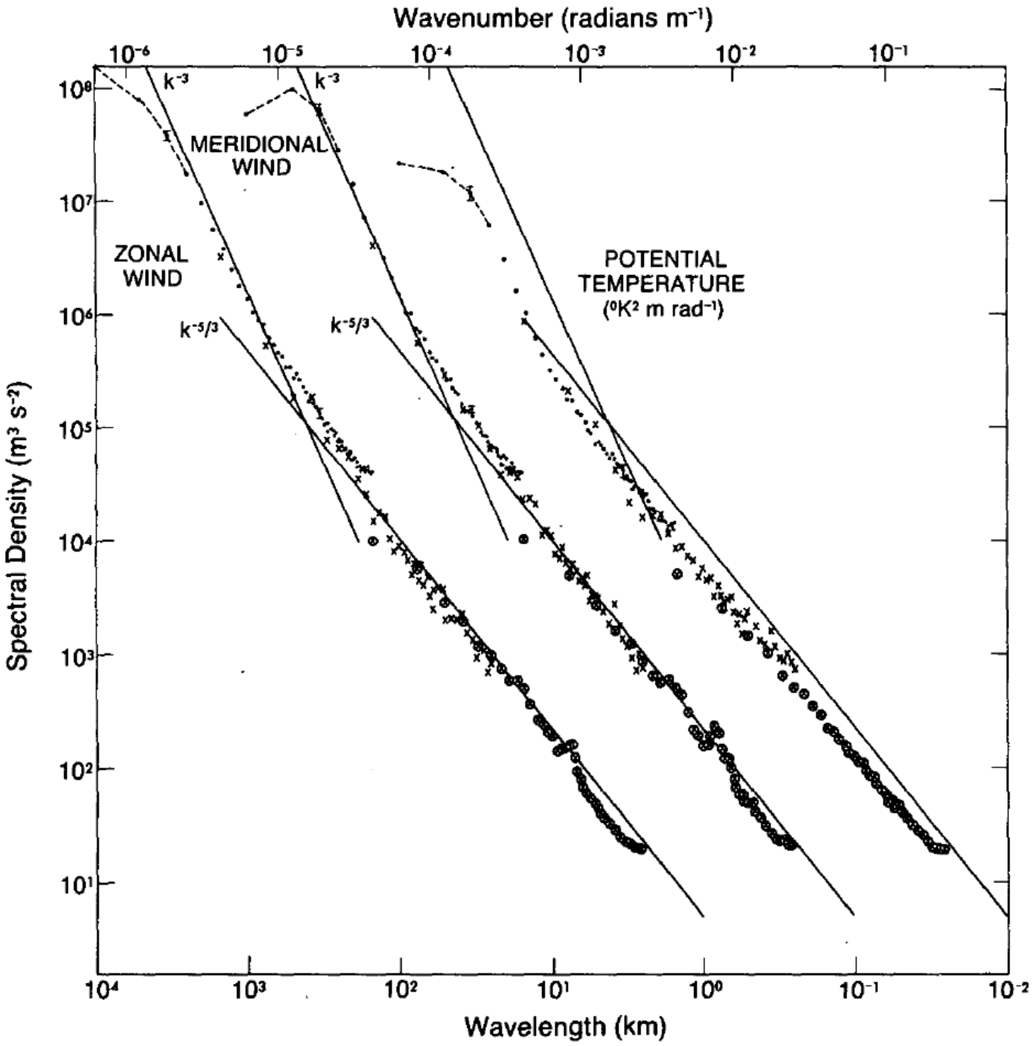
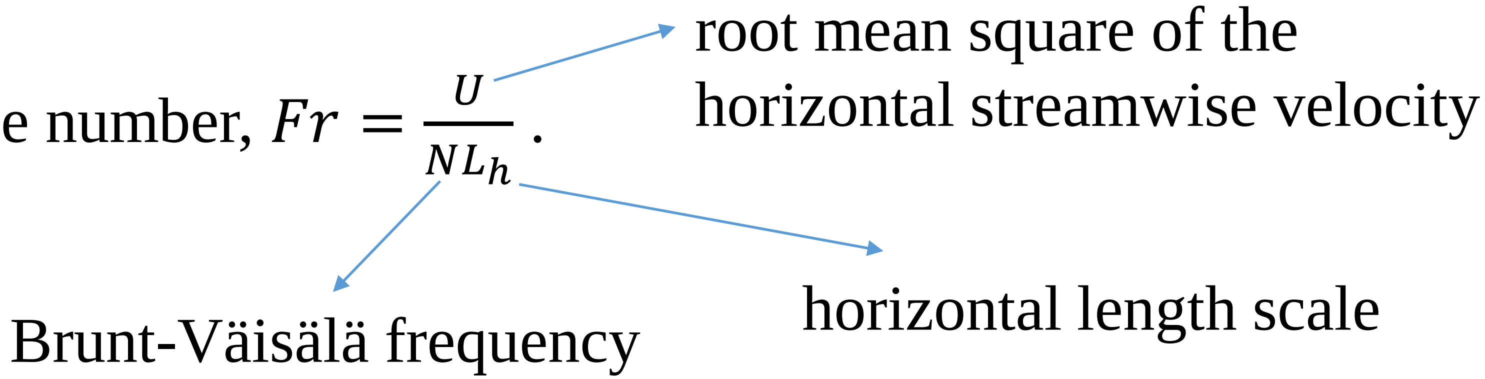


FIG. 3. Variance power spectra of wind and potential temperature near the tropopause from GASP aircraft data. The spectra for meridional wind and temperature are shifted one and two decades to the right, respectively; lines with slopes −3 and −⁵⁄₃ are entered at the same relative coordinates for each variable for comparison.

Figure from Nastrom and Gage (1985).

Important variables



- Horizontal Froude number, $Fr = \frac{U}{NL_h}$.

root mean square of the horizontal streamwise velocity

Brunt-Väisälä frequency

horizontal length scale

- Buoyancy length scale $L_b = 2\pi \frac{U}{N},\ k_b = \frac{2\pi}{L_b}$ (Billant and Chomaz, 2001) .

- Dougherty-Ozmidov length $L_O = 2\pi \left(\frac{\epsilon}{N^3}\right)^{\frac{1}{2}}, k_O = \frac{2\pi}{L_O}$ (Dougherty, 1961; Ozmidov, 1965).

- $k_a z = 1$ denotes effects of wall on turbulence spectra (Townsend,1976; Katul et al. 2014).

Theoretical basis: Spectra of “locally inertial” turbulence

- “locally inertial” assumption: *w* and *u* spectrum are comparable (Reiter & Burns 1965).
- Weinstock (1978) derived stratified turbulence spectra **without considering wall effects**.
- The spectral TKE balance equation (Lumley and Panofsky, 1964; Phillips, 1965).

$$\underbrace{\frac{\partial E(k)}{\partial t}}_{\substack{\text{spectral kinetic}\\ \text{energy density}}} + \overbrace{\frac{\partial Q(k)}{\partial z}}^{\substack{\text{vertical transfer of}\\ \text{turbulent energy}}} = \underbrace{S(k)}_{\substack{\text{spectrum of Reynolds}\\ \text{stress } -\overline{uw}}} \frac{\partial U_0}{\partial z} - \overbrace{\frac{\partial \epsilon(k)}{\partial k}}^{\substack{\text{net rate of spectral}\\ \text{energy transfer}}} + \underbrace{B(k)}_{\substack{\text{spectrum of buoyancy}\\ \text{flux } -\frac{g}{\rho_0}\overline{w\rho'}}} - \overbrace{2\nu k^2 E(k)}^{\substack{\text{rate of energy dissipation}\\ \text{by molecular viscosity } \nu}}.$$

Theoretical basis: Spectra of “locally inertial” turbulence

- Assuming steady state in the **universal equilibrium range**, Weinstock (1978) obtained

$$\frac{\partial \epsilon(k)}{\partial k} = S(k)\frac{\partial U_0}{\partial z} + B(k)\ ,$$

Kolmogorov constant

$$B(k) = -\alpha a N^2 \epsilon(k)^{\frac{2}{3}} \frac{v_m k^{-2/3}}{0.8N^2 + k^2 v_m^2}.$$

anisotropic factor

$v_m^2 = \frac{2}{3}\int_{k_m}^{\infty} E(k)dk$, k_m is the largest wavenumber in universal equilibrium range.

- $0.8N^2 \gg k^2 v_m^2$, $B(k)$ is in buoyancy subrange.
- $0.8N^2 \ll k^2 v_m^2$, $B(k)$ is in isotropic inertial subrange.

- Weinstock’s transition wavenumber k_{BW} is defined as $k_{BW} = \frac{0.8^{0.5}N}{v_m}$.

Proposed revision to Weinstock (1978)

- We **relax the “locally inertial” hypothesis**.
- The slope is shallower than -5/3 at low wavenumbers in *w* spectrum in the horizontal direction (Katul et al. 2014; Grachev et al. 2013).
- $a = \begin{cases} 1, & k \geq \max(k_a, k_O) \\ \left(\frac{k}{\max(k_a, k_O)}\right)^{\frac{2}{3}}, & k < \max(k_a, k_O) \end{cases}.$

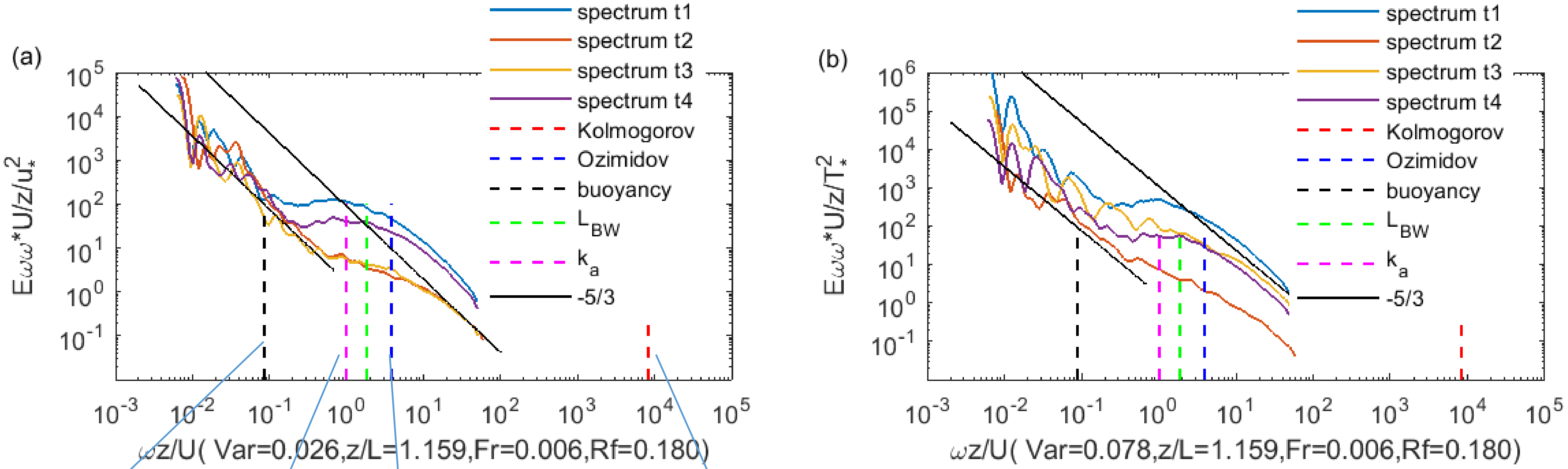
Proposed revision to Weinstock (1978)

- Horizontal wavenumber spectra for the universal equilibrium range (*Weinstock*, 1978)

$$E(k) = \alpha\epsilon_0^{\frac{2}{3}}\left[1 - \frac{5\alpha^{\frac{2}{3}}a}{12}\frac{v_m}{v_0}\left(\frac{1-R_f}{R_f}\right)\, C\left(\frac{k}{k_{BW}}\right)\right]^2 k^{-\frac{5}{3}}.$$

- $k > \max(k_a, k_O)$, isotropic inertial subrange exhibits a -5/3 slope.
- $k < k_b$, buoyancy subrange also has a -5/3 slope.
- $k_b <$ k $< \max(k_a, k_O)$, transition region will be referred to observation.

TKE and temperature spectra of Antarctica EC data



Experiment setup in Vignon et al. (2017).

buoyancy scale

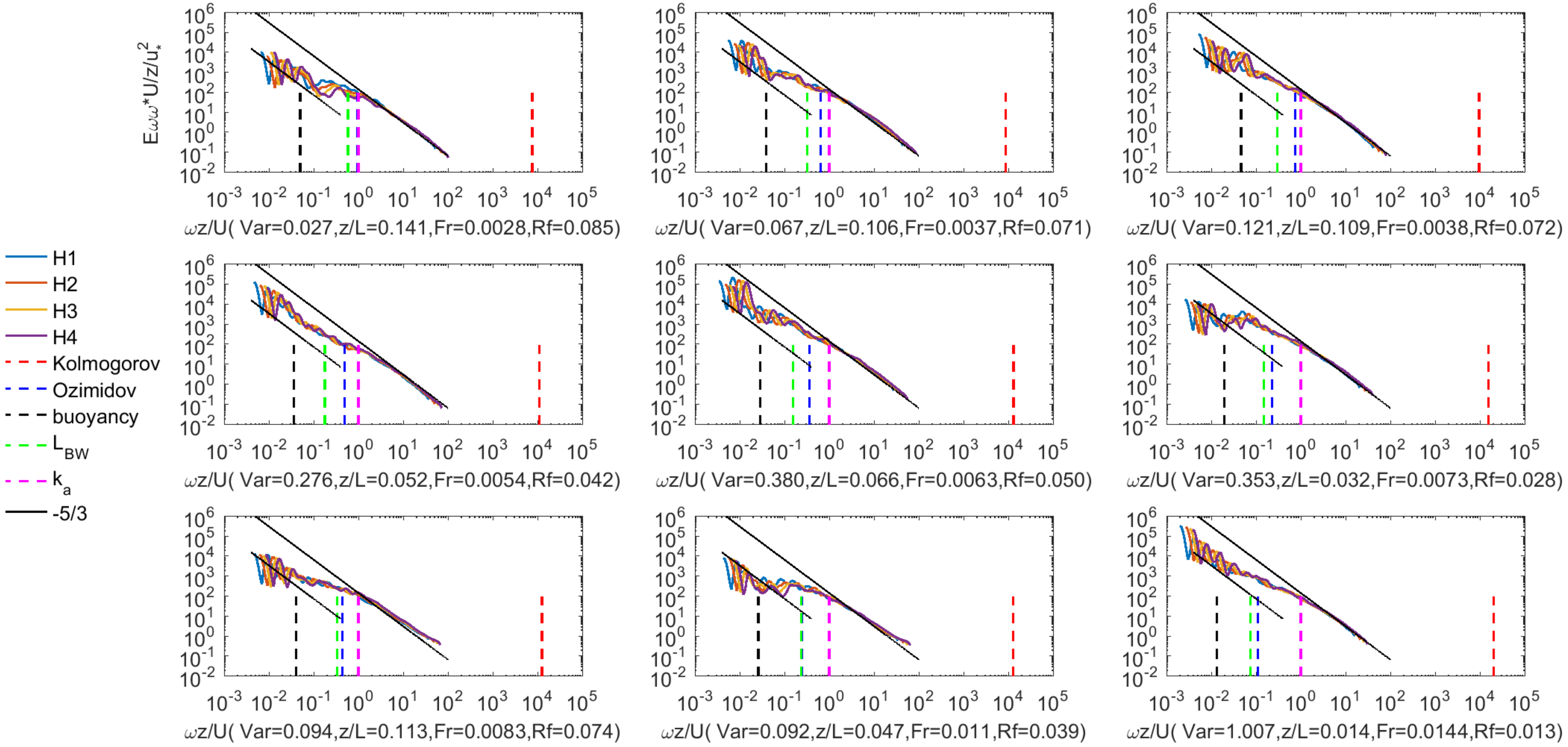
k_a

Dougherty-Ozmidov scale

Kolmogorov microscale

- “spectrum t1”, “spectrum t2”, “spectrum t3” and “spectrum t4” : 4 different stable periods.
- 3 regimes exist in turbulence TKE and temperature spectra.

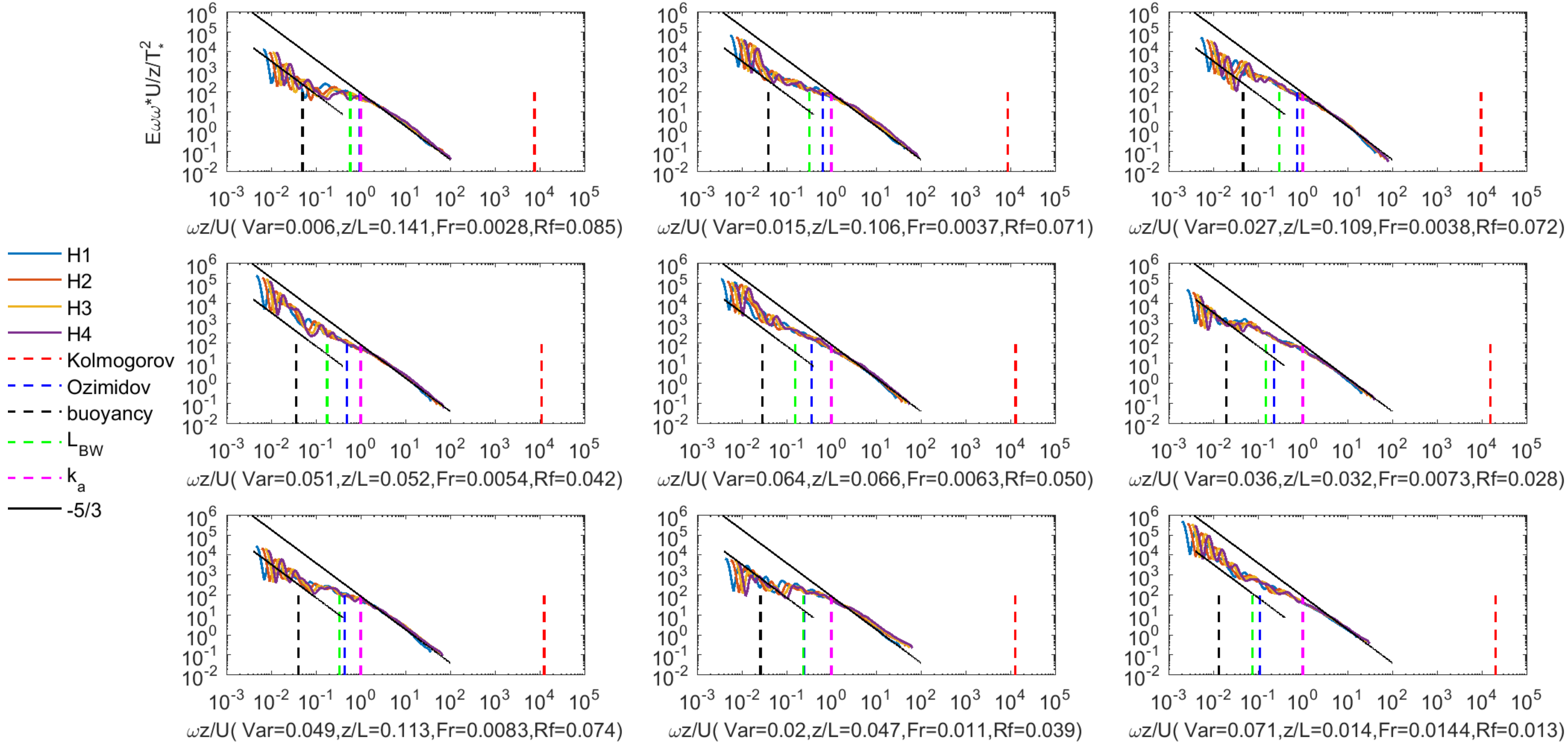
TKE spectra of Lake Geneva EC data



Experiment setup Bou-Zeid et al. (2008), Li & Bou-Zeid (2011) and Vercauteren et al. (2008).

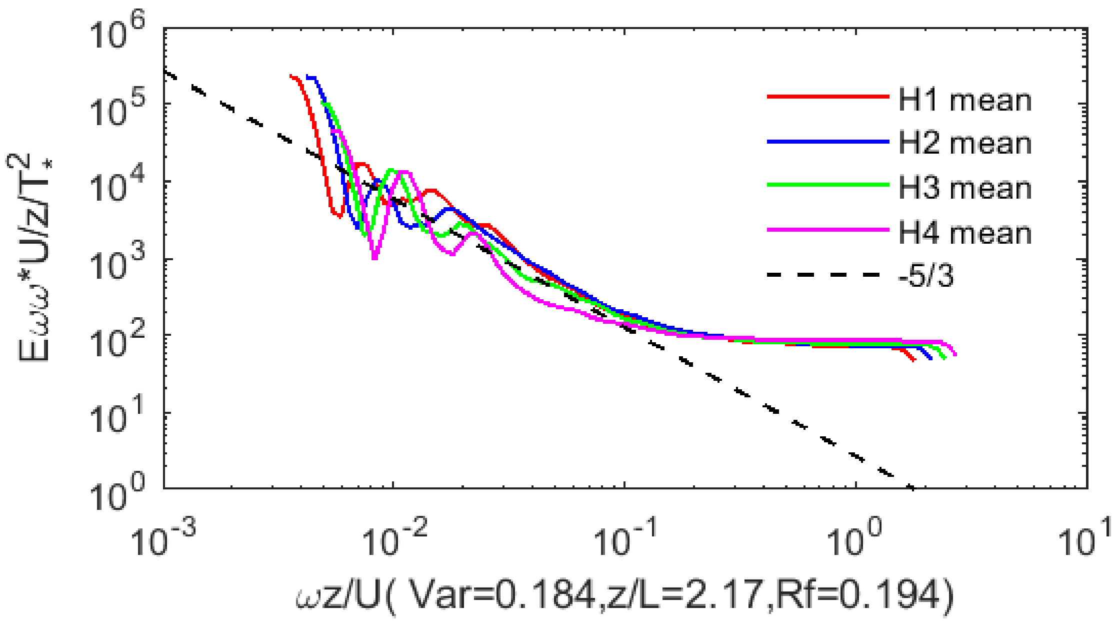
- 4 heights are 1.66m, 2.31m, 2.96m and 3.61m above the lake.
- 3 regimes are in the TKE spectra.

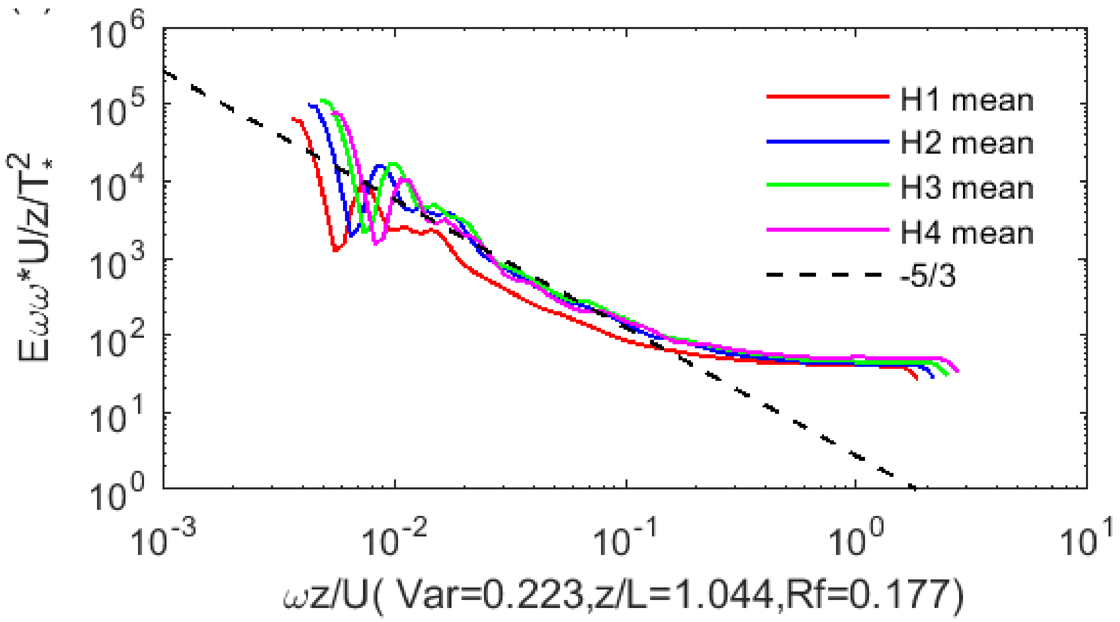
Temperature spectra of Lake Geneva EC data



3 regimes are in temperature spectra.

Temperature spectra of DTS data at Oklahoma

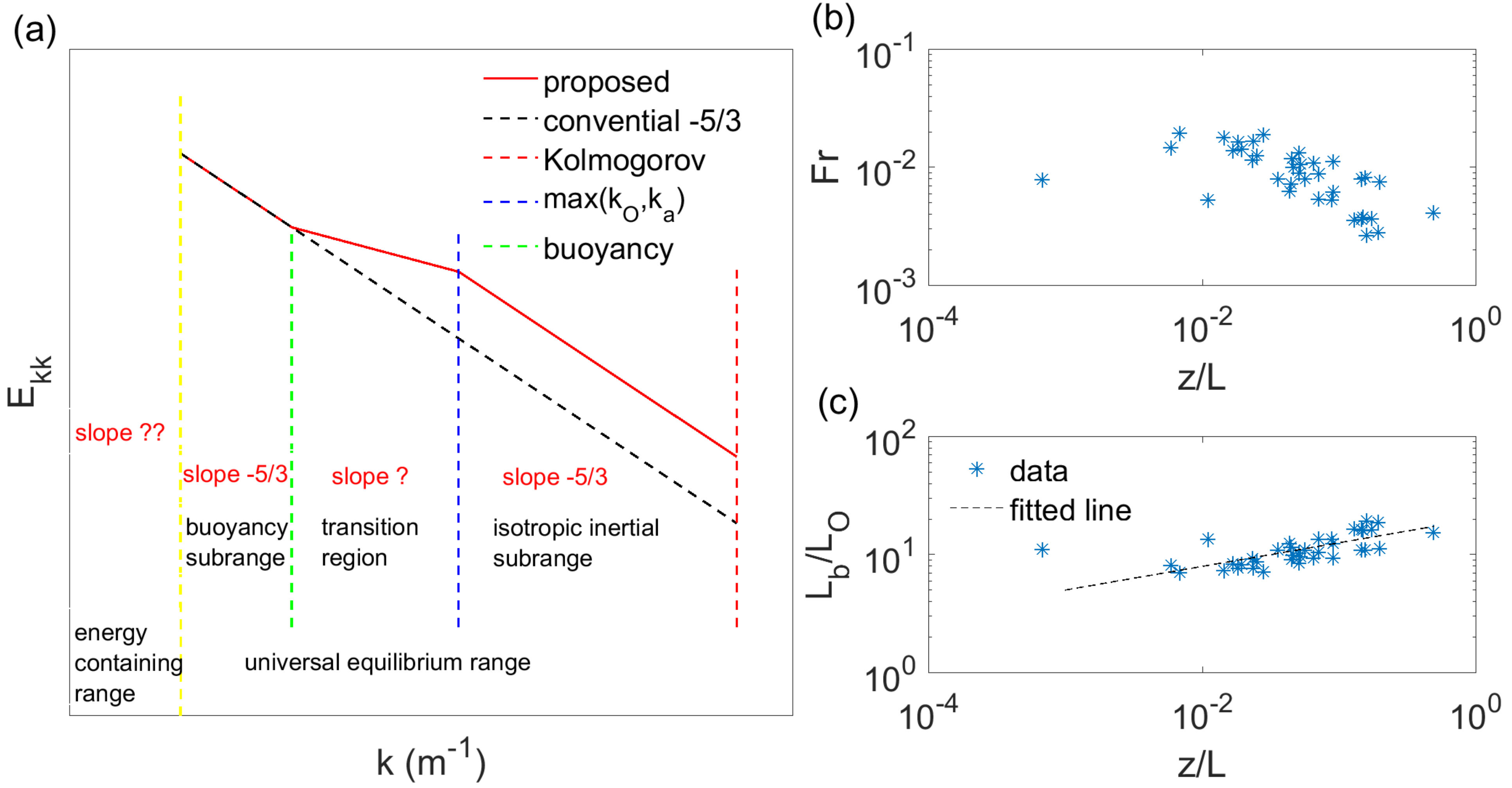




Experiment setup in Cheng et al. (2017).

- 4 heights are 1.00 m, 1.25 m, 1.50 m and 1.75 m above ground.
- **Only the buoyancy subrange and transition region are resolved**.

Proposed spectra

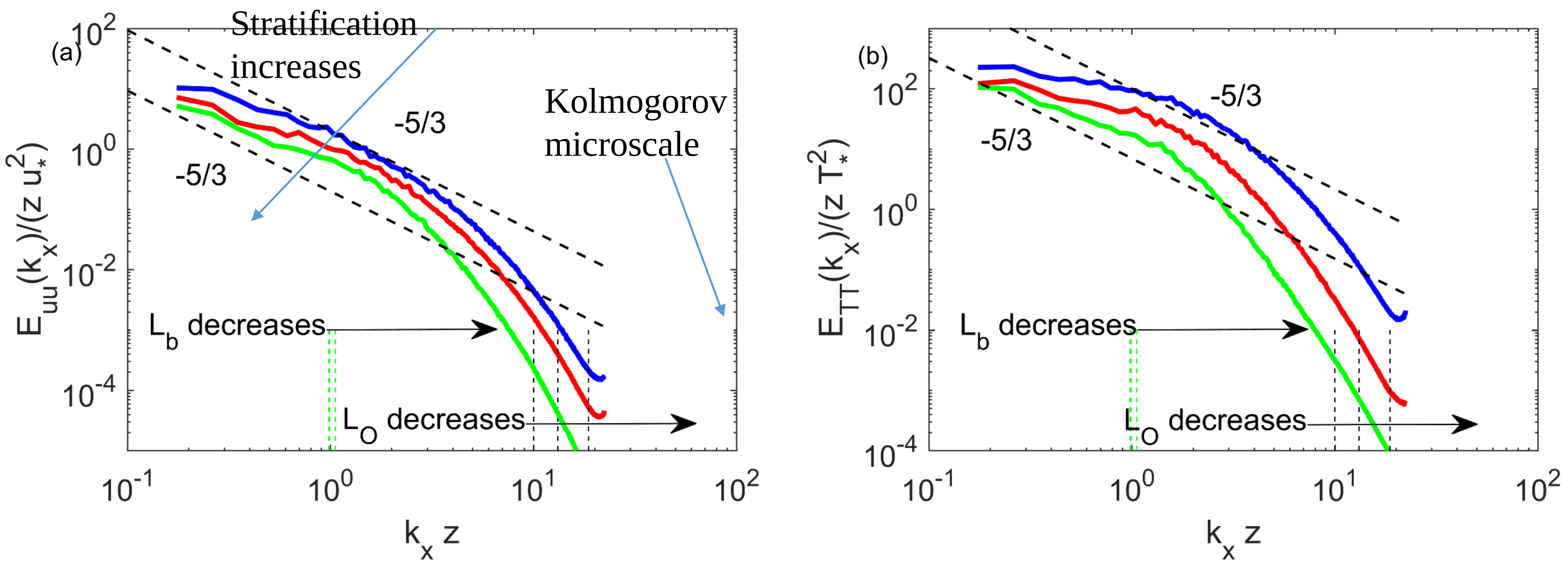


(a) Proposed TKE spectra.

(b) *Fr* plotted against *z/L* in Lake data.

(c) Relative length of transition region plotted against *z/L* in Lake data.

Limits of simulations: example of DNS of stably stratified Ekman layer



- Computational limitation: not enough scale separation.
- Same for LES: in this case too coarse for Dougherty-Ozmidov scale.

Conclusion

- 3 regions in spectra of stable atmospheric boundary layer.
- Spectra between buoyancy and Dougheryt-Ozmidov scale have implications for the limits of simulations.
- Transition region spectra generates departure from MOST.

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